### **AI-ECON (NCCU) Lectures on**

### **Financial Market Behaviour with Heterogeneous Investors**

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#### Lecture 1: Dynamics of Moving Averages

Despite the pervasiveness of the efficient markets paradigm in the academic finance literature, the use of various moving average (MA) trading rules remains popular with financial market practitioners. In this lecture I will present a dynamic financial market model in which demand for traded assets has both a fundamentalist and a chartist component. The chartist demand is governed by the difference between current price and a (long-run) MA. Both types of traders are boundedly rational. The stability and bifurcation properties of the underlying deterministic model are characterized via the reaction coefficient of the fundamentalists, the extrapolation rate of the chartists and the lag length used for the MA. Various rational routes to randomness for different MA rules are examined by increasing the intensity of choice to switching strategies. The price dynamics of the moving average rule are also examined and it is found that an increase of the window length of the MA rule can destabilize an otherwise stable system, leading to more complicated, even chaotic behaviour. The analysis of the corresponding stochastic model shows that the interaction of the MA and market noises can lead to the tendency for the market price to take long excursions away from the fundamental. The model reveals various market price phenomena, the coexistence of apparent market efficiency and a large chartist component, price resistance levels, long memory and skewness and kurtosis of returns.

This lecture will be based on my two joint papers with Carl Chiarella and Cars Hommes (2006*a*, 2006*b*).

# Lecture 2: Market Behaviour and Statistic Properties of a Market Fraction Model of Heterogeneous Agents

This lecture contributes to the development of the recent literature on the explanation power and calibration issue of heterogeneous asset pricing models by presenting a simple stochastic market fraction asset pricing model of two types of traders (fundamentalists and trend followers) under a market maker scenario. It seeks to explain aspects of financial market behavior (such as market dominance, convergence of the market price to the fundamental price, and under- and over-reaction) and to characterize various statistical properties (including the autocorrelation structure and power-law behaviour in volatility) of the stochastic model by using the dynamics of the underlying deterministic system, traders' heterogeneous behavior and market fractions. The analysis shows that agent heterogeneity, risk-adjusted trend chasing through the geometric learning process, and the interplay of noisy fundamental and demand processes and the underlying deterministic dynamics can be the source of power-law distributed fluctuations. A statistical analysis based on Monte Carlo simulations shows that the long-run behavior, convergence of the market prices to the fundamental price, limiting distributions, and various under and over-reaction autocorrelation patterns of returns can be characterized by the stability and bifurcations of the underlying deterministic system. Realistic estimates of the power-law decay indices and the (FI)GARCH parameters are presented. The lecture underpins the mechanisms on various market behaviors (such as under/over-reactions), market dominance and stylized facts in high frequency financial markets.

This lecture will be based on my two joint papers with Youwei Li (2007a, 2007b).

## Lecture 3: Aggregation of Heterogeneous Beliefs and the CAPM under Mean-Variance Framework

When agents are heterogeneous in their beliefs, how the market aggregates these beliefs and determines the market equilibrium price is an important issue. Within the standard mean-variance framework, this lecture presents a procedure to aggregate the heterogeneous beliefs not only in risk preferences and expected payoffs (returns) but also in variances/covariances into a market consensus belief. Consequently, an asset equilibrium price under heterogeneous beliefs is derived. The market aggregate behavior is in principle a weighted average of heterogeneous individual behaviours. I will also present the Zero-Beta CAPM and CAPM-like equilibrium price and return relationships under heterogeneous beliefs. The impact of the diversity of heterogeneous beliefs on the market aggregate risk preference, asset volatility, equilibrium price and optimal demands of investors is examined. In particular, It is shown that the traditional two fund separation does not hold in general and, in market equilibrium, investors' optimal portfolio may not be on the market efficient frontier. As a special case, the results provide a simple explanation for the empirical relation between cross-sectional volatility and expected returns.

This lecture will be based on my recent joint work with Carl Chiarella and Roberto (2006*a*, 2006*b*) and Lei Shi (2007).

#### REFERENCES

- Chiarella, C., Dieci, R. and He, X. (2006*a*), Aggregation of heterogeneous beliefs and asset pricing theory: a mean-variance analysis, Technical Report 186, Quantitative Finance Research Center, University of Technology, Sydney.
- Chiarella, C., Dieci, R. and He, X. (2006*b*), A dynamic heterogeneous beliefs capm, working paper (draft), University of Technology, Sydney.
- Chiarella, C., He, X. and Hommes, C. (2006*a*), 'A dynamic analysis of technical trading rules in financial markets', *Journal of Economic Dynamics and Control* **30**, 1729–1753.
- Chiarella, C., He, X. and Hommes, C. (2006*b*), 'Moving average rules as a source of market instability', *Physica A* **370**, 12–17.
- He, X.-Z. and Li, Y. (2007*a*), 'Heterogeneity, convergence and autocorrelations', *Quantitative Finance* to appear.
- He, X.-Z. and Li, Y. (2007*b*), 'Power law behaviour, heterogeneity, and trend chasing', *Journal of Economic Dynamics and Control* to appear.
- He, X.-Z. and Shi, L. (2007), Zero-beta heterogeneous CAPM, working paper, School of Finance and Economics, University of Technology, Sydney.