The Financial Instability Hypothesis: a Stochastic Microfoundation Framework

C. Chiarella and C. Di Guilmi*

School of Finance and Economics - University of Technology, Sydney
PO Box 123, Broadway, NSW 2007, Australia.

Preliminary draft
June 29, 2009

Abstract

This paper examines the dynamics of financial distress and in particular the mechanism of transmission of shocks from the financial sector to the real economy. The analysis is performed by modelling the linkages between microeconomic financial variables and the aggregate performance of the economy by means of a new stochastic aggregation framework. This methodology overcomes some of the restrictions of the representative agent hypothesis which seems to be unsuitable for a context where different financial conditions of firms, and consequently different reactions to external shocks, impact on the macroeconomic dynamics. The model is solved both numerically and analytically, by means of a stochastic approximation that is able to replicate the numerical solution.

1 Introduction

Minsky (1977) defines financial fragility as "...an attribute of the financial system. In a fragile financial system continued normal functioning can be disrupted by some not unusual event". The two key points highlighted by this definition are the "not unusual event" that may stop the normal functioning of a financial system and that the system in question must display a certain degree of fragility. As regards the former point, there is no shortage of interpretations in this sense of the crises that, at progressively shorter intervals, have hit the capitalist economies in the last quarter of century (Kindleberger, 2005). The idea of an intrinsic instability of the capitalist financial system dates back to Minsky (1963) and gained increasing attention. As regards the second point,

*Corresponding author: Corrado Di Guilmi. School of Finance and Economics - University of Technology, Sydney - PO Box 123, Broadway, NSW 2007, Australia. Ph.: +61295147743, Fax: +61295147711. E-mail: corrado.diguilmi@uts.edu.au.

the identification of the degree of systemic fragility, according to Minsky, involves a micro-level analysis, being dependent on the share of financially sound and distressed firms in the economy. More precisely, in his famous 1963 essay, Minsky classifies firms into hedge, speculative or Ponzi type. The first are the sound firms that can repay their debt and the interest on it. The second type are the ones able to meet only the interest due on outstanding debt while, for the Ponzi firms, their cash flow is insufficient to fulfil either the repayment of capital or the interest due on outstanding debts.

As Taylor and O'Connell (1985) point out "Shifts of firms among classes as the economy evolves in historical time underlie much of its cyclical behavior. This detail is rich and illuminating but beyond the reach of mere algebra". According to them, this is the main reason for which Minksy's work has been so far either neglected or formulated in aggregate terms rather than being microfounded.

Such is no longer the case. In recent years a consistent stream of research has started to deal with the microfoundation of macroeconomics with heterogeneous and evolving agents. Significant results in terms of replication of empirical stylized facts has been reached through the numerical solution of agent based models[1]. From an analytical perspective, the most relevant contribution has been provided by Aoki[2]. His framework seems to allow a comprehensive analytical development of Minsky's theory that satisfactorily encompasses its essential microeconomic foundation. Aoki adopts analytical tools originally developed in statistical mechanics. In his view, as the economy is populated by a very large number of dissimilar agents, we cannot know which agent is in which condition at a given time and whether an agent will change its condition, but we can know the present probability of a given state of the world. This approach hence focuses in particular of the evolution of agents' characteristics through time. The basic idea consists in introducing a meso-level of aggregation, obtained by grouping the agents in clusters according to a measurable variable. The dynamics of the number of firms in each cluster defines as well the evolution of the whole economy, which is identifiable by specifying some general assumptions on the stochastic evolution of these quantities. For example, assuming their dynamics to be a Markov process, it is possible to describe the stochastic evolution of these occupation numbers using the master equation which is a standard tool in statistical mechanics. Interaction among agent is modelled by means of the mean-field approximation (Aoki, 1996) that, basically, consists in reducing the vector of observations of a variable over a population to a single value. The usefulness and the potential of this approach for Minsky's theoretical structure appears to be promising.

The aim of this paper is to propose a financial fragility model, along the lines of Minsky (1975) and Taylor and O'Connell (1985), with heterogeneous and interacting firms, using first a numerical solution for the agent based model and then comparing this solution with the one obtained by means of the stochastic

[1]See by way of example Axelrod (2003); Axtell et al. (1996); Delli Gatti et al. (2005).

[2]Namely, Aoki (1996, 2002); Aoki and Yoshikawa (2006), with a further development provided by Di Guilmi (2008)

dynamic aggregation technique above mentioned. Besides the technical contribution, such an improvement should allow a deeper insight into the mechanism by which shock are transmitted from the financial sector to the real economy. This latter aspect, which is central in Minsky's approach, is not the main focus of Taylor and O'Connell (1985). In their view the market valuation of shares may differ from the present value of capital, with the difference being absorbed by net worth. Given the substitutability of assets, a shift of investor preferences impacts on firms' net worth via a different evaluation of capital assets. Therefore, investor expectations of future profits influence, on the one hand, the prices of firms' equities on the stock market and, on the other hand, the current value of firms' assets. For example, if the market forecasts a rise in the demand for a certain product, there will be an increase in the evaluation of the machines that produce that good and a contemporaneous rise in the price of shares for the firms that sell them (Wray and Tymoigne, 2008). These two effects shape firms' decisions on investment and, as a consequence, output and employment levels. At the aggregate level then the economy may experience periods of growth, depression or fluctuations due solely to changes in the market mood and not to its actual productivity. This mechanism, first studied by Keynes (1936), has been subsequently formalized by Kalecki (1971) and Minsky (1975). Taylor and O'Connell (1985) introduce into the original analysis of Minsky an exogenous variable that expresses the level of confidence of the market, isolating the effect of investors' expectations on the value of a firm's assets.

We bring two main modifications to this original framework: the first is methodological, as we relate the equations of the model to effects at the microeconomic level. We then study two different macro-dynamics: the first being an agent based one, with the highest degree of heterogeneity, and a stochastic approximation, obtained by means of Aoki's aggregation tools. The second derives from the observation that, from our perspective, the evaluation of capital assets comes from the stock market, in which investors display heterogeneous expectations about firms' future profits. In particular we consider as endogenous the new variable introduced by Taylor and O'Connell (1985), linking it to the predominant strategy in the stock market.

The outline of the paper is as follows. Section 2 describes the general features of the model and outlines the basic structure of firms in the agent based framework. The behavioural hypotheses and equations are the same for the dynamics and for the stochastic approximation. In this section they are referred to each single firms, without limitation in the endogenous heterogeneity, while the stochastic dynamics consider a representative firm for each cluster. Section 3 defines the hypotheses for investors and capital market. Section 4 discusses the stochastic approximation to a high degree heterogeneous model. Section 5 uses simulation to contest the outcomes of the two dynamics, the agent based model and the stochastic approximation. Section 6 concludes with some discussion of the questions that can be addressed using the framework developed here.

2 Firms

This section presents the structure of the agent based model's . Variables are written with the superscript j when they refer to a generic firm, with the subscript $z = 1, 2$ when referring to a microstate, and without any sub- or superscript when indicating aggregate values. The model is set up in continuous time. The hypotheses of the model are:

- Due to informational imperfections in capital markets (Myers and Majluf, 1984; Greenwald and Stiglitz, 1990), firms prefer to finance their investments I^j with retained earnings F^j and, only if they are not sufficient, by the emission of new equities E^j or with new debt D^j.

- Firms are classified into two groups, clustering together the speculative and Ponzi firms of the Minsky (1963) taxonomy. Analogously to Lima and de Freitas (2005), and in order to ease the calculations, the threshold level of debt is set to 0. Therefore, the classification defines as *speculative* (type 1) the firms that have to finance their investment with debt or new equity and as *hedge* (type 2) the firms that can finance their investments with retained profits and do not need external sources. Thus firms can be classified into two states, depending on whether or not they display a positive debt in their balance sheet:

  - state 1: $D^j(t) > 0$,
  - state 2: $D^j(t) = 0$.

  A generic firm is indicated with the superscript j; variables referring to one of the two states are identified by the subscript $z = 1, 2$. Within the two clusters firms are identical.

- A firm decides the level of investment $I^j(t)$ based on the shadow-price (Minsky, 1975; Kalecki, 1971) of its capital $P_k^j(t)$, so that:

  $$I^j(t) = aP_k^j(t), \tag{1}$$

  where a is a parameter measuring the sensitivity of firms to the current value of capital assets and the shadow price P_k is specified below. This formulation recalls the one adopted by Delli Gatti et al. (1999), while the model of Taylor and O'Connell (1985), very much in line with Minsky (1975), takes into account the price differential between the shadow price and the price of furnishing new investment goods. Our choice in 1 is motivated by the opportunity of keeping the computational mechanism as simple as possible. Moreover, the solution adopted by Taylor and O'Connell (1985) would add a factor that might result noisy for the identification of the effects of financial markets fluctuations on investment.

- The selling price of the final good is obtained by applying a mark-up τ on the direct production costs according to

  $$P = (1 + \tau)wb, \tag{2}$$

where w is the nominal wage and b is the labour-output ratio, assumed equal for all firms.

- All wages are consumed and the market for the final good is assumed to be in equilibrium.
- Assuming that the firms adopt a technology with constant coefficients, the amount of labour requested is residually determined once the optimal level of investment, and hence of capital, is quantified. In particular, the production function for all firms is

$$X^j(t) = G(K^j(t), L^j(t)) \tag{3}$$

with K and L representing, respectively, physical capital and labour. Assuming G to be a homogeneous function of L we have

$$1/b = G(K^j/L^j, 1) \equiv g(K^j) \tag{4}$$

For the sake of simplicity L is normalised to 1.

- The rate of profit r^j is given by

$$r = r^j(t) = \frac{\tau}{1+\tau}\frac{X^j(t)}{K^j(t)} , \tag{5}$$

which is set equal across firms since they are assumed to apply the same mark-up and use the same technology. Final production and physical assets are priced at the level P, as in Taylor and O'Connell (1985), and all profits are retained.

- P_k^j is determined according to

$$P_k^j(t) = \frac{(r(t) + \rho^j(t))P}{i(t)} , \tag{6}$$

where i is the interest rate and ρ^j is the expected difference of return to capital for the firm j with respect to the average level r. The variable ρ is introduced by Taylor and O'Connell (1985) in their analysis of the original Minsky model in order to link investors' expectations to the investment decision; it plays a decisive role in their treatment as well as in the present one. Here we make it endogenous, considering it as a function of the prevailing strategy on financial market. This quantity is therefore the key variable in the mechanism of transmission of shocks from the financial markets to the real economy. This mechanism by which the process occurs is fully detailed in section 3.

- Firms finance the part of investment that cannot be covered with internal funds by a fraction $\phi\, i(t)$ with equities, where $\phi > 0$ is a parameter[3], and

[3]In the simulations a control is introduced in order to ensure that $\phi i \leq 1$.

then the rest with debt, the dependence on the interest rate reflecting the fact that in periods with high interest rate equities would be preferred. The price of the new capital goods is assumed to be equal to the final goods price P. The sum of retained profits is indicated by F^j. To simplify the notation and without loss of generality P is normalized to 1. Thus, the variations of E^j and D^j at an instant of time are given by

$$dE^j(t) = \phi\, i(t) \left[\frac{I^j(t) - F^j(t)}{P_{e1}}\right] dt \tag{7}$$

$$dD^j(t) = \big[1 - \phi\, i(t)\big]\big[I^j(t) - F^j(t)\big] dt \tag{8}$$

where P_{e1} is the price of equities for speculative firms to be defined in the following section.

- The timeline of the whole process over successive time intervals is shown in figure 2.

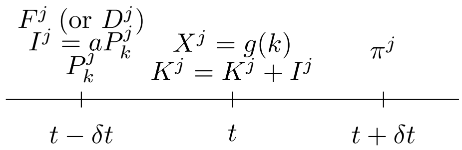

Figure 1: Timeline of the investment process.

- Combining (1) with (6) we obtain:

$$I^j(t) = a\left[\frac{(r + \rho^j(t))P}{i(t)}\right] \tag{9}$$

- The balance sheet of a typical firm has the structure shown in table 1. We use A to indicate the difference in the market evaluations of as-

Assets	Liabilities
	$P_e E^j$
$\frac{r+\rho}{i} K^j$	A
	D^j (or F^j)

Table 1: Structure of a generic firm's balance sheet

sets and shares, less the eventual debt. Adopting the terminology of

Taylor and O'Connell (1985) we term it as net worth. Actually, in terms of accountability, retained profits are a component of firms net worth and therefore they should be summed up to the latter. We indicate them separately in order to quantify the cash flow that can be used to finance future investment.

- Capital depreciates each period at a constant rate.
- Profits are given by:

$$\pi^j(t) = \tau w b X^j(t) - i(t) D^j(t) \tag{10}$$

- Accordingly, the variation in retained profits, or cash flow, for a hedge firm is:

$$\frac{F^j(t)}{dt} = \pi^j(t) - P_k^j I^j(t) \tag{11}$$

If, at time t, $F^j(t) < I^j(t)$, the firm becomes speculative and $D^j(t) = F^j(t) - I^j(t)$ will be financed with new equities and debt according to equations (7) and (8).

- The debt of a speculative firm evolves according to:

$$\frac{D^j(t)}{dt} = [1 - \phi\, i(t)][I^j(t) - F^j(t)] \tag{12}$$

If $D^j(t)(1+i(t)) - \pi^j(t) \leq 0$, then the firm becomes hedge with $F^j(t+\delta t) = -D^j(t)$.

- A firm fails if

$$D^j(t) > c\, K^j(t) \tag{13}$$

with $c > 1$. The probability for a new firm of entering is directly proportional to the variation in the aggregate production with respect to the previous period.

- Finally, asset prices are subject to changes due to variation in investors' expectations and strategies. Using P_e to denote the price of assets, at the aggregate level we can express the fundamental equation of capital accumulation in the economic system as:

$$\begin{aligned} \tfrac{d(P_k(t)K(t))}{dt} &= P_k(t)I(t) + \tfrac{dP_k(t)}{dt}K(t) \\ &= \tfrac{dP_e(t)}{dt}E(t) + P_e\tfrac{dE(t)}{dt} + \tfrac{dD(t)}{dt} + \tfrac{dF(t)}{dt} + \tfrac{dA(t)}{t} \end{aligned} \tag{14}$$

where E, D, F and A are respectively the value of the assets, the debts and of net worth of all firms. Equation (14) illustrates that the variation in the value of total physical capital of the economy also shows up as a variation in the amount or in the value of equities and/or a modification in the total net worth and debt of firms.

3 Investors

3.1 Behavioural hypothesis

Even though a complete modelling of stock markets goes beyond the aim of the present analysis, some behavioural assumptions on investors are needed for the internal consistency of the framework. The preferences of investors are modelled in a Keynesian fashion, considering a share of wealth kept liquid. Minsky (1975) gives to the usual Keynesian motives (transaction, precautionary, speculative) an analytical representation, modelling the demand of money as a function of income, interest rate, assets price, firms debt and near money supply. We model demand of money accordingly. Moreover, we consider the financial operators acting according to a bounded rationality paradigm. Consequently, we classify them in the two broad categories of chartists and fundamentalists, within an approach that has an established tradition in literature. It has been demonstrated (Aoki and Yoshikawa, 2006, ch. 9) that this classification gives account for almost the totality of different possible strategies. We adopt this assumption that comes out to be particularly suitable in this framework. Indeed we can reasonably assume that fundamentalists, focusing on the real value of firms, will favour investment on hedge firms, while chartists, based on extra-balance sheet information, may prefer riskier equities. We assume that all investors maximize a CARA utility function in order to avoid the distinction between chartists' and fundamentalists' wealth. Since our focus is on how changes in investors' expectations impact the real economy, we may assume that variations in the proportion of the types of operators are not dependent on firms' performance and are simply governed by a stochastic law. This also allows for a wider range of possible outcomes and behaviours as a result of the multiplicity of exogenous factors (not related to the economy) that influence the markets.

3.2 The determination of ρ

As anticipated, the variable ρ plays a key role in all the story, as it incorporates expectations that emerge in financial markets into the decision process of firms about investment. Taylor and O'Connell (1985) introduce it in order to better isolate the effect of the difference between the anticipated return and the current profit rate, an effect that in the original treatment of Minsky (1975) is directly incorporated in the shadow price P_k. They are not interested on the impact of financial markets and hence they do not explicitly calculate ρ, assuming independence between the behaviours of investors and firms. On the contrary in our perspective, mainly focused on the transmission of shocks from the financial sector, the role of ρ recalls the Tobin's q (Tobin, 1969), that is connected to equity values. In this sense our work constitutes a bridge between the two and an extension of them.

Two basic assumptions are at the root of the formulation of ρ: the first is its dependence on the relative proportion of chartists and fundamentalists in the market; the second concerns the formation of expectations. Since fundamentalists look at the balance sheet of firms while chartists focus only on the evolution of returns, we can assume that an increment in the proportion of chartists fuels the expectations about indebted firms that, on the contrary, are penalized when the share of fundamentalists is bigger. Accordingly, ρ^j is differently determined if a firm is in state 1 or in state 2, namely:

$$\begin{aligned} f_1(n^c) = \rho_1^j &= \frac{n^c}{\tilde{\varpi}^j} \\ f_2(n^c) = \rho_2^j &= \frac{1-n^c}{\tilde{\varpi}^j} \end{aligned} \tag{15}$$

where $\tilde{\varpi}^j$ is an idiosyncratic random variable. Since this random variable has the same support for each firm, on average a bigger fraction of trend chasers in the market leads firms in state 1 to increase their investments, their production and their debt. At the same time, the growing demand of credit puts pressure on interest rates. Therefore the system experiences a debt driven expansion that makes it vulnerable to sudden changes in investors expectations (a diminishing in the number of chartist and firms' bankruptcies, in the present treatment).

3.3 Equilibrium in the capital market

The two different types of firms emit equities which can be correspondingly sorted in two classes with different associated risk. In order to set up an allocation criteria for investors' wealth, we assume that they consider in their choice the mean-field values of the ρ_z^js, namely ρ_1 and ρ_2.

The wealth W of investors is the sum of owned shares, bonds and money:

$$W(t) = P_{e1}(t)E_1(t) + P_{e2}(t)E_2(t) + D(t) + M(t) \tag{16}$$

where $M(t)$ is the nominal demand of money. It evolves over time according to:

$$\frac{dW}{dt} = \frac{dP_{e1}}{dt}E_1(t) + \frac{dP_{e2}}{dt}E_2(t) + P_{e1}\frac{E_1}{dt} + P_{e2}\frac{E_2}{dt} + \frac{dD}{dt} + \frac{dM}{dt} \tag{17}$$

An initial endowment of money is assumed and variations in wealth are then given by capital gains.

Investors allocate their wealth among equities, firms' bonds and money according to the functions: $\epsilon_1(i, \rho_1, \rho_2, \psi), \epsilon_2(i, \rho_1, \rho_2, \psi)$, $\beta(i, \rho_1, \rho_2, \psi)$ and $\Psi(i, \rho_1, \rho_2, \psi)$, with the constraint $\epsilon_1 + \epsilon_2 + \beta + \Psi = 1$. The parameter ψ reflects the sensitivity of investors to the near money activities; it is constant over time[4]. The proportions of the two kinds of strategies influence ρ and through this the allocation

[4]Introducing ψ we can provide a functional form for the demand of money according to the formulation of (Minsky, 1975, chap. 4). Namely in his treatment it is given by the combined liquidity effects of the income Y, the interest rate r and the shadow price of capital P_k, the firms' debt F and the supply of near money activities NM:

$$M = L_1(Y) + L_2(r, P_k) + L_3(F) - L_4(NM).$$

of wealth between the two assets. The equilibrium conditions on equities and credit markets are (time indexes are omitted)[5]:

$$\begin{cases} \frac{\epsilon_1(i,r+\rho_1)}{P_{e,1}}W = E_1 \\ \frac{\epsilon_2(i,r+\rho_2)}{P_{e,2}}W = E_2 \\ \beta(i, r+\rho_1)W = D \\ \Psi(i, W, \psi)W = M \\ W = P_{e1}E_1 + P_{e2}E_2 + D + M \end{cases} \tag{18}$$

The system (18) returns the value of asset prices, interest rate, demand of money and aggregate rentiers' wealth.

4 Stochastic dynamics

We discussed so far in terms of single firms, referring all the variables to the agents level, and only in the last section we introduced the mean-field approximations ρ_z. These variables allow us to set up the tools for the analytical solution of the model. The equations (1) and (6) can be computed starting from the mean-field values ρ_z in order to calculated the variables I_z that refer to two representative firms, one for each state. With this approximation, using the techniques of Aoki (2002) and Di Guilmi (2008), it is possible to obtain an exhaustive analytical description of system's dynamics, starting from the micro level probabilities. Therefore, as explained in section 5, the model is able to generate dynamics in two different ways: an *agent based* approach with N different agents and a *stochastic approximation*, with two different firms: one "good" and one "stressed".

4.1 Transition probabilities

The probability for a firm of transitioning from state 2 to state 1 depends upon its level of investment and retained profits. A hedge firm becomes speculative if its level of net worth does not cover the desired investment. Therefore the probability ζ for a firm to move from state 2 to state 1 is equal to

$$\begin{aligned} \zeta(t) &= Pr\left[I_2(t) \geq F_2(t)\right] = \\ &= Pr\left[a\frac{[r+f_2(n^c)]P(t)}{i(t)} \geq F_2(t)\right] \end{aligned} \tag{19}$$

As regards speculative firms, they can move to state 2 if they are able to generate a level of profit sufficient to repay their debt; so that the relative probability of

[5] Actually, in each period, only speculative firms issue equities, given that hedge firms can finance all their investment with retained profits. Anyway in the market there are also the equities of firms that were speculative and became hedge, that are assessed differently by investors.

transition ν is given by:

$$\begin{aligned}\nu(t) \quad &= Pr\left[\tau wbX_1(t) \geq D_1(t)(1+i(t))\right] = \\ &= Pr\left[\tau wbg(K_1(t)) \geq D_1(t)(1+i(t))\right] = \\ &= Pr\left\{\tau wbg\left[K_1(t-\delta t) + a\tfrac{(r+f(n^c))P(t)}{i(t)}\right] \geq D_1(t)(1+i(t))\right\}\end{aligned} \tag{20}$$

Let us denote with η the *a-priori* probability for a firm to be in state 1, taking it as exogenous at this stage. The transition rates will be then given by:

$$\lambda(t) = (1-\eta)\zeta(t) \tag{21}$$

$$\mu(t) = \eta\nu(t) \tag{22}$$

4.2 System dynamics

We assume that firms switch from one state to another according to a Markov jump process. We already defined the micro-states of the process, that correspond to states 1 and 2 for the firms. In order to define the macro dynamics we are interested in the occupation numbers, i. e. in the number of firms that are in one of the states at a given time. These occupation numbers identify the macro-states of the process, that, accordingly, are given by all the possible combination of N_1 and N_2 with the constraint $N_1 + N_2 = N$. In this way, their stochastic dynamics can be conveniently described by a master equation. Using N_z to denote the occupation number for the state z, the master equation can be expressed as:

$$\frac{dPr(N_z,t)}{dt} = \lambda Pr(N_z-1)(t) + \mu Pr(N_z+1)(t) - \left[(\lambda+\mu)Pr(N_z)(t)\right] \tag{23}$$

where $Pr(N_z)(t)$ indicates the probability to observe an occupation number equal to N_z in state z at time t. This ordinary differential equation for $Pr(N_z)(t)$ allows us to describe the stochastic dynamics of the occupation numbers by identifying the component of the stochastic process that governs their evolution. To this end Aoki (2002) suggests to split the state variable N_z into drift (m) and diffusion (s) components, according to

$$N_z(t) = Nm + \sqrt{N}s \tag{24}$$

At this stage it is possible to apply the method detailed in Di Guilmi (2008) and Landini and Uberti (2008) to obtain the dynamics for m and s. First, by means of lead and lag operators, probability fluxes in and out the states can be treated as homogeneous. Then, the Taylor series expansion of the modified master equation identifies a Fokker-Planck equation for the transition density of the spread $Q(s,\tau)$ depending on the trend and the diffusion of the process according to:

$$\frac{\partial Q}{\partial \tau} - N^{1/2}\frac{dm}{d\tau}\frac{\partial Q}{\partial s} \approx \left[-N^{1/2}\frac{\partial}{\partial s}\alpha_1(m) + \frac{1}{2}\left(\frac{\partial}{\partial s}\right)^2 \alpha_2(m)\right] Q(s,\tau) \tag{25}$$

where:

- $Q(s, \tau)$ is the transition density function[6] of the spread s denoted with respect to τ, which denotes the time rescaled by the factor N, so that $\tau = tN$;
- α_n is the n^{th}-moment of the stochastic process for s;
- $m = \frac{N_z}{N}$ is the state variable, indicating the proportion of firms of type z in the total population of firms.

The asymptotic solution of the (25) leads to the following system of coupled equation:

$$\frac{dm}{d\tau} = \lambda m - (\lambda + \mu)m^2 \tag{26}$$

$$\frac{\partial Q}{\partial \tau} = [2(\lambda + \gamma)m - \lambda] \frac{\partial}{\partial s}(sQ(s)) + \frac{\left[\lambda m(1 - m) + \gamma m^2\right]}{2} \left(\frac{\partial}{\partial s}\right)^2 Q(s) \tag{27}$$

where the first is an ordinary differential equation the solution of which is the drift of the process N_z, while the partial differential equation (27) describes the evolution of density of the random spread s around the drift. As one can see, dynamics (26) is convergent to the steady state value m^* given by

$$m^* = \frac{\lambda}{\lambda + \mu} \tag{28}$$

Then, directly integrating equation (26) we find that

$$m(\tau) = \frac{\lambda}{(\lambda + \mu) - \omega e^{-\vartheta \tau}} \quad : \quad \begin{cases} \omega = 1 - \frac{m^*}{m(0)} \\ \vartheta = \frac{(\lambda+\mu)^2}{\lambda} \end{cases} \tag{29}$$

Equation (29) describes the evolution of fraction m of firms and we see that it is fully dependent on transition rates. Solution of the equation for the density of the spread component yields the limit distribution function $\bar{Q}(s) = \lim_{\tau \to \infty} Q(s, \tau)$ for the spread s, determining, in this way, the long run probability distribution of fluctuations, namely:

$$\bar{Q}(s) = C \exp\left(-\frac{s^2}{2\sigma^2}\right) \quad : \quad \sigma^2 = \frac{\lambda \mu}{(\lambda + \mu)^2} \tag{30}$$

Equation (30) is a Gaussian density whose parameters are dependent on the transition rates.

4.3 Solution

We are able at this point to identify the two dynamical variables that drive the dynamics of the economy: the first is capital accumulation that reflects investors' expectations and animal spirits, and the second is the underlying stochastic dynamics of the proportion of speculative firms. These two dynamical

[6] See equation (6.43) in Di Guilmi (2008, 73).

variables are connected since the transition rate λ is a function of the level of investment i_2 and the aggregate investment depends on the shares of the two types of firms. Taking as state variable the share of speculative firms $n_1 = \frac{N_1}{N}$:

$$\begin{cases} dn_1(t) = (\lambda n_1(t) - (\lambda + \mu)[n_1(t)]^2)dt + \sigma\, dW \\ dK(t) = I(t)dt = N\left\{[aP_{k1}(t)]n_1(t) + [aP_{k2}(t)][1 - n_1(t)]\right\}dt \end{cases} \tag{31}$$

where $\sigma\, dW$ is the stochastic fluctuation component in the proportion of speculative firms, coming from the distribution (30). These dynamics can then also identify the evolution of employment and aggregate output.

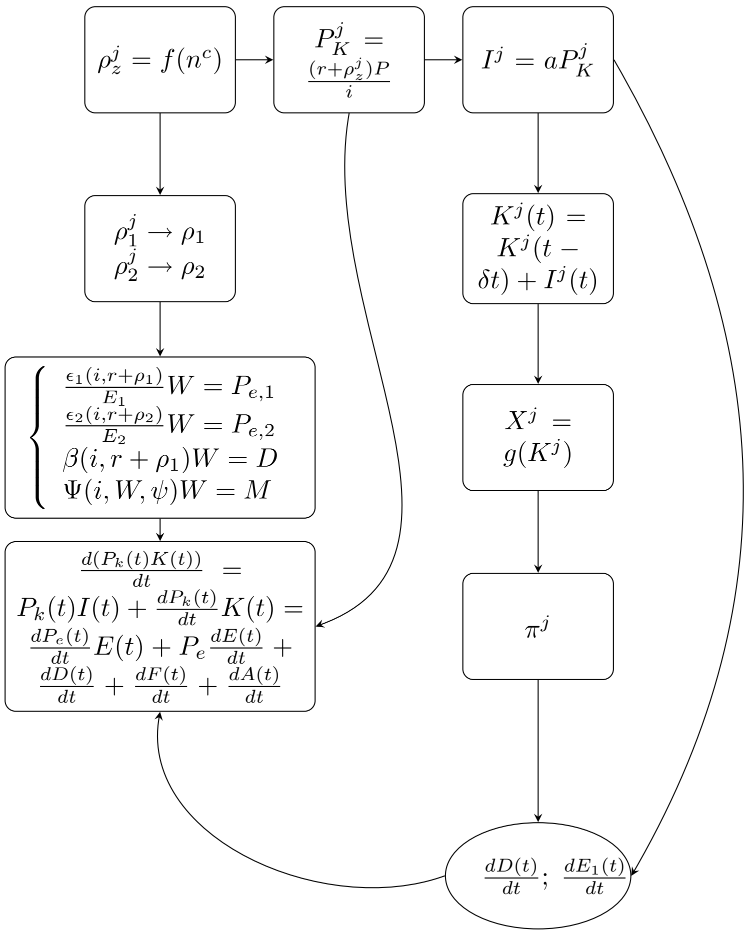

Figure 2: Flowchart of the model.

5 Simulations

5.1 Specification of functional forms

The intensive production function (4) is assumed to be of the form

$$X^i(t) = \varphi\, K^i(t) \tag{32}$$

with $\varphi > 0$ as a constant parameter.

The random variable n^c and $\tilde{\varpi}$ are assumed to have a uniform distribution. As a consequence of these hypotheses the transition probabilities can be specified in term of the known probability function of n^c as

$$\zeta(t) = F(n^c_\zeta) = Pr\left\{ n^c(t) \leq r\,\varpi - \frac{F\,i\,\varpi}{p\,a} + 1 \right\} \tag{33}$$

$$\nu(t) = 1 - F(n^c_\nu) = Pr\left\{ n^c(t) > \varpi \left[\frac{i}{P\,a} \left(\frac{D(t)(1+i)}{\tau w b \varphi} - K_1(t-1) \right) - r \right] \right\} \tag{34}$$

where $\varpi = \mathbb{E}[\tilde{\varpi}]$. The functions ϵ_z and β of system (18) are formulated in the following way:

$$\epsilon_1(t) = \frac{1}{1 + e^{i(t)+\rho_2(t)+\psi-\rho_1(t)}} \tag{35}$$

$$\epsilon_2(t) = \frac{1}{1 + e^{i(t)+\rho_1(t)+\psi-\rho_2(t)}} \tag{36}$$

$$\beta(t) = \frac{1}{1 + e^{\rho_1(t)+\rho_2(t)+\psi-i(t)}} \tag{37}$$

$$\Psi(t) = \frac{1}{1 + e^{i(t)+\rho_1(t)+\rho_2(t)-\psi}} \tag{38}$$

$$\tag{39}$$

The parameter ψ is kept fixed, representing an exogenous factor.

Equations (35) and ff. are substituted the system (18) that becomes:

$$\begin{cases} P_{e1}(t)E_1(t) = \frac{W(t)}{1+e^{i(t)+\rho_2(t)+\psi-\rho_1(t)}} \\ P_{e2}(t)E_2(t) = \frac{W(t)}{1+e^{i(t)+\rho_1(t)+\psi-\rho_2(t)}} \\ D(t) = \frac{W(t)}{1+e^{\rho_1(t)+\rho_2(t)+\psi-i(t)}} \\ M(t) = \frac{W(t)}{1+e^{i(t)+\rho_1(t)+\rho_2(t)-\psi}} \\ W(t) = P_{e1}(t)E_1(t) + P_{e2}(t)E_2(t) + D(t) + M(t) \end{cases} \tag{40}$$

All firms have an initial endowment of internal funds.

The mechanism of entry of new firms is stochastic. In every period a random number drawn from a uniform distribution with support $[0, 1]$ is assigned to each potential new firms; if this number is bigger to the normalized variation of aggregate output observed in the previous period the firm becomes active. The variation is normalized such as a variation of $+5\%$ is equal to 0 and a -5% is equal to 1. The configuration of parameters is (where non differently indicated):

- $n^c \in [0, 1]$;
- The values of ρ_z are the mean of the ρ^js included between the 40^{th} and the 90^{th} percentiles within each cluster of firms;
- $\tilde{\varpi} \in [0.01, 1.99]$;
- $a = 10$;
- $\phi = 2$;
- $i(0) = 0.12$;
- $c = 25$;
- $\psi = 0.2$.

5.2 Simulations results

Simulations are performed by implementing two separate procedures, one agent based and another for the stochastic dynamics, that produce their own dynamics of proportions of firms and capital accumulation. They are linked as the agent based procedure provides the mean-field variables ρ_z, with $z = 1, 2$, calculated as the mean of the ρ^j in each cluster of firms (excluding the first and the last decile). These values are the inputs for the stochastic approximation. Then the procedure is replicated with the two representative firms for each state, obtaining a dynamics driven by system (31). The transition probabilities are normalized taking the theoretical maximum and minimum values of the r.h.s. of the inequalities (33) and (34). The bankruptcy condition provides an upper limit for debt, while it is not possible to set a limit for k_z and F_z. We performed simulations for different numbers of periods and the results have been tested running 1000 Monte Carlo replications for each simulation. A period can be considered as a year. The average interest rate for each replication is about 10%.

The results are shown and compared in figures 3 and 5 that report the dynamics of capital and speculative firms, respectively, for a single replication and the Monte Carlo simulation. For both the variables in exam the stochastic approximation satisfactorily mimics the results produced by the agent based simulation. The dynamics of capital (and consequently of aggregate production) displays a long term upward trend. Within this trend, big cycles of a duration of 60/70 periods and smaller variations (from a period to another) are identifiable. The length and the amplitude of cycles are determined by the underlying debt cycle. During periods of growth, the proportion of speculative firms and aggregate debt rise. Consistently with Minsky's model, growth and the accumulation of debt increase until the most indebted speculative firms begin to fail, reducing the amount of capital and the aggregate wealth in the system. The amount of available credit reduces causing the demise of other speculative units. This downward spiral interrupts when all the firms with the relatively worst financial condition collapsed, letting the cycle start again. The cycles are reproduced in the financial market as shown in figure 4. It is noticeable that, during growth phases, the equity prices soar, raising the aggregate wealth of investors. With reference to the long run trend of the system, the length of the cycles is around 60 period from peak to peak, consisting of about 40 periods of growth and 20 of contraction. In the expansion period the dynamics of capital follows an upward trend with oscillations. Figure 7 highlights how the growth is led by positive expectations which impact the shadow price P_k. During this phase of *euphoria* the number of speculative firms and, more noticeably, their debt grow. When firms begin to fail, the trend reverts and the prey is the financial wealth and the predator is capital. Along this path economy shrinks, investors' wealth declines leading to a fall in aggregate capital. Around these trends the economy experiences fluctuations of smaller degree. Figure 4 reports the evolution of aggregate wealth and of its component. The dynamics of debt appear as relatively smoother if compared to the oscillations recorded for money and equities.

5.3 Some notes about economic policy

The model reveals that without other channels through which debt can be shifted, the economy may experience long period of steady declining activity. In the real world these channels are provided by governments, foreign capital markets and households[7]. This pattern is being very well exemplified in the current financial crisis. In recent decades and until the crisis, all developed economies recorded a remarkable growth of household debt. The present process of deleveraging forces state governments to directly take on part of the private sector's liabilities or to sustain demand, shifting the private debt to public sector.

The study of the impact of the institutional parameters provides some indications about possible alternative economic policies to shorten or mitigate the downward trends. As the interest rate is endogenously determined in the model on the credit market, the monetary policy may intervene by changing the parameter ψ that, according to Minsky's perspective, represents the availability of near money financial products. In particular a bigger availability of this kind of assets reduces the interest rate, impacting the length and the amplitude of the phases within the cycle. On the other direction, a positive variation of the interest rate reduces the weight of financial assets with respect to the real sector, favouring a smoothing of the cycle and a less severe contraction. The other parameters that impact the cycle are the bankruptcy parameter c and the sensitivity of firms to the shadow price a. The former quantifies the contribution of debt to growth (bigger is c, longer and wider are the phases). The latter has two opposite effects: it determines the velocity of accumulation of capital, and then the level of profits, and the amount of debt for firms unable to finance investments with internal funds. Simulations show that a greater a reduces the amplitude of fluctuations, indicating that the first effect overrules the second.

[7]Note also that in the present model the productivity is assumed to be constant.

As illustrated by figure 6 this parameter influences the distribution of firms: the median becomes larger and the distribution more dispersed as a grows, for the faster accumulation of capital.

6 Concluding remarks

In this work we study the transmission of shocks from the financial sector to the real economy, along the lines of Minsky and Taylor and O'Connell (1985). Using a recently developed aggregation method, suitable for heterogeneous agents frameworks, we further develop these original models, introducing an appropriate microfoundation. This solution is not possible within the traditional economic paradigm. Indeed, even though in principle allowing a bottom-up approach, it is by construction unsuitable to deal with this problem for two main reasons. The first is the representative agent, which is hardly compatible with Financial Instability Hypothesis (that is formulated considering different types of firms with respect to their financial structure) and cannot involve phenomena like insolvency and bankruptcy. The second reason is that, in the neoclassical view, financial markets are not even potentially a factor of instability as, on the contrary, they stabilize the economy, absorbing temporary disequilibria in real markets by means of derivatives and futures. In the present paper, using a stochastic aggregation framework, we provide a consistent microfoudation for Minsky's theory. The model can generate two types of dynamics. One is agent based and allows a numerical solution; the other is a stochastic approximation that can be solved analytically. This latter demonstrates capable to satisfactorily mimic the outcomes of an agent based model with a much higher degree of heterogeneity. The framework appears then as an efficient tool to analyse the effects of instability in financial markets on the real sector of the economy and, in particular, to identify the conditions under which the system generates speculative bubbles and how they burst. This basic framework is meant to be extended including a more refined modelling of financial markets, various forms of speculative behaviour and the banking sector in the intermediation of credit. Another further development may include institutional aspects such as government policies, fiscal and monetary, and the study of the possible effects of a regulatory framework.

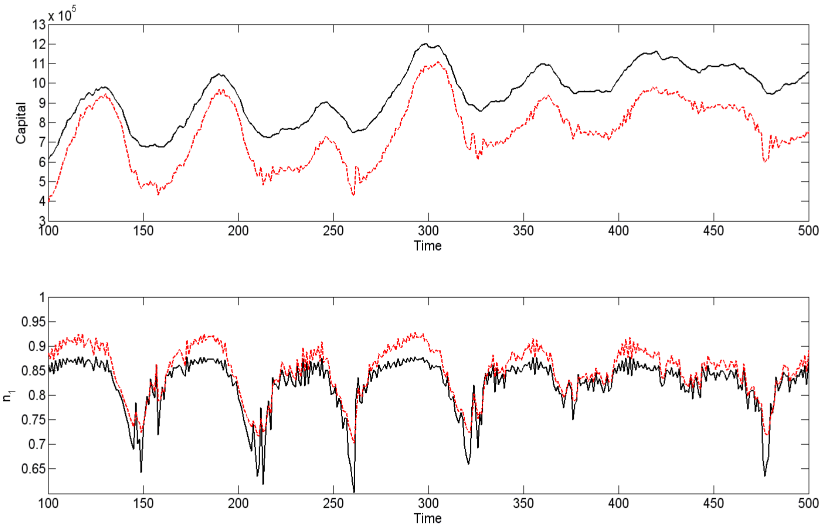

Figure 3: Different dynamics of capital (upper panel) and share of speculative firms (lower panel). Simulation of agent based model (black continuous line) and endogenous stochastic dynamics (red dashed line).

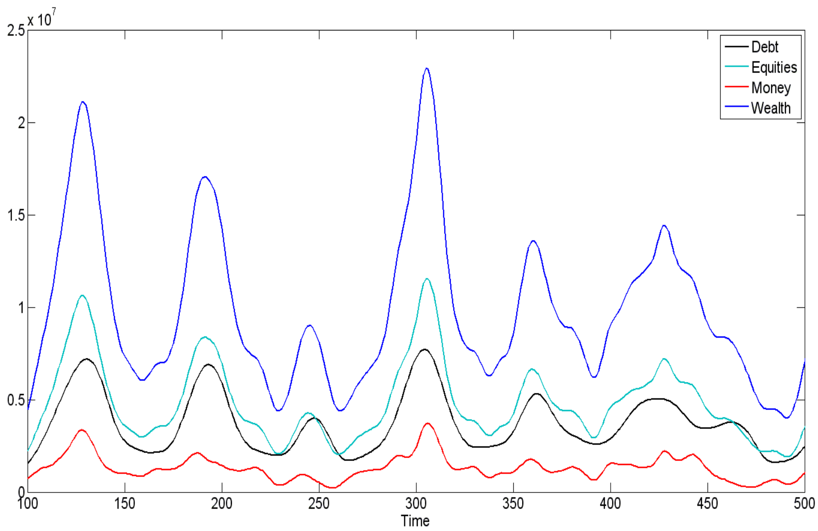

Figure 4: Aggregate wealth and its components. Simulation of agent based model and endogenous stochastic dynamics. Trend generated by Hodrick-Prescott filtering with $\lambda = 100$.

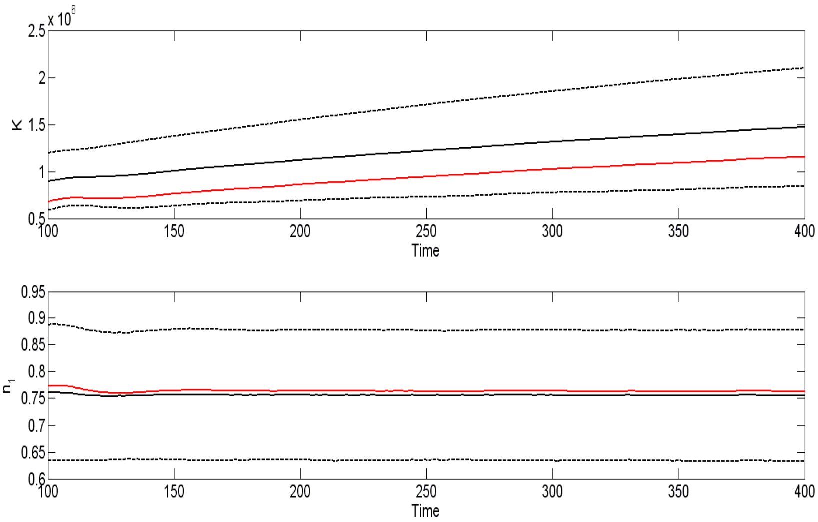

Figure 5: Different dynamics of capital (upper panel) and share of speculative firms (lower panel). Monte Carlo simulation of agent based model (black lines, mean) and endogenous stochastic dynamics (red line).

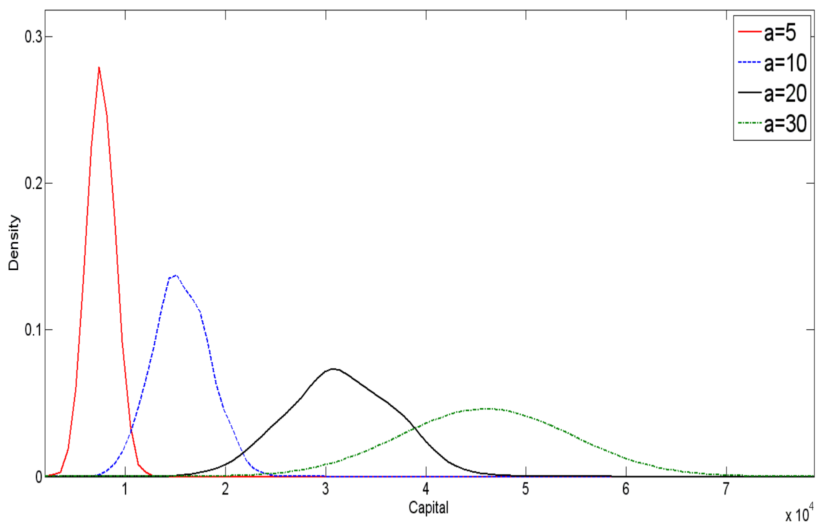

Figure 6: Distribution of capital across firms conditioned on a. Simulation of agent based model.

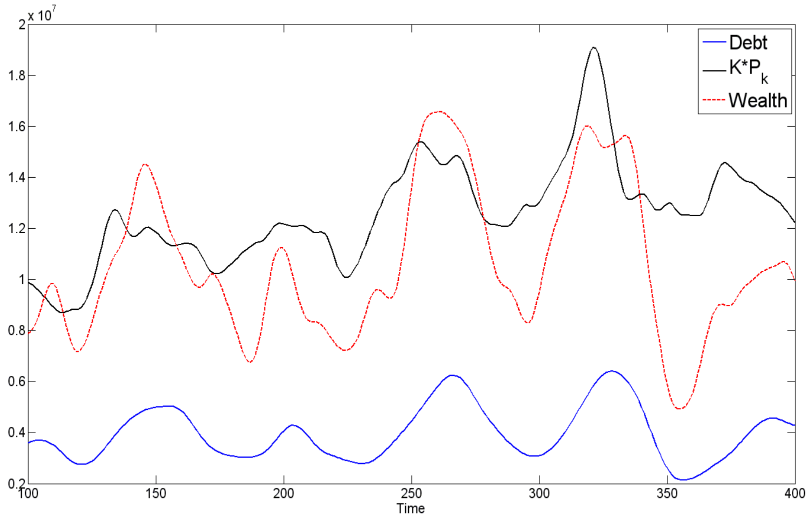

Figure 7: Dynamics of debt, capital valued at its shadow price and wealth. Simulation of agent based model. Trend generated by Hodrick-Prescott filtering with $\lambda = 100$.

References

AOKI, M. (1996): *New approaches to macroeconomic modeling.*, Cambridge University Press.

AOKI, M. (2002): *Modeling aggregate behaviour and fluctuations in economics.*, Cambridge University Press.

AOKI, M. and YOSHIKAWA, H. (2006): *Reconstructing Macroeconomics*, Cambridge University Press.

AXELROD, R. (2003): Advancing the Art of Simulation in the Social Sciences, *University of Michigan.*

AXTELL, R., AXELROD, R., EPSTEIN, J. M. and COHEN, M. D. (1996): Aligning simulation models: A case study and results, *Computational & Mathematical Organization Theory*, **1**(2): pp. 123–141.

DELLI GATTI, D., DI GUILMI, C., GAFFEO, E., GIULIONI, G., GALLEGATI, M. and PALESTRINI, A. (2005): A new approach to business fluctuations: heterogeneous interacting agents, scaling laws and financial fragility, *Journal of Economic Behavior and Organization*, **56**(4).

DELLI GATTI, D., GALLEGATI, M. and MINSKY, H. P. (1999): Financial Institutions, Economic Policy, and the Dynamic Behavior of the Economy, Macroeconomics 9903009, EconWPA, URL `http://ideas.repec.org/p/wpa/wuwpma/9903009.html`.

Di Guilmi, C. (2008): *The generation of business fluctuations: financial fragility and mean-field interaction*, Peter Lang Publishing Group: Frankfurt/M.

Greenwald, B. and Stiglitz, J. E. (1990): Macroeconomic models with equity and credit rationing., in: Hubbard, R. (ed.), *Information, Capital Markets and Investment.*, Chicago University Press, Chicago.

Kalecki, M. (1971): *Selected Essays on the Dynamics of a Capitalist Economy*, Cambridge University Press: Cambridge.

Keynes, J. M. (1936): *The General Theory of Employment, Interest and Money*, London: Macmillan.

Kindleberger, C. (2005): *Manias, Panics, and Crashes: A History of Financial Crises*, Wiley, fifth edn.

Landini, S. and Uberti, M. (2008): A Statistical Mechanic View of Macrodynamics in Economics, *Computational Economics*, **32**(1): pp. 121–146.

Lima, G. T. and de Freitas, G. G. (2005): Financial Fragility as Emergent Dynamics in a Complex System: An Agent-Based Computational Model, paper to be presented at the 10th Anniversary: WEHIA 2005 Workshop of Economic Heterogeneous Interacting Agents, University of Essex, United Kingdom, June 13-15, 2005.

Minsky, H. (1963): Can "It" Happen Again?, in: Carson, D. (ed.), *Banking and Monetary Studies*, Richard D Irwin, Homewood, reprinted in Minsky (1982).

Minsky, H. (1982): *Inflation, recession and economic policy*, New York: ME Sharpe.

Minsky, H. P. (1975): *John Maynard Keynes*, Columbia University Press: New York.

Minsky, H. P. (1977): A Theory of Systemic Fragility, in: Altman, E. I. and Sametz, A. W. (eds.), *Financial Crisis: Institutions and Markets in a Fragile Environment*, New York: John Wiley and Sons.

Myers, S. C. and Majluf, N. S. (1984): Corporate Financing and Investment Decisions when Firms Have Information that Investors do not have, *Journal of Financial Economics*, **13**: pp. 187–221.

Taylor, L. and O'Connell, S. A. (1985): A Minsky Crisis, *The Quarterly Journal of Economics*, **100**(5): pp. 871–85.

Tobin, J. (1969): A General Equilibrium Approach To Monetary Theory, *Journal of Money, Credit and Banking*, **1**(1): pp. 15–29.

Wray, L. R. and Tymoigne, E. (2008): Macroeconomics Meets Hyman P. Minsky: The Financial Theory of Investment, Economics Working Paper Archive 543, Levy Economics Institute, The.