Early Adoption in a Dynamic Social Network

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Abstract
An agent based model is developed in which a social hierarchy of leaders and followers emerges from a uniform or random social network. The formation of the social structure is driven by the desire to be an early adopter of a subsequently popular trend. The environment is related to a majority game, but introduces the importance of the timing of adoption. The proposed environment is relevant to a number of settings in which leadership and timing of decisions are important or being perceived as a trend setter is rewarded. The leadership position is attained from reinforcing behavior.

Keywords: Dynamic Network, Social Interaction, Consumer Choice

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1. Introduction

This exercise models decisions by a population for a product in which tastes are uncertain and possibly endogenous to the process. Opinion leaders (Gurus) arise naturally from the population and contribute to the agents' decisions, possibly by shaping tastes.

Preferences for certain products are subjective. For many such products, the consumer may be able to recognize difference, but may seek guidance in ranking these differentiated products.

This project focuses on issues concerning the social phenomenon by which an agent in the population becomes a leader influencing the choice of others in the population. Within this examination is the question of whether the leadership position and structure of the social network of the followers is stable. Subsequent examinations will explore feedback relationship between a leader and his or her followers and other social phenomenon related to this environment.

Social networks have recently received considerable attention in economics for explaining aggregate social behavior. In these models, individuals may be influenced globally by the aggregate behavior of the population or locally by their neighbors, a subset of the population who serve as a reference group. This project incorporates aspects of both. Economic incorporation of local interactions in decision making include Schelling (1971) and (1973) and Katz and Shapiro (1985). The network is a social structure by which discrete decisions are made or new products or technologies are adopted. The utility of a choice depends on the choices of those in an agent's peer group. Because of social interactions, products that garner no particular consumer preference, and may even be perceived to be inferior, can grow to dominate choice.

The reward structure for this project moves away from the simple majority or minority payoff to take timing into account. Agents are rewarded for being early adopters of a trend that subsequently becomes popular. Because the agents seek reward through early adoption, timing of an agent's decision must be modeled as an endogenous component of the social network, a clear deviation from the traditional mean field examinations of local interaction. A component of the reward also comes from conformity to the norm, as in Brock and Durlauf (2001)

Dynamic social network models such as Watts (2001), Bala and Goyal (2000), Jackson and Watts (2002), and Kirman et al (2007) are concerned with evolution and convergence. Bala and Goyal (2000) examine the evolution of a network in which agents create links with individuals with whom the benefit exceeds the cost of maintaining the link.

2. Model

The population consists of L agents. In each period, each agent $i \in \{1, 2, 3, ..., L\}$ chooses from among K available options, captured in notation by $k_{i,t}$. The choice represents the action of adopting a product, fad, or fashion statement for the given period. The process is repeated each period with a new set of K options.

Each period starts with each agent choosing whether to act independently or through imitation. With probability $\theta_{i,t}$, $0 \le \theta_{i,t} \le 1$, agent *i* chooses to act independently. Agent *i* chooses among the available options with equal probability,

$$\Pr(k_{i,t} = k) = p_{i,t}^k$$

with $\sum_{k=1}^{K} p_{i,t}^k = 1$.

With probability $1-\theta_{i,t}$, agent *i* choose to imitate one of the other agents in the population. Agent *i* maintains *d* non-redundant links to other agents of the population. If agent *i* maintains a link to agent *j*, then *j* is one of agent *i*'s "friends." The links are one-way in that agent *i* who maintains a link to agent *j* may choose to imitate agent *j*. Associated with each link is a weight, $w_{i,t}^{j}$. The weight $w_{i,t}^{j}$ is the probability that agent *i* imitates agent *j*, conditional on having chosen to imitate, so that

$$\Pr(\text{agent } i \text{ imitates agent } j) = (1-\theta_{i,t})w_{i,t}^{j} \tag{1}$$

with $\sum_{j} w_{i,t}^{j} = 1$. The *t* subscript on $\theta_{i,t}$ and the link weights reflect that the probabilities are adjusted over time by agent *i* employing a process to be developed below.

Each period is subdivided into rounds, indexed *r* = 1, …, *R*. Dissemination of particular choices through the population occur with the progression of rounds within a period. Prior to the first round, each agent chooses either to act independently or has chosen a friend to imitate for the period. In round 1 of period *t*, those who intend to act independently make their choice. In round two, those who have chosen to imitate those who acted in the previous round do so. If agent *i* has chosen to imitate agent *j* in period *t*, then agent *i* is a "fan" of agent *j*. The process continues for *R* rounds as trends disseminate through the population via the network of imitators. The rounds are indexed *r* = 1, …, *R*, In the final round, any agents who have not yet made a decision consider the population and choose option *k* with a probability $p_{k,t}^{R}$ that depends on the popularity of the option observed in the previous round. Let $N_{k,t}^{r}$ be the number of agents who have adopted choice *k* by round *r*. Then

$$\Pr(x_{i,t}^{R} = k \mid x_{i,t}^{R-1} \text{ unassinged}) = \frac{\exp(\rho N_{k,t}^{R-1})}{\sum_{K} \exp(\rho N_{K,t}^{R-1})}. \tag{2}$$

The parameter ρ is the *intensity of choice*, capturing how sensitive the residual population is to the relative popularity. Setting $\rho \to \infty$ produces the fully rational adoption of the most popular choice. Setting $\rho < \infty$ can be interpreted as bounded rationality or as the result of noise in the perception of relative popularity of the choice among the population. The agents choose without considering the population for ρ = 0.

The utility payoff to agent *i* derived from the adoption of *k* depends on three components,

$$\pi_{i,t} = J(N_{ki,t}^{J}) + T(N_{ki,t}^{T}). \tag{3}$$

The $J(N_{ki,t}^{J})$ component captures the social interaction term, rewarding the agent for the popularity of a choice. Here, $N_{ki,t}^{J}$, represents the total number of agents in time *t* to have chosen the particular option *k* the same as agent *i*, $N_{ki,t}^{J} = N_{k,t}^{R}$. The function *J* has $J(x) \geq 0$, $J(1) = 0$ and $J'(x) \geq 0$. Blume and Durlauf (2001) refers to this as the community effect. For the simulations, a simple linear function is employed with

$$J(N_{ki,t}^{J}) = a_J(N_{ki,t}^{J} - 1). \tag{4}$$

The $T(N_{ki,t}^{T})$ element of the payoff rewards the agent for being an early adoption. Here, $N_{ki,t}^{T}$ represents to total number of agents in time *t* who have adopted choice *k* that coincides with that of agent *i* in rounds subsequent to agent *i*'s adoption, $N_{ki,t}^{T} = N_{k,t}^{R} - N_{k,t}^{r}$. In the hierarchical structure, this would be members of the population occupying tiers below agent *i*, that is tiers $\delta_i + 1$ through Δ. Agent *j*'s inclusion in N_{ki}^{T} does not require that agent *j* be an imitator of *i*, either directly or indirectly through a chain of links, just that agent *j* makes the same decision as agent *i* in a subsequent round. The simulations are based on

$$T(N_{ki,t}^{T}) = a_T N_{ki,t}^{T} . \tag{5}$$

The coefficients a_T and a_J differentiate the payoff to agent *j* for being an early adopter from the payoff for making a popular choice. Meaningful incentives require $0 < a_J < a_T$. Normalizing $a_T = 1$ leaves a_J as the down-weighting applied to the social payoff relative to the early adoption payoff.

After the last round of the period, each agent adjusts $\theta_{i,t}$ and $w_{i,t}^{j}$ based on the perception of how successful each option would have been during the just completed period. For each of the links maintained, let $z_{i,t}^{j}$ represent the payoff earned by agent *i* by imitating agent *j* in period *t*. For those friends not selected for imitation in period *t*, the agent must form a hypothetical payoff. This is done by computing the payoff that would have been achieved had agent *i* made the same choice as agent *j* in the round following agent *j*'s selection. This is an imperfect measure since it ignores the endogenous component of agent *i*'s own action on his or her own payoff as a result of the direct and indirect fans of agent *i*. These agents too would have also switched choices if agent *i* had switched. In addition to the computational complexity involved for the agents, part of the reason for ignoring this component is that, agent *i* does not know who his or her fans are.

The weights are adjusted according to the algorithm derived from the *Experience-Weighted attraction* (EWA) learning rule suggested by Camerer and Ho (1999). Let $A_{i,t}^{j}$ be a cumulative performance measure associated with agent *i*'s imitation of agent *j*. A separate performance measure is maintained for each friend of agent *i*. The value of $A_{i,t}^{j}$ is updated according to

$$A_{i,t}^{j} = (\phi A_{i,t-1}^{j} N_{t-1} + (\delta + (1-\delta) I(s_{i,t}, j)) \pi(z_{i,t}^{j})) / N_t . \tag{6}$$

Briefly, $\pi(z_{i,t}^{j})$ is the period *t* realization of performance associated with agent *i* selecting to imitate agent *j*. The term $(\delta + (1-\delta) I(s_{i,t}, j))$ is a mechanism for down-weighting the unobserved speculative payoff associated with the friends who were not selected by *i* in the current period. Thus, the update of $A_{i,t}^{j}$ gets full weight if *j* was actually chosen by *i* in that period and has the diminished weight of δ, $0 \le \delta \le 1$, if not. Setting $\phi < 1$ decreases the weight of an observation with the passage of time. Finally, N_t follows a process

$$N_t = \rho N_{t-1} + 1 . \tag{7}$$

Various setting of N_0, ρ, and ϕ can be used to achieve a variety of weighted combinations of the previously observed $\pi(z_{i,t}^{j})$. See Camerer and Ho (1999) for more detail.

The weight that agent *i* assigns to his link to agent *j* is based on the relative performance measure. The power distribution is one option suggested by Camerer and Ho and is one of two primary mechanisms examined in this paper,[1]

$$w_{i,t}^{j} = \frac{{A_{i,t}^{j}}^{\lambda}}{\sum_{\delta=1}^{d} {A_{i,t}^{\delta}}^{\lambda}} . \tag{8}$$

[1] Other options explored were the exponential distribution

$$w_{i,t}^{j} = \exp(\lambda A_{i,t}^{j}) / \sum_k \exp(\lambda A k_{i,t}^{k}) .$$

Notice that the weights are based on (a non-linear function of) relative performance. Better performing friends have a higher weight, but all friends with a non-zero performance measure have a non-zero probability of being chosen (for finite λ). Simulations that set weights according to (8) will be referred to as EWA simulations

The alternative adjustment process employed is a Replicator Dynamic type process by which a higher performing friend attracts greater weight at the expense of poorly performing friends. The version of the Replicator Dynamics developed for this paper is modified from the *K*-choice Replicator Dynamics of Branch and McGough (2008). Let $\bar{A}_{i,t}^{W}$ be the average performance across agent *i*'s friends, weighted by current link weights, $\bar{A}_{i,t}^{W} = \sum_j w_{i,t}^j A_{i,t}^j$. Further, let a friend's performance fall into the sets "good" or "bad" according to

$$\hat{d} = \{1, \ldots, d\}, \qquad Gd = \{ j \in \hat{d} \mid A_{i,t}^j \geq \bar{A}_{i,t}^{W} \}, \qquad Bd = \{ j \in \hat{d} \mid A_{i,t}^j < \bar{A}_{i,t}^{W} \}$$

The weights evolved according to the process

$$w_{i,t}^j = w_{i,t}^j + \begin{cases} r(A_{i,t}^j - \bar{A}_{i,t}^{W}) w_{i,t}^j \text{ for } j \in Bd \\ x_i \sum_{j \in Bd} -r(A_{i,t}^j - \bar{A}_{i,t}^{W}) w_{i,t}^j \text{ for } j \in Gd \end{cases}. \tag{9}$$

with

$$x_i = \frac{\zeta / |Gd| + A_{i,t}^j - \bar{A}_{i,t}^{W}}{\zeta + \sum_{j \in Gd} (A_{i,t}^j - \bar{A}_{i,t}^{W})}$$

and $r(x) = \tanh(\lambda_{RD} x / 2)$.

Each period, those friends falling into the "bad" set are down weighted according to their performance relative to the weighted mean. The quantity of the down weighting is then distributed among those friends considered "good". The modification from Branch and McGough is to use the weighted average $\bar{A}_{i,t}^{W}$, rather than the simple un-weighted average performance. Without this modification, there is no mechanism for the highest performing friend to attract weight from other "good" friends (particularly once all of the "bad" friends reach zero weight).

The performance measure, $\pi(z_{i,t}^j)$, can reasonably be incorporated into the process of adaptation either as a straight forward measure of profits with $\pi(z_{i,t}^j) = z_{i,t}^j$ or through some non-linear or discontinuous measure based on a variety of criteria.[2] The employed method was to normalize profits by the highest paying option,

$$\pi(z_{i,t}^j) = z_{i,t}^j / \max_k (z_{i,t}^k). \tag{10}$$

For example, to give equal credit to all above average friends,

$$\pi(z_{i,t}^j) = \begin{cases} 1 \text{ for } z_{i,t}^j \geq \bar{z}_{i,t}^j \\ 0 \text{ for } z_{i,t}^j < \bar{z}_{i,t}^j \end{cases}. \tag{11}$$

The agent also adjusts the probability of acting independently, $\theta_{i,t}$. For this, the agent compares the payoff to acting independently to the best that could have been achieved through

[2] Also examined was to employ a binary measure awarding 1 to all friends realizing average performance among friends or above, zero otherwise.

imitation. Let $z_{i,t}^{ind}$ be the payoff to acting independently. If agent *i* acts independently in period *t*, then $z_{i,t}^{ind}$ is the actual realization achieved by agent *i*. If agent *i* imitates in period *t*, then the agent has to use a hypothetical payoff. Like those who actually did act independently, agent *i* selects from the available options with equal probability and uses the payoff that would have been earned had it been adopted in the first round of the period. Let $y_{i,t}^{0} = z_{i,t}^{ind}$ indicates the profits from acting independently and let $y_{i,t}^{1} = z_{i,t}^{\max(j)} = \max_h(z_{i,t}^{h})$.

The agent uses a flexible indirect measure of cumulative performance,

$$B_{i,t}^{j} = (\phi_2 B_{i,t-1}^{j} N_{2,t-1} + (\delta_2 + (1-\delta_2) I(s_{i,t}, j))\pi_2(y_{i,t}^{j})) / N_{2,t}, \tag{12}$$

$j = 0{,}1$ and $N_{2,t} = \rho_2 N_{2,t-1} + 1$.

The probability associated with acting independently evolves according to a Replicator Dynamics process,

$$\theta_{i,t+1} = \theta_{i,t} + \begin{cases} \tanh(\lambda_2 (B_{i,t}^{0} - B_{i,t}^{1})/2)(1-\theta_{i,t}) \text{ for } B_{i,t}^{0} \geq B_{i,t}^{1} \\ \tanh(\lambda_2 (B_{i,t}^{0} - B_{i,t}^{1})/2)\theta_{i,t} \text{ for } B_{i,t}^{0} < B_{i,t}^{1} \end{cases}. \tag{13}$$

Here, relative performance determines the innovation in the probabilities rather than the level. If one strategy consistently underperforms the other, it will be chosen with zero probability in the limit.

3. Analysis of the social network

Analysis of the steady state and stability properties of the dynamic social network can be found in Goldbaum (2009).

The efficiency of a hierarchy is measured based on the cumulative distance of individual agents from their efficient tier. Let $\delta_{i,t}$ represent the tier occupied by agent *i* in period *t*. Let $\delta_{i,t}^{*}$ indicate the efficient tier based on the shortest route to link to the current leader at time *t*. Efficiency, *E*, is thus measured as

$$E = \sum_i \delta_{i,t} - \delta_{i,t}^{*} \tag{14}$$

with $E = 0$ representing a fully efficient hierarchy and values of $E > 0$ represent deviations from efficiency.

A hierarchy is considered "optimal" if the agent with the greatest potential to serve as the leader does actually emerge as leader. The hierarchy is considered optimal if the agent with the greatest number of incoming links emerges as the leader.[3]

The optimality of the leader is measured by the difference in the number of potential fans of the current leader relative to the maximum number of potential fans of the individuals in the population. Let f_i be the number of agents with links to agent *i* and let $f_{0,t}$ be the number of links to the leader at time *t*. The optimality of the leader, *O*, is thus

$$O = \max(f_i) - f_{0,t} \tag{15}$$

4. Simulations

Each simulation starts with a population of individuals linked through a social network. The links are unidirectional representing a connection by which an agent can choose to imitate

[3] Other designation of a "natural leader" could be considered. The proposed definition is both reasonable and readily verifiable.

his or her friends. The baseline initiation is to randomly assign the links, but symmetrically structured social networks were examined as well. The baseline is to assign the initial weights, $w_{i,0}^{j}$, equally. Both random assignment and uniform assignment of $\theta_{i,0}$ to 0.5 were examined. The other baseline parameters of the simulation are reported in Table 1.

Table 1: Exogenous parameter setting used in the simulations

$L = 100$	Number of agents
$P = 500$	Number of periods
$R = 20$	Rounds per period
$K = 12$	Number of options per period
$d = 6$	Number of friends
$\delta_A = \delta_B = 1$	Weight on untried options
$\lambda_A = 4$	Strength of preference in choosing among friends
$\phi_A = 0.5$	depreciation rate in friendship
$\lambda_B = 0.01$	Strength of innovation in choosing between imitation and independence
$\phi_B = 0$	Depreciation rate in independence
$\underline{w} = 0$	No friend selection

From Goldbaum (2009), the equilibrium social structure for the population is a single leader with a hierarchy of followers. For everyone in the population to participate in this social structure, the total number of options available to the individuals match or exceed a lower bound, so that $K > \underline{K}$ with

$$\underline{K} = \frac{a_T(L-2)}{a_J(L-1)} + 1. \tag{16}$$

Two types of figures are employed to display the findings. The time-series figure plots a time-series of a number of model-generated variables. N(ind) is the number of agents choosing to act independently, MaxFans is the number of imitators of the agent with the highest number of fans that period, and N(r=tr) is the number of agents who make their selection in the final round. The only reason to be caught without making a choice until the final round is for the agent to have been caught in a closed circle of links, for example when two agents link with each. In this situation, no agent in the circle links out to the leader. In addition are the measures of optimality and efficiency.

The hierarchical figure plots the social structure as it exists in the final period of the simulation. Across the top of the figure are the *K* choices, labeled using capital letters. Individual agents, labeled using numbers, appear in rows below the choices, based on the round of adoption. An arrow from an individual directly to one of the choices represents a choice by an individual who has chosen to act independently for the period. An arrow from one agent to another represents a path of imitation with the arrow originating from the imitator.

4.1 Emergence of a leader

The emergence of a leader and a hierarchy of followers from the initially unstructured social network is quite robust. An example of the process of emergence is plotted in Figure 1. Figure 2 depicts the resulting social hierarchy. In this example, agent #2 emerges from the population as the leader and a hierarchy based on agent #2's leadership forms. In round 1 of each period subsequent to formation, agent #2, occupying tier 0 of the hierarchy, randomly selects one of the 12 options available for the period. In round 2, those in tier 1 of the hierarchy imitate agent #2. By round 5, agent #2's choice for the period has disseminated through the entire population.

The condition necessary to produce emergence is path dependence in the social links. In the early periods of the simulation, success by the individual is the result of random events that, in the absence of any adjustment, are transitory. For a leader to emerge, it must be that others in the population respond to the lucky individual's success by, for example, increasing the weight, $w_{i,t}^{j}$ if they are linked to someone successful and decreasing their own $\theta_{i,t}$ since the imitation offers a higher payoff. The process of observation and adjustment allows the successful agent to become empowered by his or her followers, eventually generating a hierarchy. Success breeds further success.

Efficiency in the hierarchy depends on the individual's ability to find and settle on the friend offering the shortest route the leader.

Given the emergence of a leader, simulations indicate that asymptotic efficiency in the hierarchy requires that individuals asymptotically settle on the best friend offering the most direct link to the leader (or friends if multiple friends offer equal length shortest paths). For this, eventually the random component of their decision process converges to zero, as accomplished when employing the replicator dynamic process with infinite memory.

Alternatively, allocating weight according to the EWA process, fails to produce an efficient hierarchy. The non-zero weight allows an individual follower to occasionally choose an inefficient friend. Figure 3 is the time-series output from a EWA simulation. The leadership position is stable, but those in the lower tiers of the hierarchy shuffle positions each period. The resulting hierarchy tends to have a greater number of tiers as individuals end up employing inefficient paths.

With $K \geq \underline{K}$, it is better for a non-leader to imitate than to act independently. With infinite memory in the process governing $\theta_{i,t}$ individuals learn this lesson once a leader has emerged. For the leader, $\theta_{i,t} \to 1$ quickly while $\theta_{i,t} \to 0$ for all non-leaders, making them into followers. Once a leader has emerged in simulation, that individual will retain the leadership position throughout.

A finite memory in the $\theta_{i,t}$ process tends to prevent convergence of $\theta_{i,t} \to 0$ for followers but still allows $\theta_{i,t} \to 1$ for the leader. In the extreme, with $\phi_2 = 0$, the individual is adaptive, using the previous period's payoffs as the basis for the current period's $\theta_{i,t}$. Recall that the follower evaluates the value of acting independently employing a naïve process based on considering a hypothetical choice they would have made if acting independently.

While good for creating efficient hierarchies, the robustness of the leader generated from the replicator dynamic process means that once a leader has been established, no events arise in simulation to disrupt that position allowing a new leader to emerge. Optimality of the leader tends to be poor under replicator dynamics.

While poor at generating efficiency, the EWA process is better able to occupy the leadership position with a leader of greater fitness than is the replicator dynamics. Look back at F4 and observe that the emergent leader is a natural leader with an optimality measure of zero. Incorporating a short memory in $\theta_{i,t}$ by setting $\phi_2 = 0$, combines with the movement in the social structure under EWA creates enough mixing to help the natural leader emerge as the original leader or to depose an existing sub-optimal leader, allowing a more fit leader to emerge. This can be seen in Figure 4 as agent #77 is replaced by agent #49 (in period $t = 116$), who is replaced by agent #75 (in period $t = 239$).

4.5 Non-emergence of a hierarchy

A leader can still fail to emerge from the population if agents fail to respond to an individual's successes. If there is inadequate adjustment to the social network in response to a moment of success, then the opportunity to build on individual success is lost.

In a process governed by replicator dynamics, the rate of adjustment to the social links is determined by $\lambda_{RD} \in [0,\infty)$. The greater λ_{RD}, the more responsive is the individual in sifting probability towards the higher performing friend. At $\lambda_{RD} = 0$, there is no adjustment from the initial weights. For $\lambda_{RD} = \infty$, the agents shift all the probability weight away from the lowest performing friend. In simulation with $\lambda_{RD} = 0$, a leader does emerge but the hierarchy based upon the leader is itself transient as individuals often fail to link to the leader, directly or indirectly.

To build a network of fans, an agent needs to be visible. If an agent is lucky enough to choose a trend early, he or she needs to be observed doing so in order to attract fans. Setting $\delta_A < 1$ reduces the successful agent's exposure. Setting $\delta_A = 0$ or close to zero prevents the emergence of a leader and thus the formation of a social structure. Without exposure, there is no response and thus no emergence. In order to prevent emergence, δ_A had to be set very low. Though the process is slowed compared to the baseline simulation, a value of $\delta_A = 0.1$ still generates an emergent efficient hierarchy.

Setting $\delta_B = 0$ or near zero leads to a stable pool of agents acting independently. They do so suboptimally, settling on a strategy based on insufficient information. Figure 5 results from setting $\delta_B = 0.01$.

5. Conclusions

An efficient hierarchy emerges from the population. The emergent hierarchy is efficient when individuals respond and adjust towards high performing strategies, driving the probability of following an inefficient strategy to zero. Such a process favors stability as well as efficiency and as a result tends to propel an individual from the population towards leadership based on luck rather than advantage.

An adjustment process that allows individuals to occasionally pursue inferior strategies will produce less efficient hierarchies and fluidity in the hierarchy. Through this instability, a more fit leader, measured by the number of social links, tends to emerge.

The hierarchy emerges because early transitory events become embedded in the social network to favor those with early success. An individual's good luck leads to adjustment in the social network that perpetuates the success by attracting followers and followers lead to greater success.

Certain environments can hinder the process of hierarchy formation. If individuals are less observant of the individual success around them, then they fail to respond adequately, thereby denying the successful individuals the opportunity to perpetuate his or her success through social evolution. Stubbornness on the part of individuals when adjusting strategies will also undermine the social process that produces hierarchies.

Mindful of the Kirman *et al* (2007) criticism that endogenous network models tend to require unreasonably high knowledge regarding the full social network, hierarchy formation is achieved with no knowledge of the existing or potential social structure.

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Figure 1: Time-series of population characteristics. Replicator Dynamics, $K = 12$, $\underline{w} = 0$, $\phi_A = 0.5$, $\phi_B = 1$, $\lambda_{RD} = 1$. The leader emerges as sole leader, the only agent acting independently. Soon after, the efficiency of the hierarchy increases to zero. The leader is not a natural leader. Once established, there are no disruptions to the hierarchy.

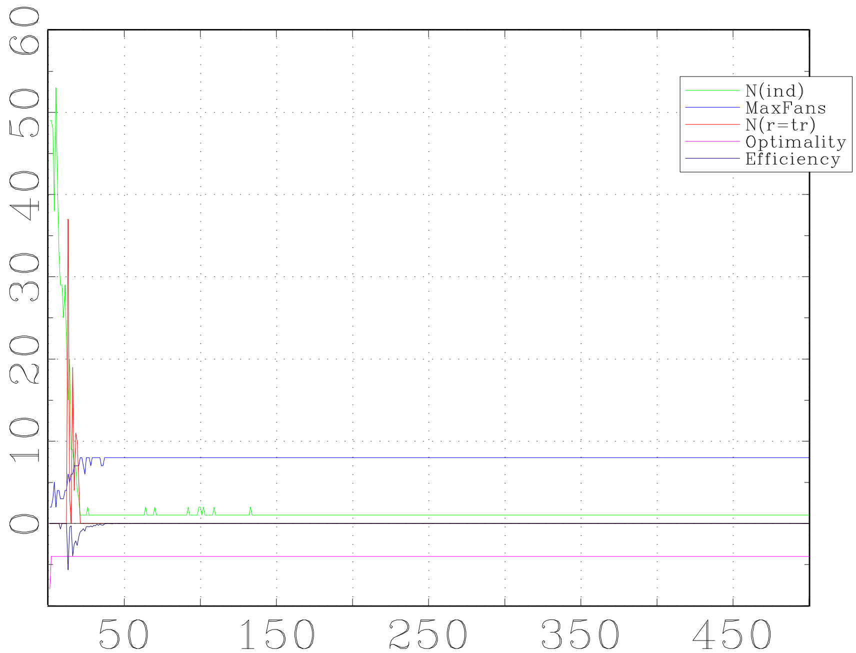

Figure 2: Structure of the social hierarchy in the final period (t = 500). Replicator Dynamics, K = 12, λ = 1, $\underline{w} = 0$, $\phi_A = 0.5$, $\phi_B = 1$, $\lambda_{RD} = 1$. Efficient hierarchy based on Agent #2, who emerged from the population as the leader.

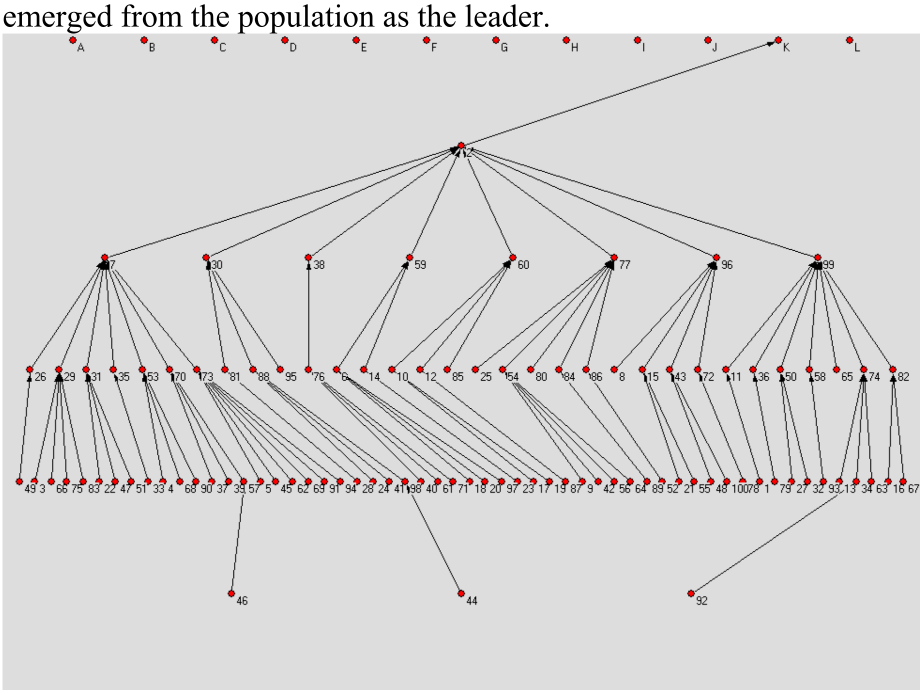

Figure 3: Time-series of population characteristics. Experience Weighted Attractor, $K = 12$, $\underline{w} = 0$, $\phi_A = 0.5$, $\phi_B = 1$, $\lambda = 0.1$. The leader emerges as sole leader, the only agent acting independently. Efficiency does not arise in the hierarchy. There is fluidity in the position of the followers in the hierarchy. The leader is a natural leader.

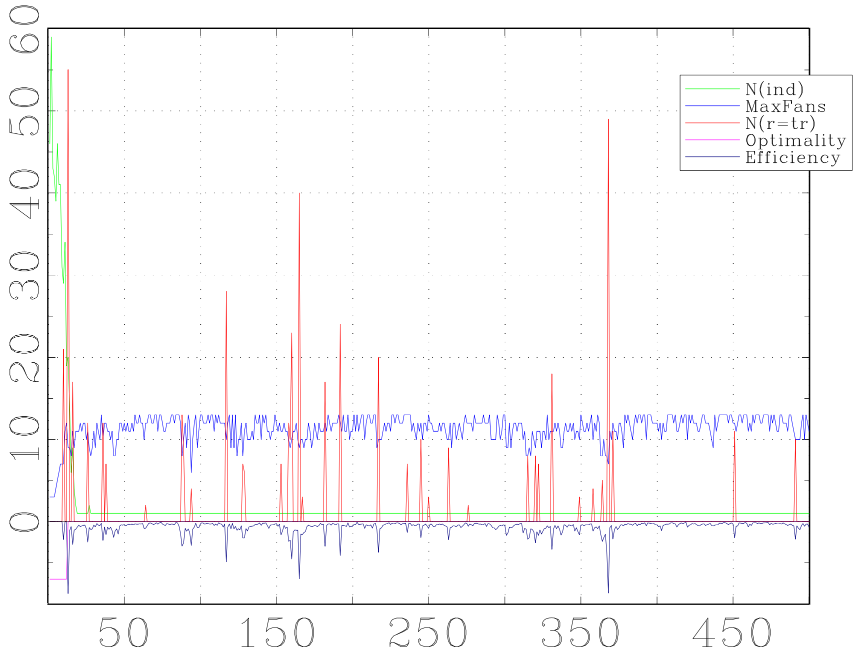

Figure 4: Time-series of population characteristics. Experience Weighted Attractor, $K = 12$, $\underline{w} = 0$, $\phi_A = 0.5$, $\phi_B = 0$, $\lambda_1 = 3.5$. In most periods, there is one leader. There are two changes in leadership (at t=116 and t=239).

Figure 5: Time-series of population characteristics. Replicator Dynamics, $K = 12$, $\underline{w} = 0$, $\phi_A = 0.5$, $\phi_B = 1$, $\lambda_{RD} = 1$, $\delta_B = 0.01$. A lack of information leads a population of agents to suboptimally act independently.