Optimized two-stock portfolio using time dependent mean-variance analysis

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A strategy for managing a two-stock portfolio is introduced using time dependent mean-variance analysis that incorporates both the long term optimum and short term price fluctuations. This strategy is tested with real data of the Hang Seng Index. The testing data cover the period between Jan 10, 2007 and July 21, 2009 for 24 blue chip stocks from the Hang Seng Index. The strategy is compared to the average return of these 24 stocks as well as to the Hang Seng Index in the same period. We conclude that our strategy has a positive return over most of the days of the testing period, including a very stable positive performance in the period of market crash. The variation of the cumulative return of our strategy is less than both the average returns of the chosen stocks or the Hang Seng Index, thereby providing a portfolio with a smaller risk but still attractive return. This strategy is therefore a good investment scheme for conservative investors who prefer stable return even during market downturn.

1. Introduction

Portfolio management of stocks is an optimization problem on complex systems, and is of great interests for financial institutes, researchers in mathematics and physics, as well as individual investor. The underlying rational for the selection of stocks and timely transaction, usually involves information that are generally unavailable to outsiders, but the mathematical machinery on historical data on stocks can still give some guidance on the scheduling of buy and sell to provide a good return. One famous illustration of the power of mathematical analysis is provided by Markowitz [1], with his rather intuitive application of mean–variance analysis on two-stock portfolio. In a recent work in our group [2], we have investigated a multi-agent system of stock traders, each making a two-stock portfolio [2] using the mean-variance analysis. The results of this work show that there indeed exist portfolio with low risk and high return, in spite of the random nature of the stock price and unknown mechanism between the price variations of individual stock. Indeed, in all works on portfolio management involving stocks, a common goal is to pursue high return, low risk and consistent performance [3-5]. In this paper, this goal is extend our previous work to take into account of time dependence on our trading strategy, so that the return is high, risk is low, and most important of all, do not incur great loss even in crash. The time dependence nature of our trading strategy also should avoid frequent transaction, as we must be aware of the great loss incur by the transaction cost in the long run. To meet all these requirements, we design our investment strategy by taking into account of the “Sharpe ratio” using mean

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variance analysis in the long time scale, as well as the stock price fluctuation in the short term. A critical value of our trading decision mechanism is set at a rather high value to avoid frequent transactions that accumulate large loss in the long run, thereby providing a low risk portfolio even during the financial tsunami in 2008. Our investment strategy give good performance (better than the Hang Seng Index) in a test period between 2007-2009, covering both the bull market and the crash.

2. Description of the strategy

2.1 Mean-variance analysis

Following the standard mean variance analysis of Markowitz[2], we consider a pair of stocks denoted as stock 1 and 2. By definition, the expected return $U(t)$ and variance $Var(t)$ for each stock are expressed as:

$$U(t) = \frac{1}{SampleSize} \sum_{k=t-SampleSize+1}^{t} r(k) \tag{1}$$

$$Var(t) = \frac{1}{SampleSize-1} \sum_{k=t-SampleSize-1}^{t} \left(r(k) - U(t)\right)^2 \tag{2}$$

where $r(t) = \dfrac{p(t) - p(t-1)}{p(t-1)}$ is the daily rate of return and $p(t)$ is the daily closing price of the stock. The sample size is chosen to be 50 days. In our study of a two-stock portfolio, the expected return and variance for stock pair (1,2) is given by

$$U_{12}(t,x) = U_1(t)x(t) + U_2(t)y(t) \tag{3}$$

$$Var_{12}(t,x) = Var_1(t)x^2(t) + Var_2(t)y^2(t) + 2Cov_{1,2}x \cdot y \tag{4}$$

where x and y are the fraction of the portfolio invested in stock 1 and in stock 2 respectively. Note that the constraint $x + y = 1$, and $x, y \in (0,1)$ imply that these quantities are function of *t* and *x* only. The covariance $Cov_{1,2}$ of the two stocks is defined as

$$Cov_{1,2}(t) = \frac{1}{SampleSize} \sum_{k=t-SampleSize-1}^{t} \left(r_1(k) - U_1(t)\right)\left(r_2(k) - U_2(t)\right) \tag{5}$$

while the standard deviation of the two-stock portfolio is expressed as:

$$\sigma_{12}(t,x) = \sqrt{Var_{12}(t,x)} \tag{6}$$

To analyze this two-stock portfolio, we make use of a version of the Sharpe ratio defined as:

$$F_{12}(t,x) = U_{12}(t,x) / \sigma_{12}(t,x) \tag{7}$$

Note that this ratio is a function of *x*, so that we can find its maximum in the range of $x \in (0,1)$.

In order to achieve maximum return per unit fluctuation or risk, we maximize $F_{12}\left(t,x\right)$ with respect to x and denote this maximum value as $F_{12}^{*}\left(t\right)$ and the corresponding resource allocation at $\left(x(t),y(t)\right)=\left(x^{*},y^{*}=1-x^{*}\right)$. Note that this resource allocation of the portfolio refers to time *t* and stock pair (1,2).

2.2 Short term stock price analysis

In the mean-variance analysis described above, we have chosen the sample size for the evaluation to be 50, corresponding to a rather long period for stock price prediction. In general, the value of the fluctuation (risk) is more reliable when the sample size is large. However, the reliability of the value of mean return for the prediction of the actual return at time *t+1,* will decrease with the sample size. In actual price prediction, usually the most important data for reference are the stock price in the past few days. This kind of short term price analysis is absent in our mean-variance analysis. To handle the price during this short time scale, we propose to consider the following values of the portfolio, assuming that the resource allocation vector $\left(x(t),y(t)\right)=\left(x^{*},y^{*}\right)$ remains unchanged. Specifically, we consider the value of the portfolio on two days before the trading day:

$$r_{12}\left(t-1\right)=r_{1}\left(t-1\right)x^{*}\left(t\right)+r_{2}\left(t-1\right)y^{*}\left(t\right) \tag{8}$$

$$r_{12}\left(t-2\right)=r_{1}\left(t-2\right)x^{*}\left(t\right)+r_{2}\left(t-2\right)y^{*}\left(t\right) \tag{9}$$

where r_{12} is the estimated profit of the two-stock portfolio for the stock pair (1,2) using the proportion $\left(x^{*},y^{*}\right)$ determined in section 2.1 using long term mean variance analysis at time *t*. We will use these two terms, $r_{12}\left(t-1\right)$ and $r_{12}\left(t-2\right)$, as reference to reflect the price values at short term, assuming that during these past two days, the portfolio allocation remains unchanged at $\left(x^{*},y^{*}\right)$. The key question for the trading strategy involves the following decision making process:

(1) Should we change the selected choice of stock pair (1,2) to some other pair of stock?
(2) What is the new portfolio allocation, in case of same stock pair, or in case of the new stock pair?

We now discuss the decision making process based on a combination of long term mean-variance analysis and a short term price trend.

2.3 Decision on each trading day

On each trading day, each stock pair *(i,j)* has its best "Sharpe ratio", denoted as $F_{ij}^{*}(t)$ with corresponding resource allocation vector $v_{ij}^{*}(t)=\left(x_{ij}^{*}(t),y_{ij}^{*}(t)=1-x_{ij}^{*}(t)\right)$. Each stock pair also has the estimated profits for the past two days given by $r_{ij}(t-1)$ and $r_{ij}(t-2)$, as in Eq.(8),(9). We now introduce the combined factor $s_{ij}(t)$ defined by

$$s_{ij}(t)=F_{ij}^{*}(t)\exp\left(\left(r_{ij}(t-1)/U_{ij}^{*}(t)\right)\left(r_{ij}(t-2)/U_{ij}^{*}(t)\right)\right) \tag{10}$$

where $U_{ij}^{*}(t)=U_{ij}\left(t,x_{ij}^{*}(t)\right)$. The exponential factor in Eq.(10) can be considered as a correction factor to the ratio $F_{ij}^{*}(t)$ by the price trend in the short term of the past two days. The price trend factor is itself a normalized form by the normalization factor $U_{ij}^{*}(t)$, which is the long term expected return of the stock pair *(i,j)* provided by mean variance analysis.

On each trading day *t*, we can compute the value of $s_{ij}(t)$ for a given stock pair (*i,j*). For a given set of N important stocks, we can form M=N(N-1)/2 pairs. Therefore we can have a set K of M values of $s_{ij}(t)$ and we can form a distribution of these values. Let the mean and standard deviation of this distribution of $s_{ij}(t)$ be $\bar{s}(t)$ and $\sigma_s(t)$. Let the maximum value of this set K be

$$G(a,b,t,x_{ab}^{*})=\max\left(s_{ij}(t)\middle|\, i=1,...,N;\, j<i,\right) \tag{11}$$

Note that this G is given by Eq.(10) for the stock pair (a,b) and the corresponding portfolio allocation vector $v_{ab}^{*}(t)=\left(x_{ab}^{*}(t),y_{ab}^{*}(t)\right)$.

The decision criterion is now given by the critical value

$$G_c(t)\equiv\bar{s}(t)+3\sigma_s(t) \tag{12}$$

When $G(a,b,t,x_{ab}^{*})>G_c(t)$, meaning that the switch to the new pair of stocks (a,b) is likely to give a higher "Sharpe ratio", we will change our portfolio from the original to the new one. The resource allocation is also set by the new value $v_{ab}^{*}(t)$. Of course, if there is the stock pair (a,b) is the same as the original stock pair, no action is required. If we do not have any stock, we will use all our cash to buy the stock pair (a,b) with $x_{ab}^{*}(t)$ for stock a and $\left(1-x_{ab}^{*}(t)\right)$ for stock b. On the other hand, if $G(a,b,t,x_{ab}^{*}) < G_c(t)$, we should be safer to keep cash and we will sell all stocks if we have stocks, or in the case that we have cash, we only keep cash and no action is required.

In this trading strategy, we have reduced the number of transaction substantially, thereby avoiding the transaction cost which can reduce the return of the portfolio greatly in the long run. Trading action is required only when the optimum “Sharpe ratio” $G(a,b,t,x_{ab}^{*})$ is greater than the critical value, $G_c(t)$. When we begin our investment with a given cash reserve, we only buy stock and form a portfolio with stock pair (a,b), which has a high return and low risk. As soon as the optimum value $G(a,b,t,x_{ab}^{*})$ falls below the critical value $G_c(t)$, we will keep cash. Therefore, we expect our trading strategy to yield high profit with low risk some time. When there is no such opportunity, cash is preferred. Our strategy is generally more conservative than many stock portfolio that keep cash less often.

3. Simulation Result

To perform numerical test of our theory, we select 24(=N) stocks that make up the Hang Seng Index. These 24 stocks are listed in Table 1. They are all listed for a period longer than the testing period between Jan 10, 2007 and July 21, 2009. With these 24 stocks we have a collection of M=276 distinct pairs of stocks that can be candidate of the optimum two-stock portfolio in the context of mean variance analysis.

Table 1. The stocks whose real price data are used in the study

0001.HK	0002.HK	0003.HK	0004.HK
0005.HK	0006.HK	0011.HK	0012.HK
0013.HK	0016.HK	0019.HK	0023.HK
0066.HK	0101.HK	0144.HK	0267.HK
0291.HK	0293.HK	0330.HK	0494.HK
0762.HK	0883.HK	0941.HK	1199.HK

The initial condition for the simulation is that all the money is in cash. The cost is 0.1% of the stock price for each selling or buying transaction. We numerically calculate the resource allocation of our money on optimum stock pair over the period of Jan 10, 2007 and July 21, 2009, which consists of a maximum of 600 days for possible trading. To make proper comparison, we also compute the simple averaged value of the stock price for the 24 stocks as follow:

$$c_i(t)=\sum_{k=0}^{t} r_i(k) \tag{13}$$

$$\bar{c}(t)=\frac{1}{N}\sum_{i=1}^{N} c_i(t)$$

where $c_i(t)$ is the cumulative price evaluation for stock *i*. By averaging the stock price variation as in Eq. (13), one gets the curve shown in Fig.1 with black open squares. We also include the Hang Seng Index (which consist of these 24 stocks in additional to some other stocks) in the testing period. The portfolio cumulative value is shown in Fig.1 by the blue open triangles.

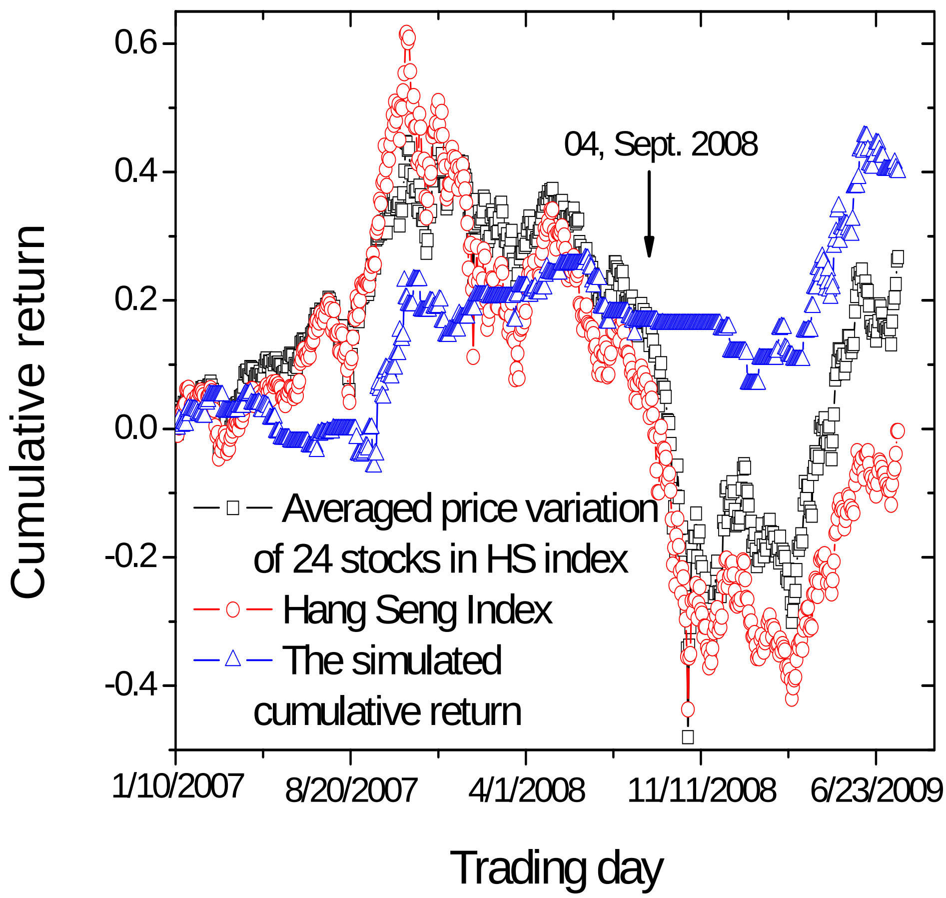

Fig.1 The cumulative return by the proposed trading strategy in comparison with the stock average price variation and the Hang Seng Index over the same period.

From Fig.1, we see that before 04/09/2008, the cumulative return, overcoming the 0.1% transaction cost, does not deviate much from the averaged stock price variation. The return is less than a portfolio that tracks the Hang Seng Index, since we tend to keep cash while the stock market rises rather sharply. Nevertheless, the return is still impressive, making about 30% increase in mid 2008 before the tsunami. The shape of the blue curve looks similar to the Hang Seng Index, though with generally less fluctuation. This implies that the proposed strategy can predict the portfolio with above-average return at least in most of the days. Indeed, if our strategy has no intelligence, for example, in the case of a random transaction, the transaction cost will incur great lost (each day when trading happens, one loses 0.1%, and over a period of 600 days, the loss can be really great). During the months of financial tsunami (September-October 2008), the stock price and Hang Seng Index nosedive while our portfolio more or less is immune to the loss, because we mainly keep cash. Our strategy initiates a timely withdrawal from the stock market, so that large loss is avoided. From November 2008 onward, the cumulative return is growing in pace with the stock price variation, indicating that during that rising period, the strategy can keep up with an early sign of bull market and take profits by investing in pair of good stocks. As a result, the overall performance of this strategy is better than the stock price variation as well as the Hang Seng Index, over this period of 600 days.

4. Conclusion

We proposed a rather conservative strategy of investment using the time dependent mean variance analysis on a two-stock portfolio. The time dependence covers both the long term aspect of the pair of stocks, as well as the return of the stocks over the past two days. The long term aspect is covered by computing the "Sharpe ratio" at time *t*, while the short term stock price return provide a correction factor. The combined effect on the performance of the two-stock portfolio is a modified time dependent "Sharpe ratio". By setting a critical value for the trading threshold, we can avoid loss by placing emphasis on cash and take profit only when there is a clear indication of a pair of stocks with low risk and high return. Numerical simulation of this trading strategy with real data on a set of blue chips in the Hang Seng Index indicates good return on bull market and small loss on bear market. The overall performance of our strategy beats the performance of the chosen set of stocks as well as the Hang Seng Index. This strategy should be suitable to conservative investors.

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References

[1] CHEN CHEN,RONG TANG,K.Y. SZETO† "Optimized Trading Agents in a Two-stock Portfolio using Mean-variance Analysis", Proceeding of 2008Winter WEHIA & CIEF 2008,pp61-67, 2008

[2] H. Markowitz, J. of Finance, Vol.7, 77, (1952)

[3] K.Y. Szeto, K.H. Cheung, "Multiple time series prediction using genetic algorithms optimizer," *Proceedings of the International Symposium on Intelligent Data Engineering and*

Learning, Hong Kong, IDEAL'98, 127-133, 1998.

[4] K.Y. Szeto, L.Y. Fong, “How adaptive agents in stock market perform in the presence of random news: a genetic algorithm approach,” LNCS/LNAI, Vol. 1983, Ed. K. S. Leung et al. Spriger-Verlag, Heidelberg, 2000, IDEAL 2000, 505-510,
2000.

[5] Alex L.Y. Fong and K.Y. Szeto, Rule Extraction in Short Memory Time Series using Genetic Algorithms; European Physical Journal B Vol.20, 569-572(2001)