Lecture One

Dynamics of Moving Averages

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AI-ECON (NCCU) Lectures on

Financial Market Behaviour with Heterogeneous Investors

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Outline

- **Related Literature and Motivation**
- **An Asset Pricing Model with A Market Maker**
- **Stability and Bifurcation Analysis**
- **Dynamics of the Nonlinear Deterministic System**
  - **The effect of the switching intensity—Rational Routes to Randomness**
  - **The effect of the lag length—Dynamics of the Moving Average**
- **Dynamics of the Stochastic System**
  - **Multiplicative noise**
  - **Additive noise**
- **Conclusion and Remarks**

1 Related Literature and Motivation

- **Technical Analysis (TA)**
  - **TA attempt to forecast prices by the study of patterns of past prices and a few other related summary statistics about security trading**
  - **Shifts in supply and demand can be detected in charts of market movements**
  - **TA is inconsistent with the (weak form) market efficiency**
  - **Practical use of TA seems to be widespread amongst financial markets traders**

- **Empirical Studies**
  - **Exchange Rate Markets:**
    - **US dollar exchange rate in eighties: Frankel and Froot (1986, 1990),**
    - **Survey: Allen and Tylor (1990) and Taylor and Allen (1992)**
  - **Predictability in Stock Markets:**
    - **Sources and asymmetric nature: Brock et al (1992),**
    - **Evidence: Pesaran and Timmermann (1994, 1995)**
  - **Profitability of TA: Lo et al (2000), Boswijk et al (2000), and Goldbaum (2003)**
  - **Genetic programming (e.g. Neely et al 1997) and Neural networks (e.g. Gencay, 1998)**
- **Problems**
  - **More focus on empirical studies BUT less on analytical studies**
  - **Simplified demand functions based on the trading signals**

- **Questions**
  - **How to determine the amount to buy/sell?**
  - **How the market prices are affected following these buy/sell actions?**
  - **How the length of moving average window affect the price dynamics?**
- **Heterogeneous Agent Models (HAM)**
  - **Heterogeneity among agents (investors)**
  - **Boundedly rationality**
  - **Survey papers:**
    - **Theoretical: Hommes (2006)**
    - **Computational: LeBaron (2006)**
    - **Computational intelligence: Chen (1983-now)**

- **Aims of this lecture**
  - **To present a market of financial market dynamics in which demand for traded assets has both fundamentalist and chartist components**
    - **fundamental demand: mean reversion to the fundamental price**
    - **chartist demand: based on the difference between current price and a moving average**
  - **To examine the impact of the moving average trading rules on the market dynamics when agents switch among two strategies based on certain fitness measure**
  - **To analyze** *Rational Routes to Randomness* **in relation to the moving average trading strategies**
  - **To demonstrate that the model is able to generate various market behaviours (e.g. price resistance, bull/bear markets, bubbles and crashes) and stylized factors (e.g. skewness, kurtosis, volatility clustering and long memory).**

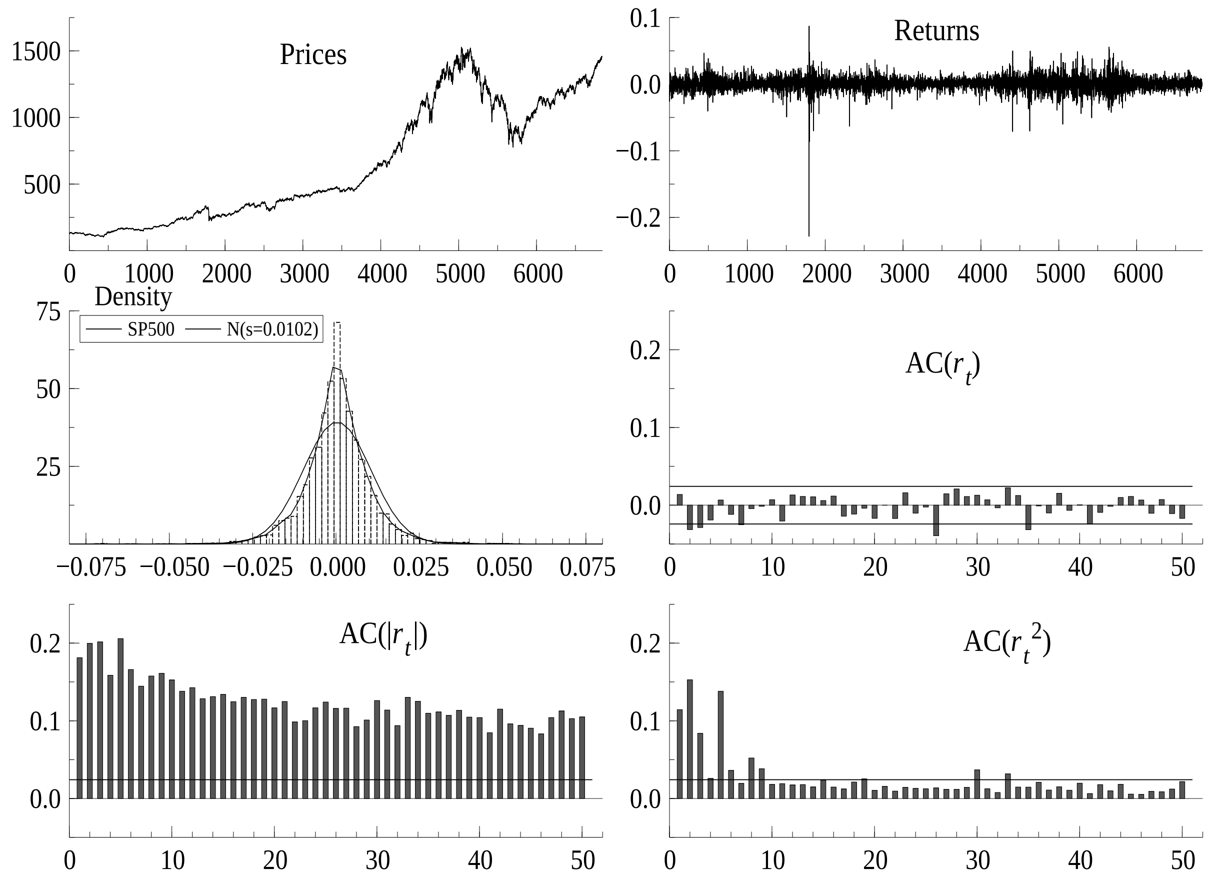


Figure 1: The daily $S\&P500$ date, 5/12/1980-23/2/2007—Daily price and returns, return r_t distribution, the ACs for the returns $AC(r_t)$, the absolute $AC(|r_t|)$ and squared $AC(r_t^2)$ returns

2 An Asset Pricing Model with a Market Maker

- **Heterogeneous Agent Model**
  - **Brock and Hommes framework**
  - **Optimal demand, utility maximization and trading strategies**
  - **Market maker via Walrasian scenarios**
  - **This lecture focuses on the price dynamics of the trading rules with a market maker scenario**

- **Notation**
  - P_t: the price (cum dividend) of the risky asset at time t.
  - N: the total number of traders (assumed to be a constant) in the market, among which there are $N_{h,t}$ of type h traders at time t with $h = 1, 2, \cdots, H$ and $\sum_{h=1}^{H} N_{h,t} = N$
  - The market fractions of different types of traders at time t can be defined as

$$n_{h,t} = N_{h,t}/N, \qquad h = 1, 2, \cdots, H.$$

  - The population weighted aggregate excess demand at time t is given by

$$D_t = \sum_h n_{h,t} D_{h,t}.$$

- **A Market Maker Mechanism**

$$P_{t+1} = P_t[1 + \sigma_\epsilon \tilde{\epsilon}_t] + \mu \sum_h n_{h,t} D_{h,t},$$

  - $\tilde{\epsilon}_t \sim \mathcal{N}(0, 1)$ **captures a random excess demand process**
  - $\mu > 0$ **measures the speed of price adjustment of the market maker to the excess demand.**

- **A Model of Two Types of Traders—Fundamentalists and Chartists**
  - **Market fractions**

$$n_{f,t} = N_{f,t}/N, \quad n_{c,t} = N_{c,t}/N, \quad m_t = n_{f,t} - n_{c,t} \in [-1, 1].$$

  - **The excess demands of the fundamentalist and chartist:** D_t^f **and** D_t^c
  - **The market price of the risky asset**

$$P_{t+1} = P_t[1 + \sigma_\epsilon \tilde{\epsilon}_t] + \frac{\mu}{2}[(1 + m_t)D_{f,t} + (1 - m_t)D_{c,t}].$$

- **Fundamentalists**
  - **Excess demand:**
$$D_t^f = \alpha(P_t^* - P_t).$$
  - **Fundamental price**
$$P_{t+1}^* = P_t^*[1 + \sigma_p \epsilon_t], \qquad \epsilon_t \sim \mathcal{N}(0, 1)$$
  - $\alpha > 0$ **is a combined measure of the aggregate risk tolerance of the fundamentalists and their reaction to the** *mis-pricing*

- **Chartists**
  - **Moving average**

$$ma_t^k = \frac{1}{k}\sum_{i=0}^{k-1} P_{t-i}, \quad (k \geq 1)$$

  - **Trading signals:**

$$\psi_t^{S,L} = ma_t^S - ma_t^L, \qquad L \geq S$$

In the following, we choose $S = 1$ and let $\psi_t^L = \psi_t^{1,L}$.

  - **Excess demand:**

$$D_t^c = h(\psi_t^{S,L}), \qquad h(0) = 0, \qquad h'(x) > 0, \qquad xh''(x) < 0$$

  - **In this lecture, we choose**

$$h(x) = \tanh(ax), \qquad a = h'(0) > 0$$

- **Fitness Measure and Population Evolution**
  - **The realized capital gains**

$$\pi_{f,t} = D^f_{t-1}(P_t - P_{t-1}) - C_f, \qquad \pi_{c,t} = D^c_{t-1}(P_t - P_{t-1}) - C_c$$

  - **The discrete choice model:**

$$n_{f,t} = \frac{e^{\beta U_{f,t}}}{e^{\beta U_{f,t}} + e^{\beta U_{c,t}}}, \qquad n_{c,t} = \frac{e^{\beta U_{c,t}}}{e^{\beta U_{f,t}} + e^{\beta U_{c,t}}}.$$

    * **Fitness:**

$$U_{f,t} = \pi_{f,t} + \eta U_{f,t-1}, \qquad U_{c,t} = \pi_{c,t} + \eta U_{c,t-1}$$

    * $\eta \in [0, 1]$ **measures the memory of the cumulated fitness function**
    * $\beta \geq 0$ **measures the switching intensity among the two strategies.**
      · $\beta = 0$: $m_t = 0$
      · $\beta = \infty$.

- **A Complete Asset Pricing Model**
  - **The market price of the risky asset is determined according to**

$$P_{t+1} = P_t[1+\sigma_\epsilon \tilde{\epsilon}_t] + \frac{\mu}{2}\left[(1+m_t)\alpha(P_t^* - P_t) + (1-m_t)h(\psi_t^L)\right]$$

  - **The difference of population fractions m_t evolves according to**

$$m_t = \tanh\left[\frac{\beta}{2}(U_t - C)\right]$$

  - **The cumulated fitness function satisfies**

$$U_t = [D_{t-1}^f - D_{t-1}^c][P_t - P_{t-1}] + \eta U_{t-1},$$

  - **The deterministic system: $\sigma_\epsilon = \sigma_p = 0$**

$$P_{t+1} = P_t + \frac{\mu}{2}\left[(1+m_t)\alpha(P^* - P_t) + (1-m_t)h(\psi_t^L)\right]$$

3 Stability and Bifurcation Analysis

- **The deterministic model is an $L + 2$ dimensional nonlinear difference system**
- **Steady state equilibrium: Assume $\eta \in [0, 1)$. Then there exists a unique steady state**

$$(P_t, m_t, U_t) = (P^*, m^*, 0)$$

where P^* is the constant fundamental price and $m^* = \tanh(-\beta C/2)$.

- **Let**

$$n_f^* = (1+m^*)/2, \quad n_c^* = (1-m^*)/2, \qquad \bar{\alpha} = \alpha\mu n_f^*, \quad \bar{a} = a\mu n_c^*$$

Then $n_f^* = n_c^* = 0.5$ when $C = 0$. In general, $n_c^* \geq n_f^*$ when $C \geq 0$.

- **Local Analysis**
  - **if $\bar{\alpha} = 1 + \bar{a}$, then the steady state price P^* is locally stable for $0 < \bar{a} < L$. At $\bar{a} = L$, there occurs a $1 : L + 1$ resonance bifurcation**
  - **a necessary condition for the steady state price to be stable is given by**

$$0 < \bar{a} < L, \qquad 0 < \bar{\alpha} < \begin{cases} 2 + \bar{a} & \text{for } L = 2l; \\ 2 + \frac{L-1}{L}\bar{a} & \text{for } L = 2l + 1. \end{cases}$$

  - **for all L, P^* is LAS if $(\bar{\alpha}, \bar{a}) \in D_S(\bar{\alpha}, \bar{a}) := \{(\bar{\alpha}, \bar{a}); 2\bar{a} < \bar{\alpha} < 2\}$**
  - **for sufficiently large L, P^* is unstable if $\bar{a} > \bar{\alpha}$**
  - **Stability region D_S and boundaries in population weighted reaction coefficients $\bar{\alpha}$ and $\bar{a}$.**

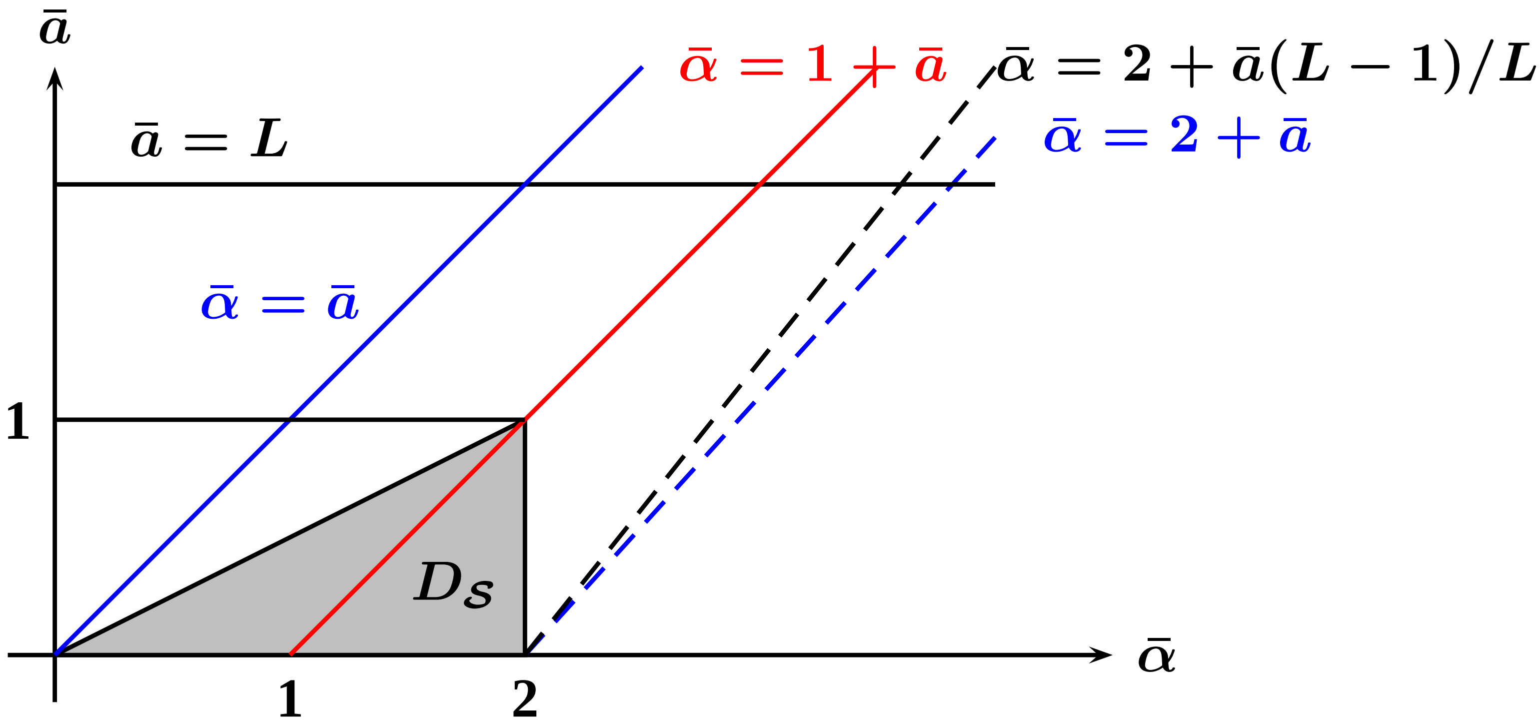
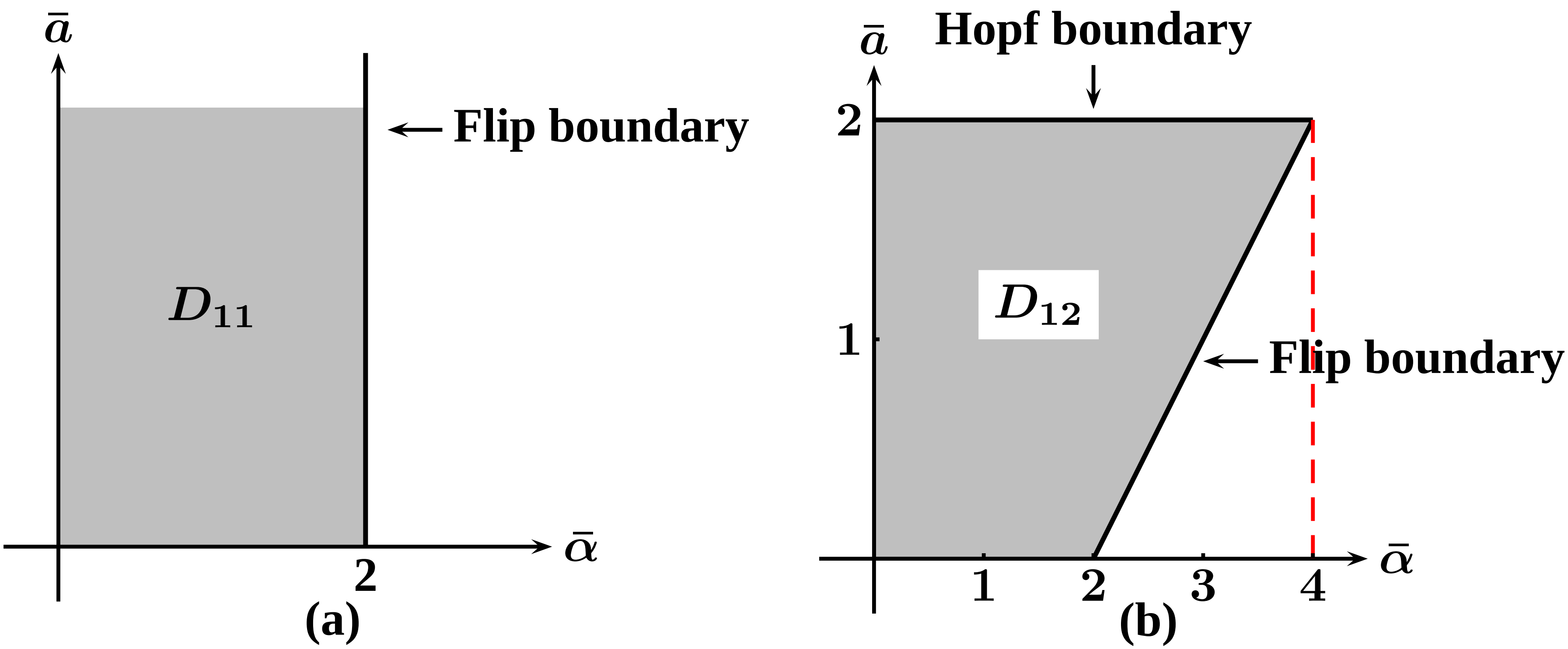
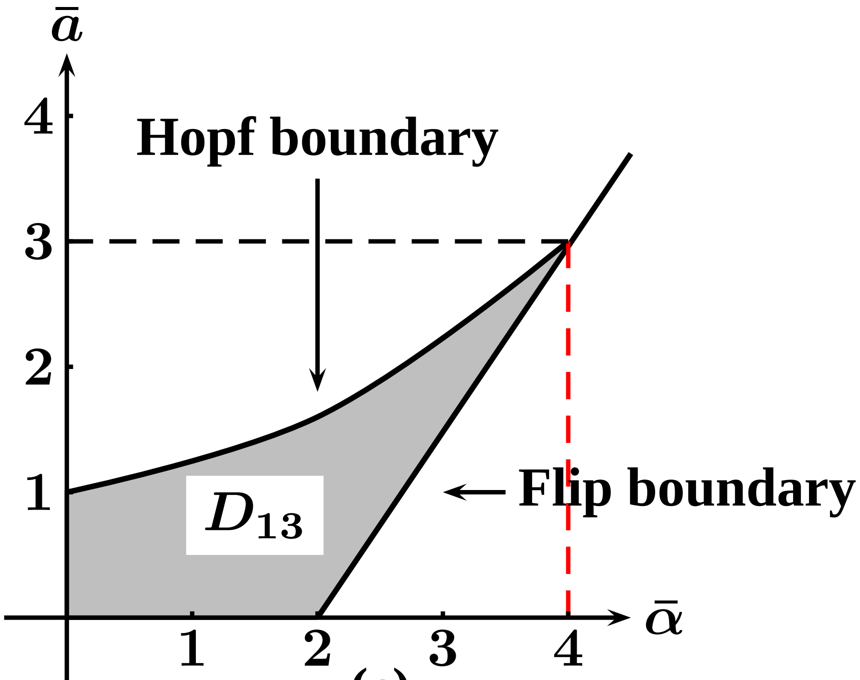


Figure 2: The common stability region $D_{\mathcal{S}}$ for general lag length L and necessary stability boundaries $\bar{a} = L$, $\bar{\alpha} = 2 + \bar{a}$ for even lag L and $\bar{\alpha} = 2 + \bar{a}(L-1)/L$ for odd lag L. On the horizontal (vertical) axis we have the population weighted reaction coefficient of the fundamentalists (chartists) at the steady state, i.e. $\bar{\alpha} := \alpha\mu n_f^*$, $\bar{a} := a\mu n_c^*$.

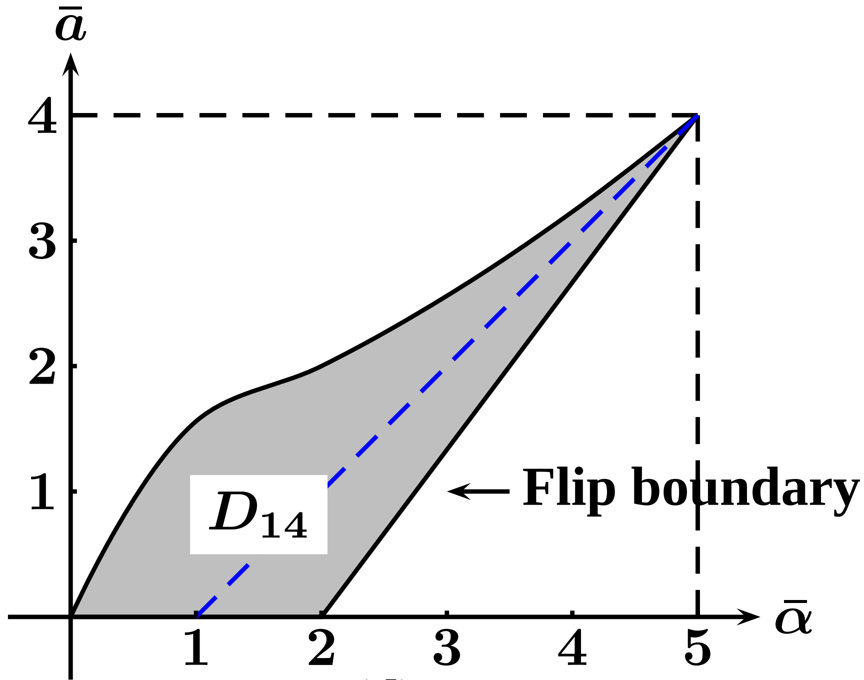
- **A comparison of the stability regions $D_L = D_{1L}$ for $L = 1, 2, 3$ and 4:**





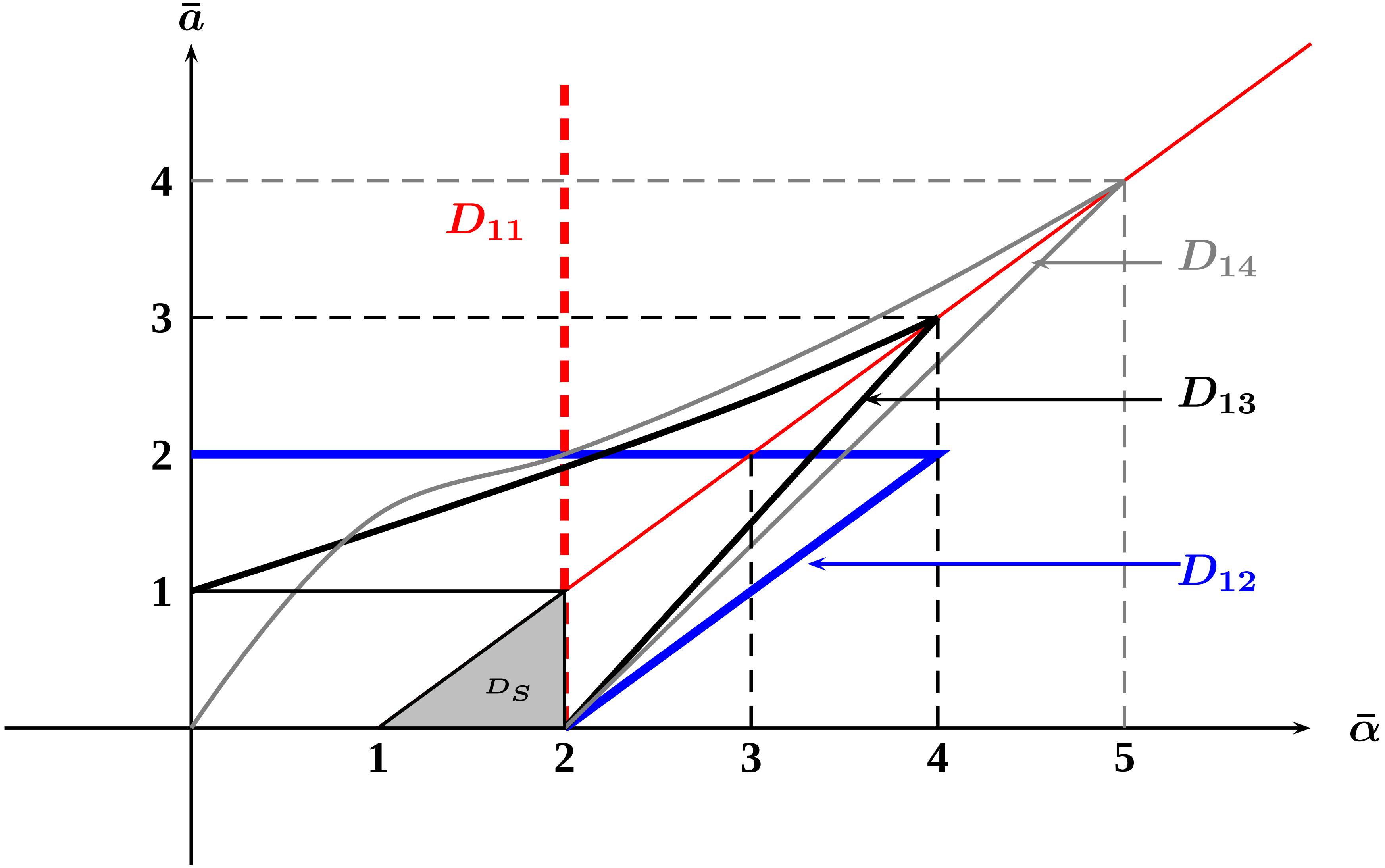
$\bar{a}$
4
3
2
1
Hopf boundary
D_{13}
Flip boundary
1
2
3
4
$\bar{\alpha}$

(c)



$\bar{a}$
4
3
2
1
D_{14}
Flip boundary
1
2
3
4
5
$\bar{\alpha}$

(d)


$\bar{a}$
4
3
2
1
D_{11}
D_{14}
D_{13}
D_{12}
D_S
1
2
3
4
5
$\bar{\alpha}$

- **Observations:**
  - **Along the line $\bar{\alpha} = 1 + \bar{a}$, for general L, the stability region is proportionally enlarged as the lag length of the MA process (L) increases. Intuitively, the linearization reduces to**

$$X_{t+1} = [1 + \bar{a} - \bar{\alpha}]X_t - \frac{\bar{a}}{L} \sum_{i=0}^{L-1} X_{t-i}. \tag{1}$$

  - **The stability region is NOT enlarged as the lag length L increases**
  - **Destabilizing effect: When the chartists become more active (so that $\bar{a} > \bar{\alpha}$), increasing in L destabilizes the system. Intuitively, as L increases, the MA becomes more smoother and sluggish and a small change in price leads to a relatively large change of chartists demand.**

- **Conjecture**— *as lag L increases, the stability region tends to shrink towards, but stretch along, the line $\alpha = 1 + \bar{a}$ with common stability region D_S.*

- **Other destabilizing effects**
  - **increase of the switching intensity β when $C > 0$**
  - **increase of C**

4 Dynamics of the Nonlinear System

4.1 The effect of switching intensity—rational routes to randomness

- **Parameters selection:**

$$\alpha = 1, \mu = 2, \eta = 0.2, a = 1, C = 1$$

- **Case $L = 4$**
  - P^* **is LAS for** $\beta = 0$
  - **Choose** $\beta = 0.2, 0.3, 0.49, 0.52, 0.555$ **and** 0.57
  - **As** β **increases,** P^* **becomes unstable, leading to complicated and even chaotic price dynamics**

- **Case $L = 100$**
  - **As conjectured, P^* is unstable even for $\beta = 0$**
  - **Choose $\beta = 0.05, 0.1, 0.2, 0.3, 0.35, 0.42, 0.45, 0.46$ and 0.4652**
  - **As β increases, Lorenz-like attractors appear and then merge into strange attractors**
  - **Time series:**
    - **Regular boom and bear markets**
    - **High intensity leads to high volatility**
    - **Upper and lower resistance levels**

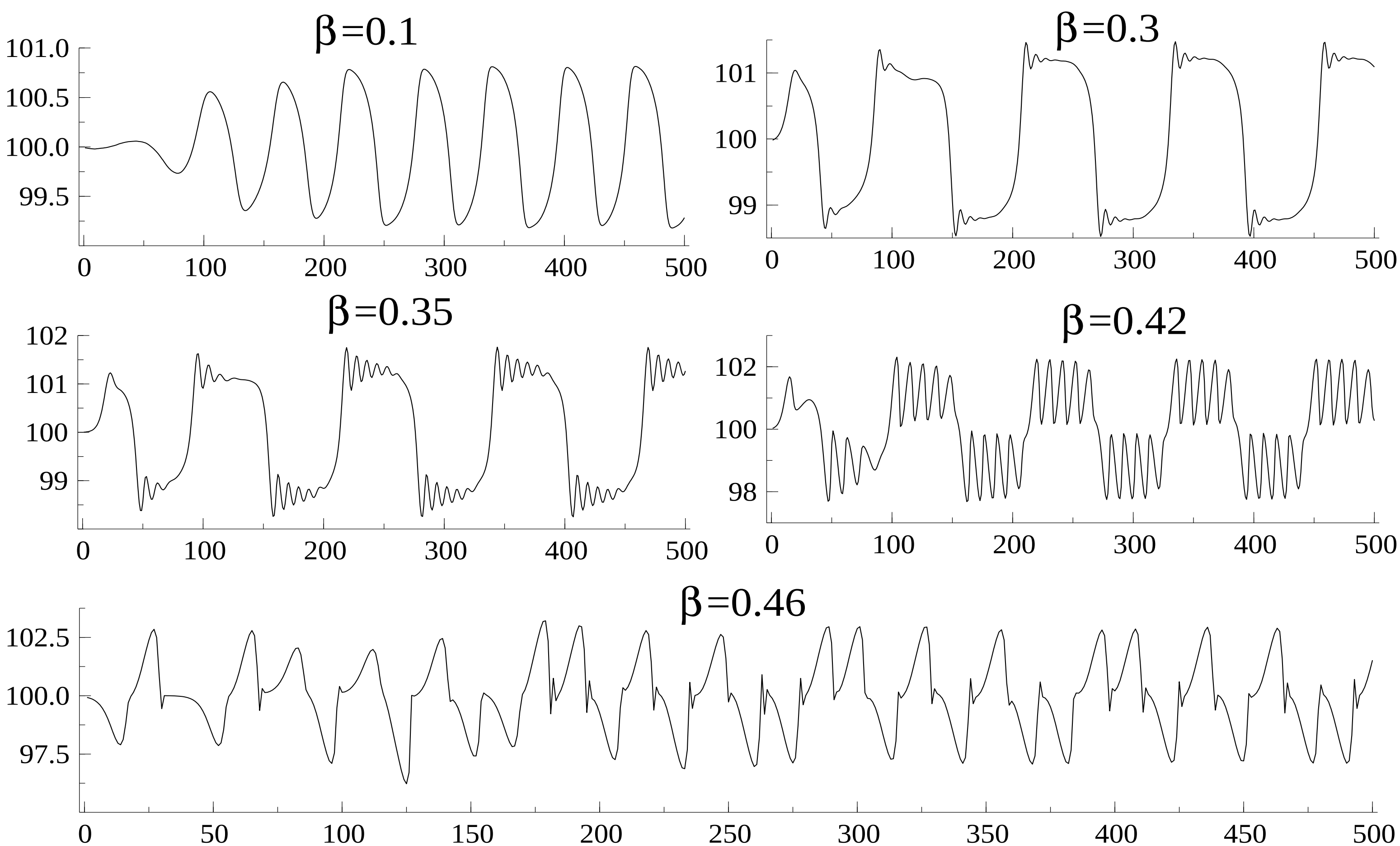


Figure 3: Price time series for $L = 100$ and $\beta = 0.1, 0.3, 0.35, 0.42, 0.46$

4.2 The effect of the lag length—Dynamics of moving average

- **Parameters selection:**

$$\alpha = 1, \mu = 2, \beta = 0.4, \eta = 0.2, a = 1, C = 0.$$

- **The fundamental price is locally stable for $L = 2, 3, 4$ and unstable for $L \geq 5$**
- **The destabilizing effect of L becomes more significant as L increases**

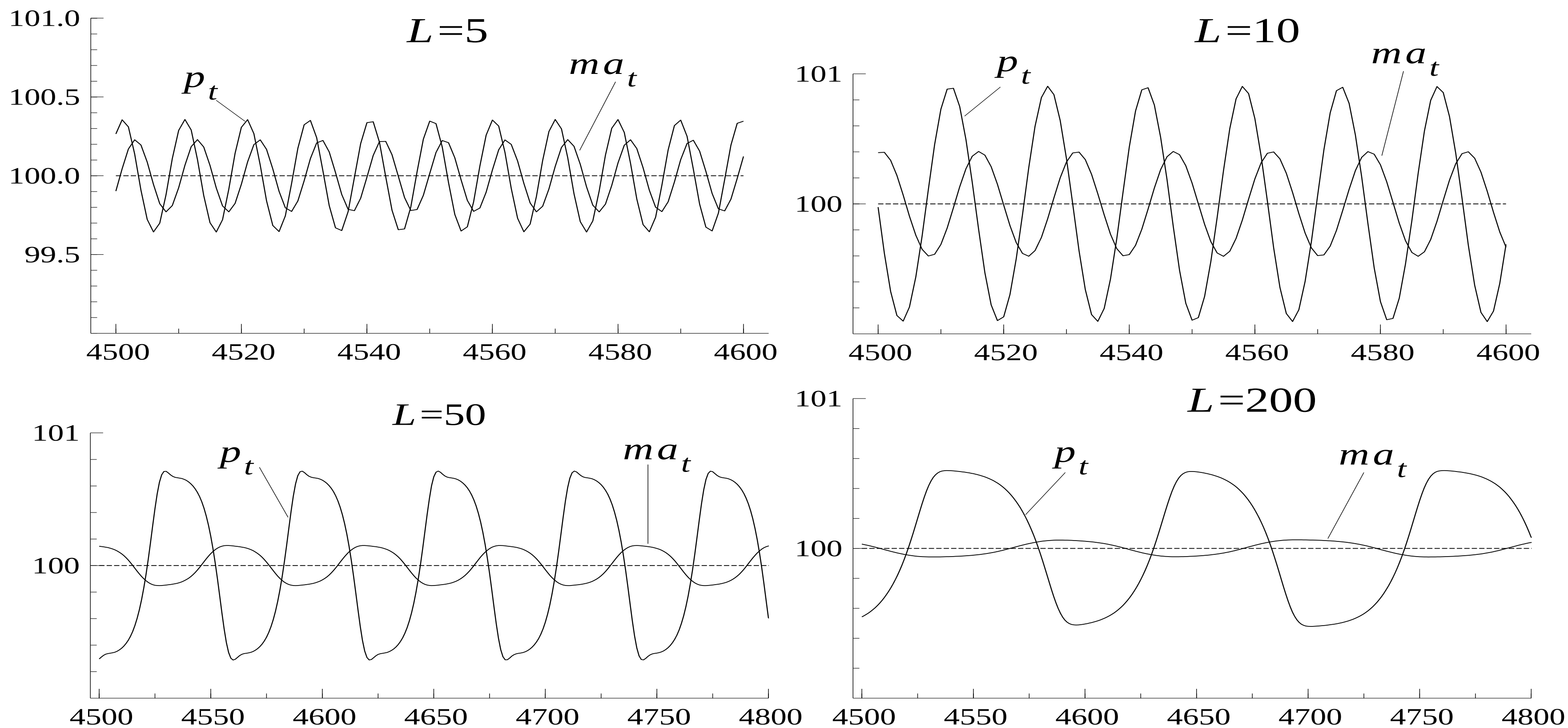


Figure 4: Price time series for fixed $\beta = 0.4$ and various $L = 5, 10, 50$ and 200.

- **Observations from time series:**
  - **Following the cross over of the long moving average and the market price, both the chartists and fundamentalists take the same long/short position initially. This push the price to either bull or bear market levels.**
  - **Limited long/short positions of the chartists reduce the marginal fitness, leading agents to switch back to the fundamental price**
  - **Over-reaction (switching) then builds an opposite trend, pushing price away from the fundamental price.**
  - **As L increases, the switching patterns between bull and bear markets become significant. Also, it takes longer time to change the price trend and bring the price towards the fundamental price level**
  - **This underlying mechanism can be used to explain the appearance of** *bubbles* **which grow and then burst.**

5 Dynamics of Stochastic System

- **The fundamental price follows a random walk**

$$P_{t+1}^* = P_t^*[1 + \sigma_\delta\, \delta_t], \tag{2}$$

where $\sigma_\delta \geq 0$ is a constant measuring the volatility of the return and $\delta_t \sim \mathcal{N}(0, 1)$. This specification ensures that relative price changes are stationary.

- **The stochastic model:**

$$P_{t+1} = P_t + \tilde{D}_t + \mu[n_{f,t}D_t^f + n_{c,t}D_t^c]. \tag{3}$$

- **Market noise: $\tilde{D}_t = \sigma_\epsilon \epsilon_t P_t$ or $\tilde{D}_t = \sigma_\epsilon \epsilon_t$**

5.1 Multiplicative noise

- **Parameter Selection:**

$$\alpha = 0.5, \beta = 0.3, a = 1, \mu = 1, \eta = 0.2, C = 1,$$

$$L = 100, P_0^* = P_0 = \$100, K = 250.$$

- **Four cases of noise combinations:**

  (a) Deterministic case: $(\sigma_\epsilon, \sigma_\delta) = (0, 0)$

  (b) Noise demand: $(\sigma_\epsilon, \sigma_\delta) = (0.5\%, 0)$

  (c) Noise fundamental: $(\sigma_\epsilon, \sigma_\delta) = (0, 5\%/K)$

  (d) Noise demand and fundamental: $(\sigma_\epsilon, \sigma_\delta) = (0.5\%, 5\%/K)$

- **Three time series: (A)** P_t^*, P_t, MA_t; **(B)** m_t; **(C)** D_t^f, D_t^c.

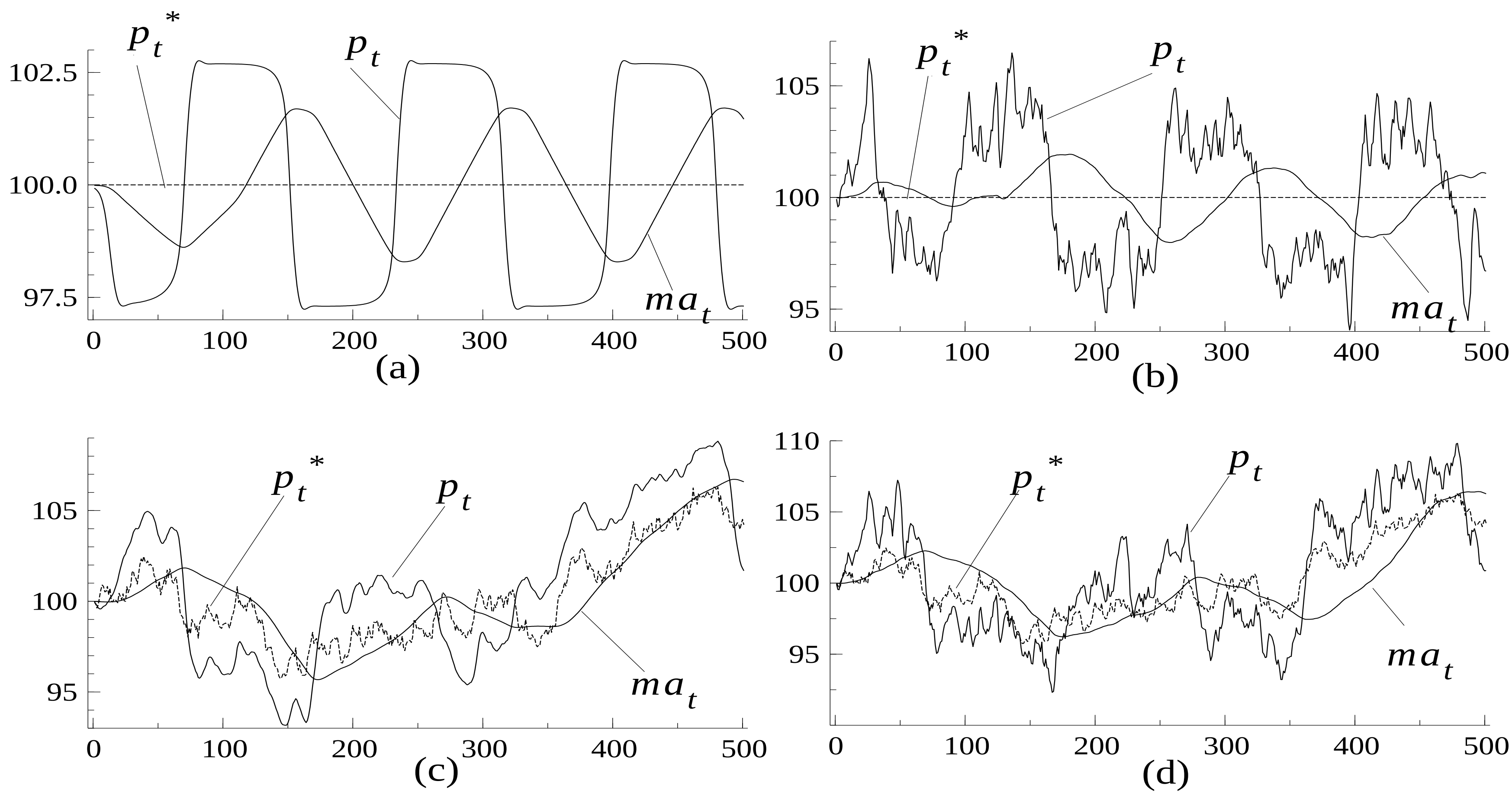


Figure 5: Time series of the prices (A)

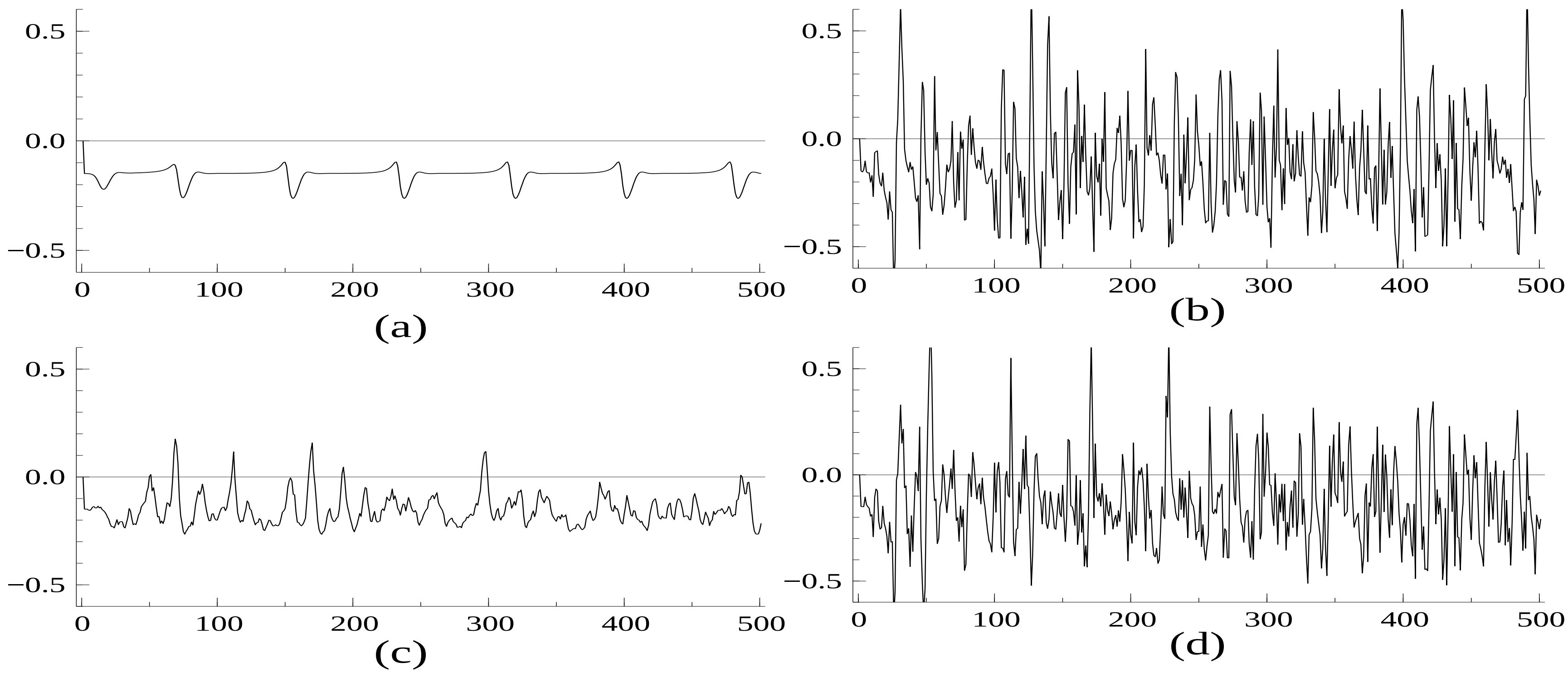


Figure 6: Population fraction differences (B)

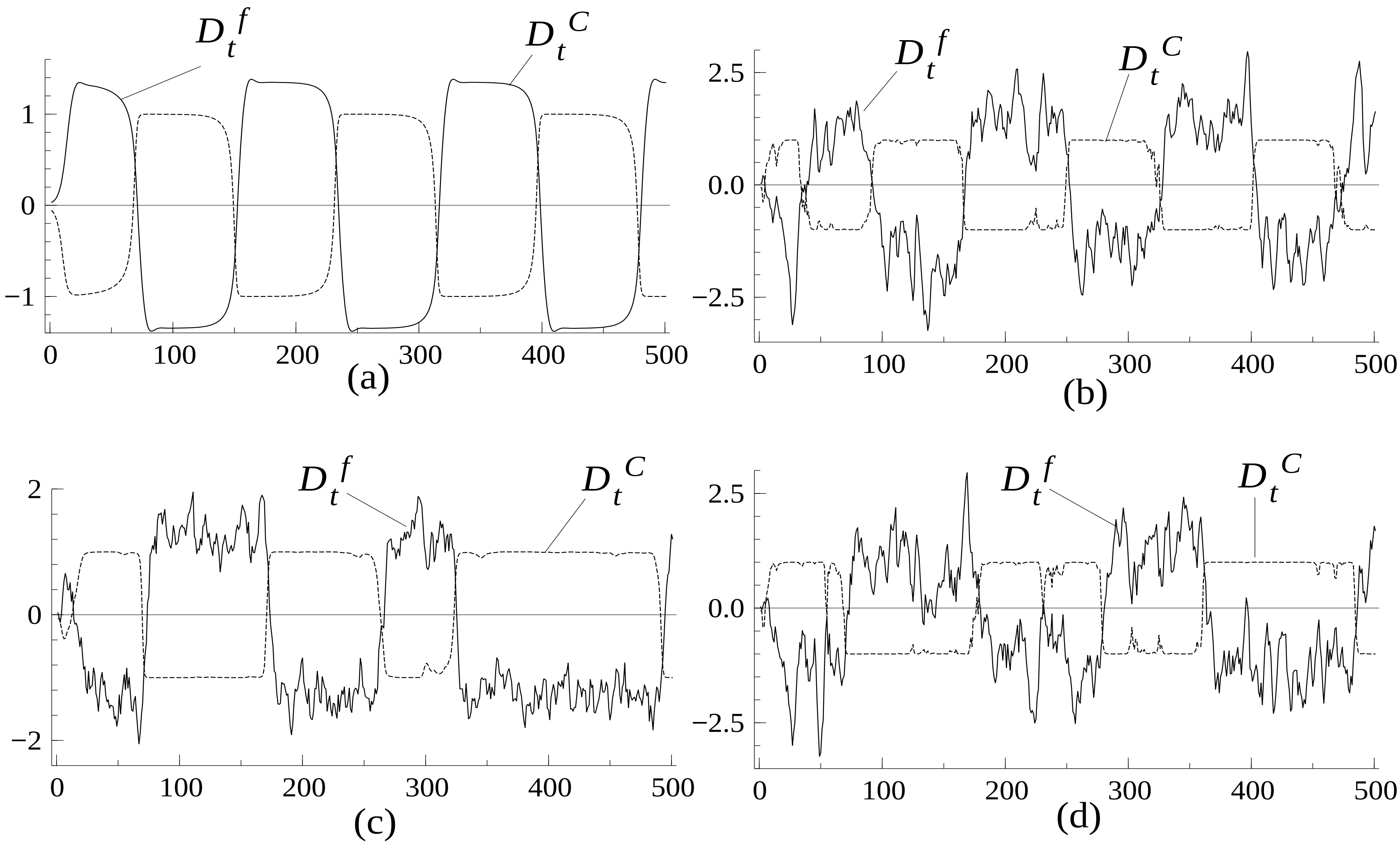
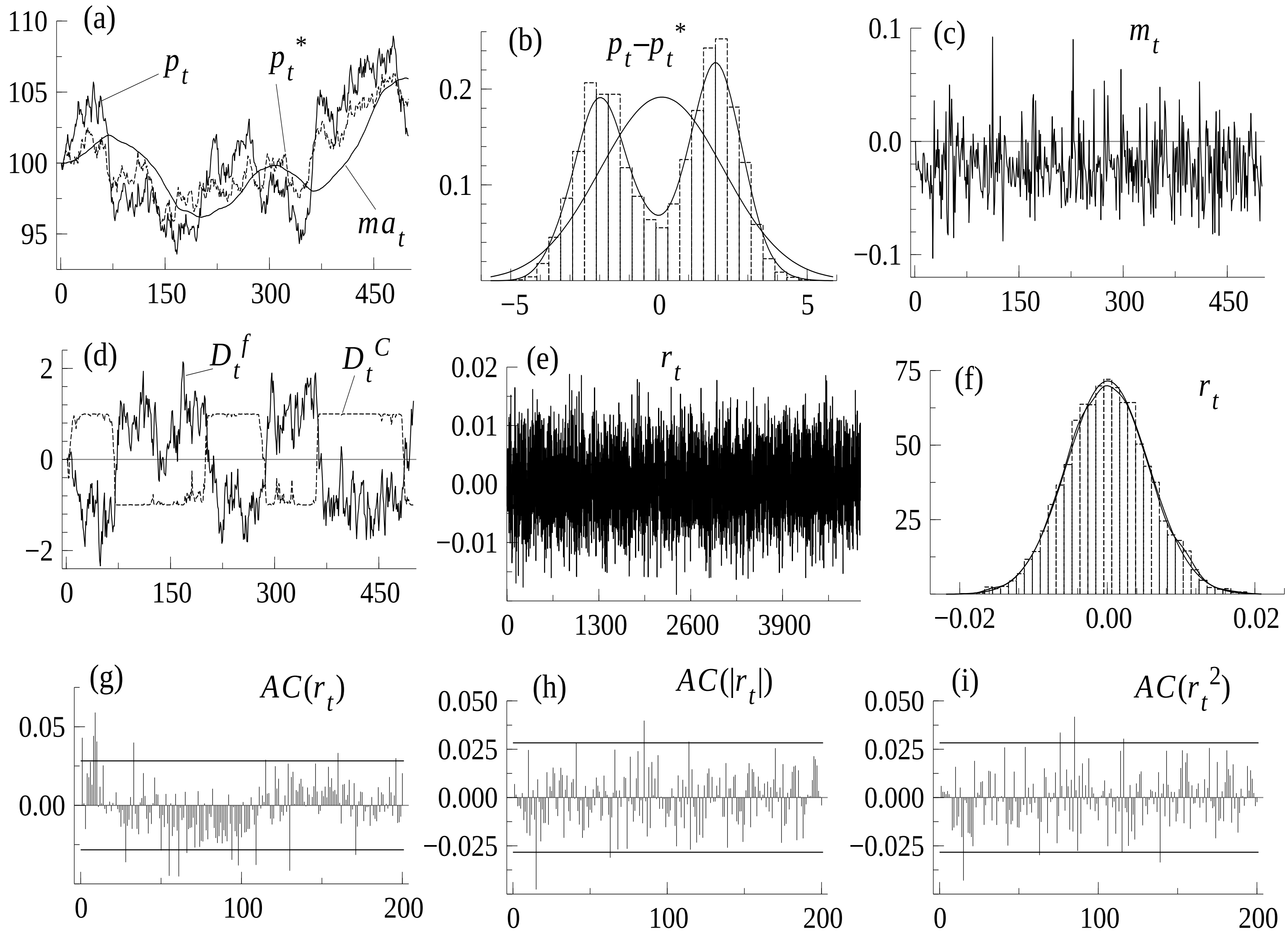


Figure 7: Demand functions (C)

- **Case (a):**
  - **The fundamentalists and the technical analysts take opposite (long/short) positions in most of the time period.**
  - **Such off-setting positions cause the price to stay bounded.**
  - **The market switches when both of them have the same position and such a transition happens very quickly.**
  - **The market is dominated by the technical analysts**
- **Case (b):**
  - **Because of the noisy demand, the market price becomes more volatile and the switching between two types of trading strategies is intensified.**
  - **The market price and the demand functions are still dominated by the underlying price dynamics of the deterministic case (a).**

- **Case (c):**
  - **The market price P_t closely follows the fundamental price P_t^***
  - **Traders tend to switch to fundamentalist analysis from time to time.**
  - **The market price is above (below) the fundamental price when the technical analysts take long (short) position.**
  - **The market price is still dominated by the technical analysts although it follows closely the fundamental price.**
- **Case (d):**
  - **Market price becomes more volatile,**
  - **It shares similar features as in the cases (b) and (c).**
  - **The market price follows the fundamental price and the market is dominated by technical analysts.**

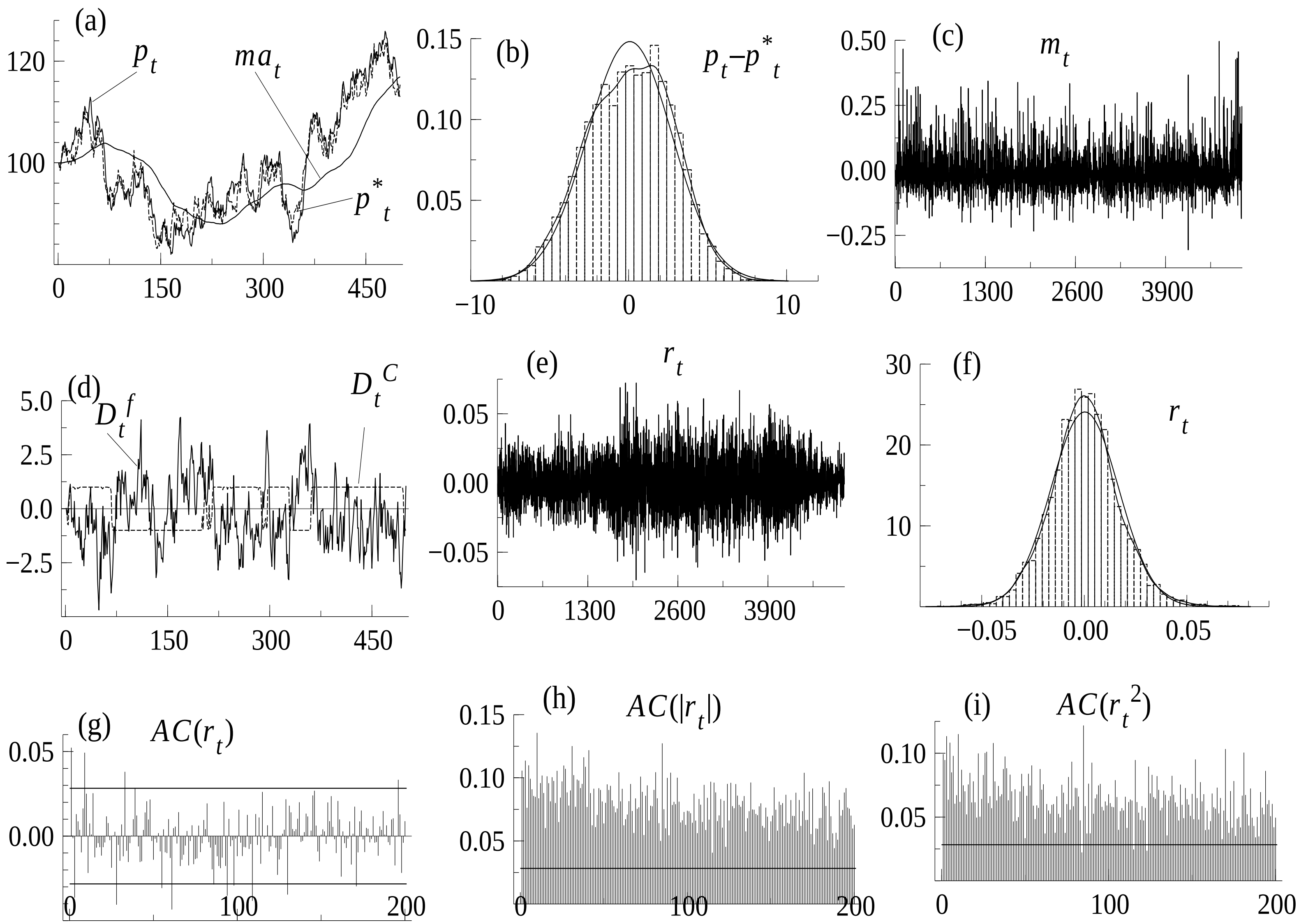
- **Observations:**
  (i) **Adding noisy demand can increase price volatility, but it has less impact on the price pattern and the market conditions of the underlying price dynamics.**
  (ii) **When the fundamental price follows a stochastic process, the market price closely follows the fundamental price.**
  (iii) **The market is mainly dominated by technical analysts. They may be the winners over short time periods (indicated by the switching), however over the whole time period they may be the losers in the sense that most of the time they seem to buy when the market prices are high and sell when the market prices are low.**
  (iv) **The switching between bull and bear markets happens when both types of traders take the same position, a very intuitive result. Such transitions can be intensified with the help of the noise traders, leading to temporary market bubbles and sudden crashes.**


(a)
p_t
p_t^*
ma_t
(b)
$p_t - p_t^*$
(c)
m_t
(d)
D_t^f
D_t^C
(e)
r_t
(f)
r_t
(g)
$AC(r_t)$
(h)
$AC(|r_t|)$
(i)
$AC(r_t^2)$

- **Time series, distributions and autocorrelations (ACs)**
  - **The market price follows the fundamental price in general**
  - **Most of the time, the fundamentalists and trend followers take opposite positions.**
  - **The cross-overs of the market price and the moving average immediately pushes the market price away from the fundamental price over long time periods (close to 100 days).**
  - **The bi-model distribution of the difference $p_t - p_t^*$.**
  - **The returns r_t are close to being normally distributed.**
  - **The market can appear to be quite efficient even though it is dominated by technical analysts and the market price is consistently pushed away from the fundamental price—a very interesting result.**
  - **It provides a basis for the existence of upper and lower price resistance levels in an apparent efficient market.**

5.2 Additive Noise and Long Memory

- **Long memory—insignificant ACs for the raw returns but significant ACs for the absolute and squared returns.**
- **In this case, the bi-model distribution disappears.**
- **For small σ_δ, for example 5%, it is found that the fundamental price is less volatile and the market returns are close to a normal distribution.**
- **However, for $\sigma_\delta = 20\%$, the fundamental price is more volatile, leading to a non-normal distribution of the market returns with some skewness and high kurtosis.**
- **A high volatility of the fundamental price can reduce the market efficiency, generate the long memory feature as well as the skewness and kurtosis observed in asset returns.**


(a)
p_t
ma_t
p^*_t
(b)
$p_t - p^*_t$
(c)
m_t
(d)
D_t^f
D_t^C
(e)
r_t
(f)
r_t
(g)
$AC(r_t)$
(h)
$AC(|r_t|)$
(i)
$AC(r_t^2)$

6 Conclusion and Remarks

- **Price fluctuations are driven by evolutionary switching between different expectation schemes.**
- **Various rational routes to randomness**
- **Intuitively one might expect that a long run MA smoothes the price dynamics and hence an increase of the lag length of the MA might be expected to stabilize the market.**
- **Surprisingly, this intuition is only true when both the reaction coefficient of the fundamentalists and the extrapolation rate of the trend followers are balanced in a certain way.**

- **In general, as the lag length L increases, the MA becomes smoother and more sluggish. A small change in the price leads to a relatively large increase of chartists demand and consequently, the lag length of the MA rule can destabilize the market price.**
- **When the fundamental price follows a stochastic process, the market price closely follows the fundamental price.**
- **The switching between bull and bear markets happens when both types of traders take the same position**

- **Different forms of the noise demand created by the noise traders can have a different impact on the price dynamics.**
- **When the noise trader demand is proportional to the market price, the price dynamics are dominated by the underlying price dynamics of the deterministic model. In addition, the market can appear efficient even though the market is dominated by technical analysts and the market price is consistently pushed away from the fundamental price.**
- **When the noise trader demand is independent of the market price, some stylized facts, including skewness, high kurtosis, and possible long memory, can be generated when the volatility of the fundamental price is high.**

- **Future research:**
  - **theoretic analysis of the interaction of the non-linear and stochastic elements, including different forms of the noise.**
  - **study a more realistic model of the market with a large number of different trading rules, in particular with agents using different MA strategies of various length, or other types of technical trading rules used in financial practice.**