Linkages Among Equity Markets Using Basket Currencies and Directed Acyclic Graphs

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Introduction

- The purpose of the present study is to examine the stability of interrelationships between nine national equity markets using different base currencies.
- Following previous literature, we use local currencies in each of these nine countries.
- We also consider the impact of using the local currency of each country as the base currency to measure equity market returns in the nine countries.
- Additionally, unlike previous studies, we apply different *basket currencies* to the measurement of equity returns.

Why Basket Currencies?

Basket currencies are advantageous in studying equity market co-movements because they are relatively stable in value over time when compared to local currencies and, therefore, mitigate the potentially confounding influence of exchange rate movements on equity market returns.

Markets and Methods

- We study equity market co-movements between the U.S., Canada, U.K., Germany, Switzerland, Italy, Belgium, Japan, and Hong Kong.

- These countries were selected because they represent a broad cross section of North American, European, and Asian markets for which daily data are available for our sample period from 1973 to early 2003.

- Following Bessler and Yang (2003), we employ the empirical method of Directed Acyclic Graphs (DAGs) to comparatively examine the structure of causality among national equity markets in contemporaneous time.

Preview of Results

- In brief, our results show that measures of equity returns using basket currencies provide more robust estimates of equity markets' volatility relative to measures obtained using individual local currencies no matter which individual base is used.

- Based on standard deviations of equity indexes in different individual currencies, we find that changes in exchange rates can understate (overstate) equity market variations depending upon whether a currency is appreciating (depreciating) over time relative to a basket of currencies.

- Studies on equity market interrelationships can avoid local currency dependent results by using basket currencies.

- The basket currency approach should provide a clearer understanding of the "true" co-movements between equity indexes by minimizing the influence of local currencies.

Basket Currencies

- After the breakdown of the Bretton Woods system in the early 1970s, many currencies of major industrial countries began to float on world currency markets. This event led to the development of currency basket indexes as a benchmark with which to evaluate local currency movements.
- For example, the U.S. Federal Reserve Board has constructed a variety of basket currency indexes, including the G-10 index, Broad index, major currency index (*MCI*), and other important trading partners (*OITP*) index.
- The International Monetary Fund (IMF) has developed well-known basket currencies, such as the Special Drawing Right (*SDR*) and the Multilateral Exchange Rate Model (MERM).

SAC

- Since basket currencies comprised of major industrial nations’ currencies are much more stable over time than individual local currencies, they are suitable for measuring local currency movements.
- Recent work by Hovanov, Kolari, and Sokolov (2004) has proposed a Stable Aggregate Currency (*SAC*) that is a basket currency constructed to have minimum variance.
- The authors employ a mean-variance framework as in Markowitz (1959) to solve for the optimal weights on local currencies in a currency basket. They find that *SAC* is more stable over time than the *SDR* from 1981-1998.

Currencies Studied

- The local currencies studied are: United States (dollar, or *USD*), Canada (dollar, or *CAD*), United Kingdom (pound sterling, or *GBP*), Germany (mark, or *DEM*), Switzerland (franc, or *CHF*), Italy (lira, or *ITL*), Belgium (franc, or *BEF*), Japan (yen, or *JPY*), and Hong Kong (dollar, or *HKD*).

- For basket currencies we employ the Fed's trade weighted index of seven major currencies *MCI* (see the Federal Reserve Board's website at www.federalreserve.gov), as well as Hovanov, Kolari, and Sokolov's (2004) recently proposed minimum variance weighted index *SAC* .

Normalized Currency Values

- Hovanov, Kolari, and Sokolov (2004 *JEDC*, page 1486) compute normalized currency values as follows:

$$NVAL_{ij}(t) = C_{ij}(t) / \sqrt[n]{\prod_{r=1}^{n} C_{rj}}(t)$$

- where $NVAL_{i,j}$ is the normalized value of a currency, $C_{i,j}$ (*t*) is a rate of exchange of the ith currency for the jth currency at time *t*, and the jth currency is the base currency.

- The denominator above is the geometric mean of values, where *n* is the number of currencies (i.e., equal to nine in this study).

- Since the normalized value in exchange $NVAL_{i,j}$ is independent of the base currency, $NVAL_{i,j}$ is set equal to $NVAL_{i}$.

- This last number essentially rescales, for example, U.S. dollars valued in terms of any local base currency (e.g., yen, pounds, etc.) to the same set of values and, therefore, is invariant to the choice of base currency.

Computation of SAC

- The composite currency will have the following variance, when weights w_i and w_k are used for combining:

$$S^2(w) = \sum_{i,k=1}^{n} w_i w_k \operatorname{cov}(i,k) = \sum_{i=1}^{n} w_i^2 s_i^2 + 2 \sum_{\substack{i,k=1 \\ i<k}}^{n} w_i w_k \operatorname{cov}(i,k)$$

- The SAC minimizes $S^2(W)$, subject to the constraint that the weights, w_i , i=1, …, n, sum to one.

- We can use the optimal weights to derive an optimal portfolio of international currencies, which will be our basket SAC currency, see Hovanov, Kolari, and Sokolov (2004) for details.

Table 1. Standard Deviations of Normalized Values for Nine Simple Currencies and Two Basket Currencies

Currency	Standard Deviation
USD	0.097
CAD	0.118
GBP	0.110
DEN	0.189
CHF	0.386
ITL	0.176
BEF	0.102
JPY	0.576
HKD	0.185
MCI	0.042
SAC	0.015

Interpretation of Table 1

Table 1 shows that over the sample period normalized values of *SAC* had the lowest standard deviation followed by the normalized *MCI*.

With the exception of *SAC* and *MCI*, the normalized values of *USD* and *BEF* are relatively stable.

By contrast, the normalized values of *JPY* and *CHF* are quite volatile compared to the other currencies.

Equity Prices

- Any equity price index (*SI*) denominated in a local currency can be redefined in terms of any numeraire (or base) currency as:

$$SI_{it}^{numeraire} = SI_{it}^{currency} \times \frac{numeraire_t}{currency_{it}}$$

- which can then be transformed into the logarithm (ln) form as,

$$\ln SI_{it}^{numeraire} = \ln SI_{it}^{currency} + \ln \frac{numeraire_t}{currency_{it}}$$

- where is the i^{th} equity price index denominated in a numeraire currency, i = 1,…,9 represents our nine countries, and t = 1,…,7870 (i.e., from January 1, 1973 to February 28, 2003).

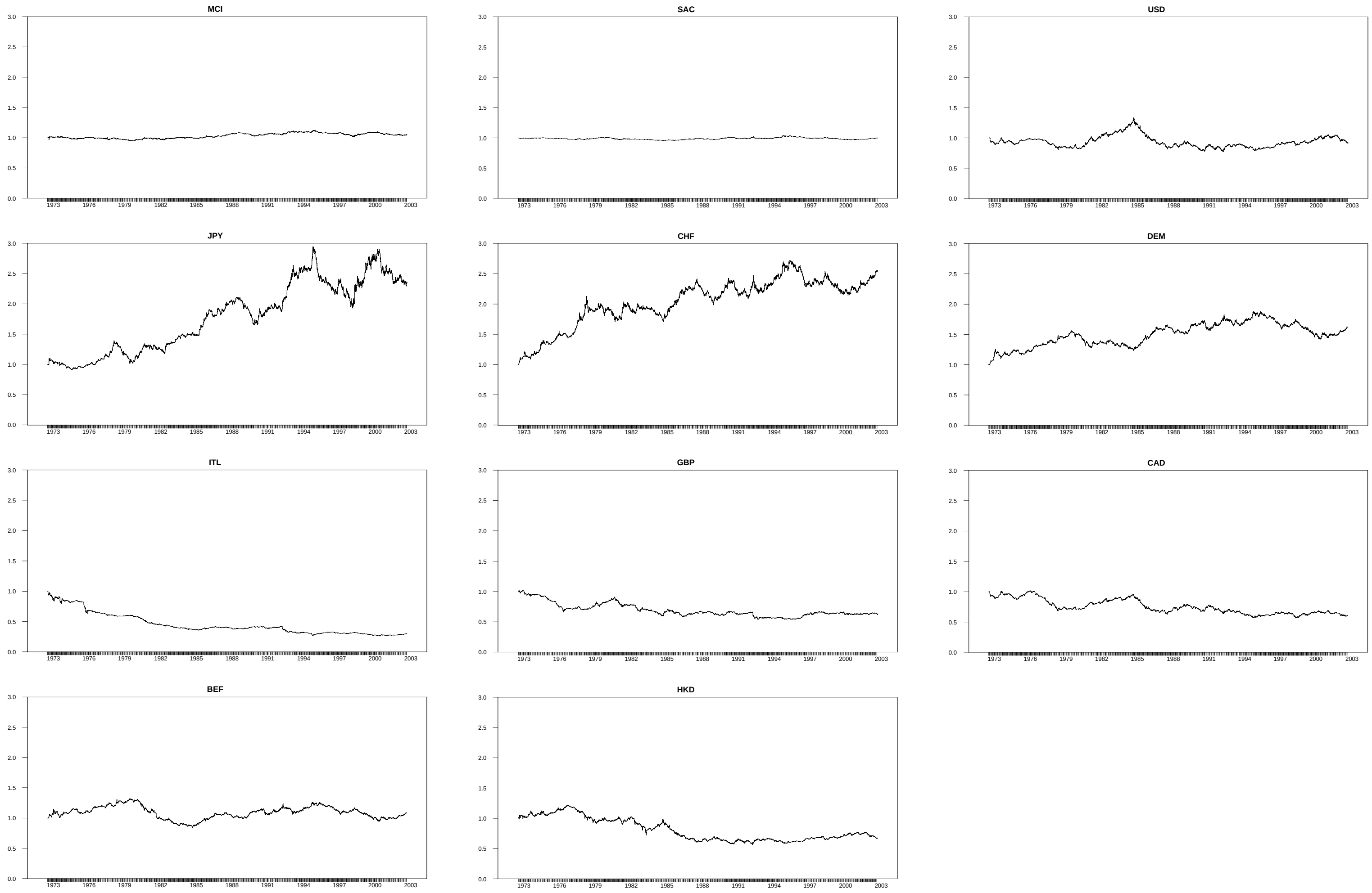
Investor's Risks

Investors who purchase a foreign equity face not only the price risk measured in terms of the local currency, but foreign exchange risk also. We can show the two risks with the variance of logarithm equity price indexes (log SI). The variance of ln SI can be written as:

$$
\begin{aligned}
Var(\ln SI_{it}^{numeraire}) &= Var(\ln SI_{it}^{currency}) \\
&+ Var(\ln(numeraire_t / currency_{it})) \\
&+ 2Cov(\ln SI_{it}^{currency}, \ln(numeraire_t / currency_{it}))
\end{aligned}
$$

The investor faces risk due to uncertainty associated with the equity valued in a local currency, uncertainty associated with his/her local currency and world currencies and their covariances.

Plots of Currencies over Time



Standard Deviations of Equity Indexes using Different Base Currencies

Equity Market	SAC	US $	HK $	JPN Y
US	0.393 (4)	0.397 (4)	0.469 (4)	0.265 (4)
UK	0.312 (5)	0.324 (5)	0.399 (5)	0.179 (5)
HK	0.476 (1)	0.487 (1)	0.555 (1)	0.343 (1)
JPN	0.402 (3)	0.416 (3)	0.495 (2)	0.273 (3)
GER	0.413 (2)	0.424 (2)	0.494 (3)	0.282 (2)

Standard Deviations of Exchange Rates using Different Base Currencies

Currency	SAC	US $	HK $	JPN Y
US $	0.048 (4)	--	0.089 (3)	0.163 (3)
UK P	0.066 (3)	0.073 (3)	0.060 (4)	0.207 (2)
HK $	0.098 (2)	0.089 (2)	---	0.238 (1)
JPN	0.148 (1)	0.163 (1)	0.238 (1)	---
SAC	---	0.052 (4)	0.111 (2)	0.133 (4)

Co-variances Between Local Equity Indexes and Exchange Rates

Co-variance	SAC	US $	HK $	JPN Y
US	-.003 (5)	---	.027 (3)	-.057 (3)
UK	-.020 (3)	-.017 (3)	.011 (4)	-.072 (2)
HK	-.045 (1)	-.039 (1)	---	-.123 (1)
JPN	.033 (2)	.036 (2)	.057 (1)	---
GER	.014 (4)	.016 (4)	.041 (2)	-.036 (4)

Tests of Significant Differences in Covariance of Returns:

Null Hypothesis is Covariance in SAC = Covariance in currency i

Currency	Box M Test Statistic	P-Value
MCI	70.82	.01
Local	776.77	.00
US $	584.82	.00
CAN $	731.42	.00
GB P	387.15	.00
DEM (GER)	316.28	.00
CHF (SWISS)	704.97	.00
BEF (BELGIUM)	402.79	.00
ITL L	411.27	.99
HK $	840.88	.00
JPN Y	858.82	.00

Interpretation of Box M-Test (previous slide)

The table shows that at the 0.005 significance level we fail to reject the null hypothesis that the covariance matrices in *MCI* and *SAC* are equal, since the test statistic value is 70.820 and the (chi-squared) critical value is about 73.128 at the 0.005 level of significance (i.e., our sample size exceeds 7000 observations).

We reject the null hypothesis that the covariance matrices in *SAC* and other base currencies are equal as suggested by the small p-values.

Directed Acyclic Graphs

- Pictures summarizing the causal flow among variables -- there are no cycles.

- Inference on causation is informed by asymmetries among causal chains, causal forks, and causal inverted forks.

A Causal Fork

For three variables X, Y, and Z, we illustrate X causes Y and Z as:

$$Y \longleftarrow X \longrightarrow Z$$

Here the unconditional association between Y and Z is non-zero, but the conditional association between Y and Z, given knowledge of the common cause X, is zero.

Knowledge of a common cause screens off association between its joint effects.

An Example of a Causal Fork

- X is the event, the student doesn't learn the material in Econ 629.
- Y is the event, the student receives a grade of "D" in Econ 629.
- Z is the event, the student fails the PhD prelim in Economic Theory.

Grades are helpful in forecasting whether a student passes his/her prelims: $P(Z \mid Y) > P(Z)$

If we add the information on whether he/she understands the material, the contribution of grade disappears (we do not know candidate's name when we mark his prelim): $P(Z \mid Y, X) = P(Z \mid X)$

An Inverted Fork

- Illustrate X and Z cause Y as:

$$X \longrightarrow Y \longleftarrow Z$$

- Here the unconditional association between X and Z is zero, but the conditional association between X and Z, given the common effect Y is non-zero:

Knowledge of a common effect does not screen off the association between its joint causes.

The Causal Inverted Fork: An Example

- Let Y be the event that my daughter's cell-phone won't work
- Let X be the event that she did not pay her phone bill
- Let Z be the event that her battery is dead

Paying the phone bill and the battery being dead are independent: $P(X|Z) = P(X)$.

Given I know her battery is dead (she remembers that she did not charge it for a week) gives some information about bill status: $P(X|Y,Z) < P(X|Y)$.
(although I don't know her bill status for sure).

$$X \rightarrow Y \leftarrow Z$$

The Literature on Such Causal Structures Has Been Advanced in the Last Decade Under the Label of Artificial Intelligence

- Pearl , *Biometrika*, 1995
- Pearl, *Causality*, Cambridge Press, 2000
- Spirtes, Glymour and Scheines, *Causation, Prediction and Search*, MIT Press, 2000
- Glymour and Cooper, editors, *Computation, Causation and Discovery*, MIT Press, 1999

Causal Inference Engine

- PC Algorithm

1. Form a complete undirected graph connecting every variable with all other variables.

2. Remove edges through tests of zero correlation and partial correlation.

3. Direct edges which remain after all possible tests of conditional correlation.

4. Use screening-off characteristics to accomplish edge direction.

Assumptions

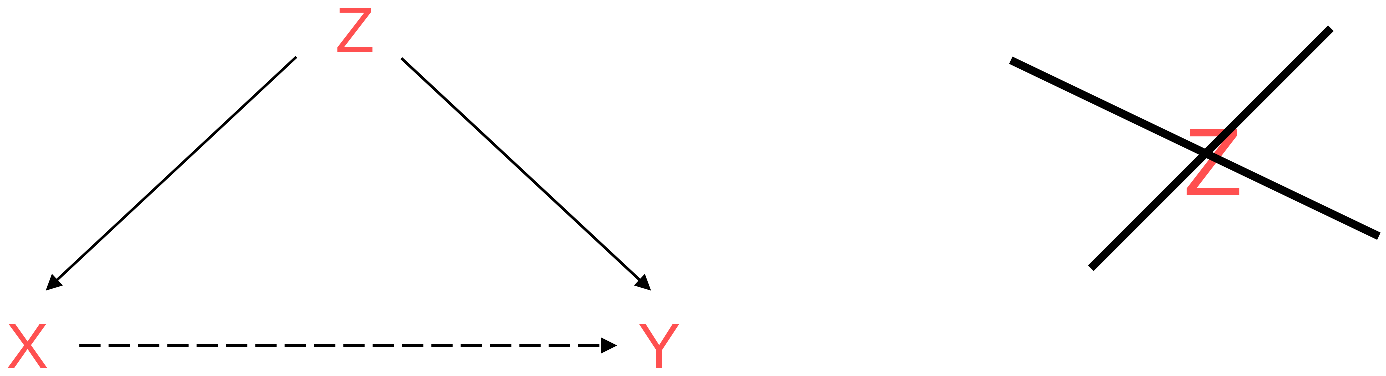
(for PC algorithm on observational data to give same causal model as a random assignment experiment)

1. Causal Sufficiency
2. Causal Markov Condition
3. Faithfulness
4. Normality

Causal Sufficiency

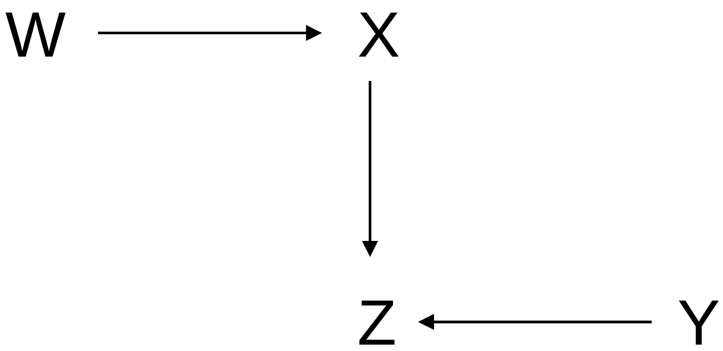
No two included variables are caused by a common omitted variable.

No hidden variables that cause two included variables.



Causal Markov Condition

The data on our variables are generated by a Markov property, which says we need only condition on parents:

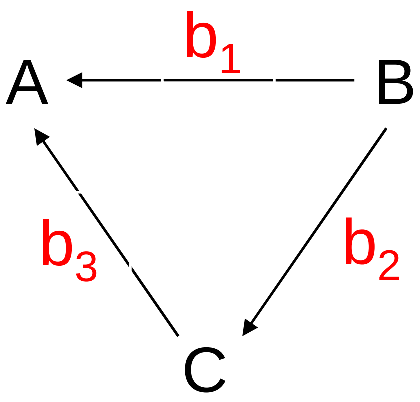


$$P(W, X, Y, Z) = P(W) \bullet P(X|W) \bullet P(Y) \bullet P(Z|X,Y)$$

Faithfulness

There are no cancellations of parameters.

$A = b_1 B + b_3 C$
$C = b_2 B$



It is not the case that: $-b_2 b_3 = b_1$

Deep parameters b_1, b_2 and b_3 do not form combinations that cancel each other.

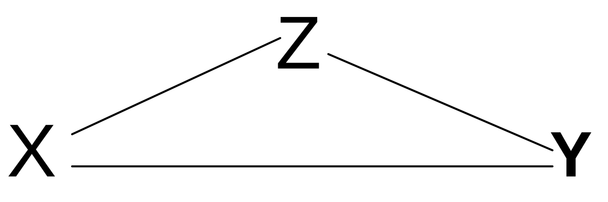
PC Algorithm

Here will present one algorithm which can be used to build directed graphs. The algorithm starts systematically from a complete undirected graph and removes edges (lines) between vertices based on correlation or partial correlation between vertices.

Spirtes, Glymour and Scheines (1993) have incorporated the notion of d-separation into an algorithm (PC Algorithm) for building directed acyclic graphs, using the notion of *sepset* (defined below).

A Complete Undirected Graph
(gets us started)

One forms a complete undirected graph G on the vertex set V. The complete undirected graph shows an undirected edge between every variable of the system (every variable in V). Edges between variables are removed sequentially based on zero correlation or partial correlation (conditional correlation).



Here X, Y, and Z are connected with lines having no arrows

Remove Edges Using Correlation or Conditional Correlation

- Each edge is subjected to tests that the correlation between its endpoints is zero:
- Ho: 👎(X, Y) = 0 ?

- If a correlation is judged to be not different from zero, we remove the edge between the two end points of the corresponding edge.
- Edges surviving these unconditional correlation tests are then subjected to conditional correlation tests:
- Ho: 👎(X, Y | Z) = 0 ?
- If these conditional correlations equal zero pick up the edge X, Y.

Fisher's Z

Fisher's *z* statistic can be applied to test for significance from zero:

$z(\rho(i,j|k)n) =$

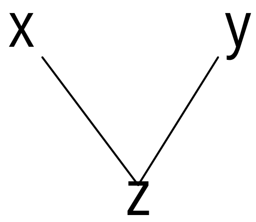
$1/2(n-|k|-3)^{1/2} \times \ln\{(|1 + \rho(i,j|k)|) \times (|1 - \rho(i,j|k)|)^{-1} \}$.

n is the number of observations used to estimate the correlations, ρ(i,j|k) is the population correlation between series i and j conditional on series k (removing the influence of series k on each i and j), and

|k| is the number of variables in k (that we condition on).

Sepset

The conditioning variable(s) on removed edges between two variables is called the *sepset* of the variables whose edge has been removed (for vanishing zero order conditioning information the *sepset* is the empty set).



If we remove the edge between x and y through unconditional correlation test, 👎(x,y)=0, then the sepset (x,y) is {♓}.

If we remove this edge by conditioning on z, 👎(x,y| z)=0 then the sepset (x,y) is z.

Edge Direction

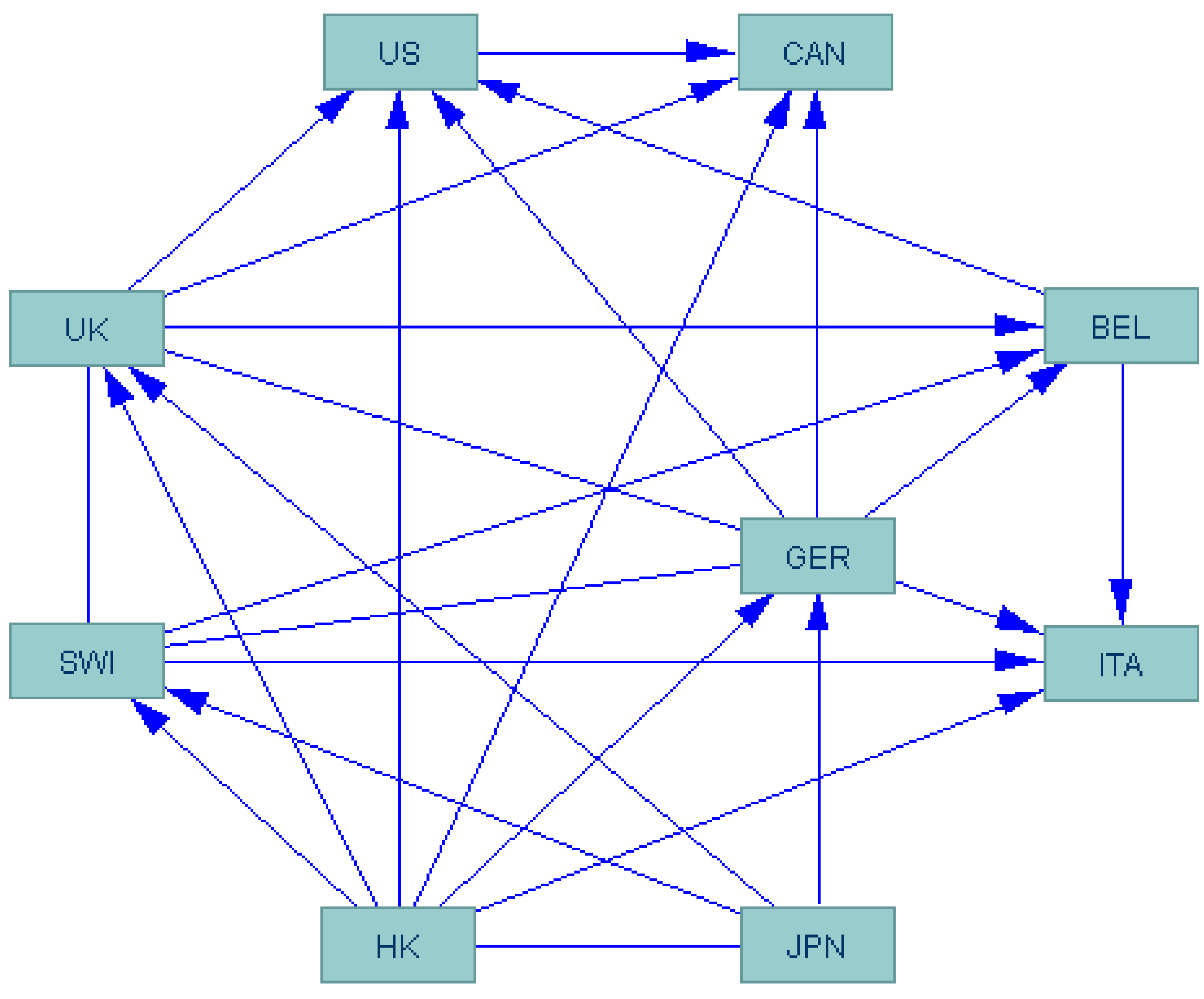
Edges are directed by considering triples, such that *X* and *Y* are adjacent as are *Y* and *Z*, but *X* and *Z* are not adjacent:

X – Y – Z.

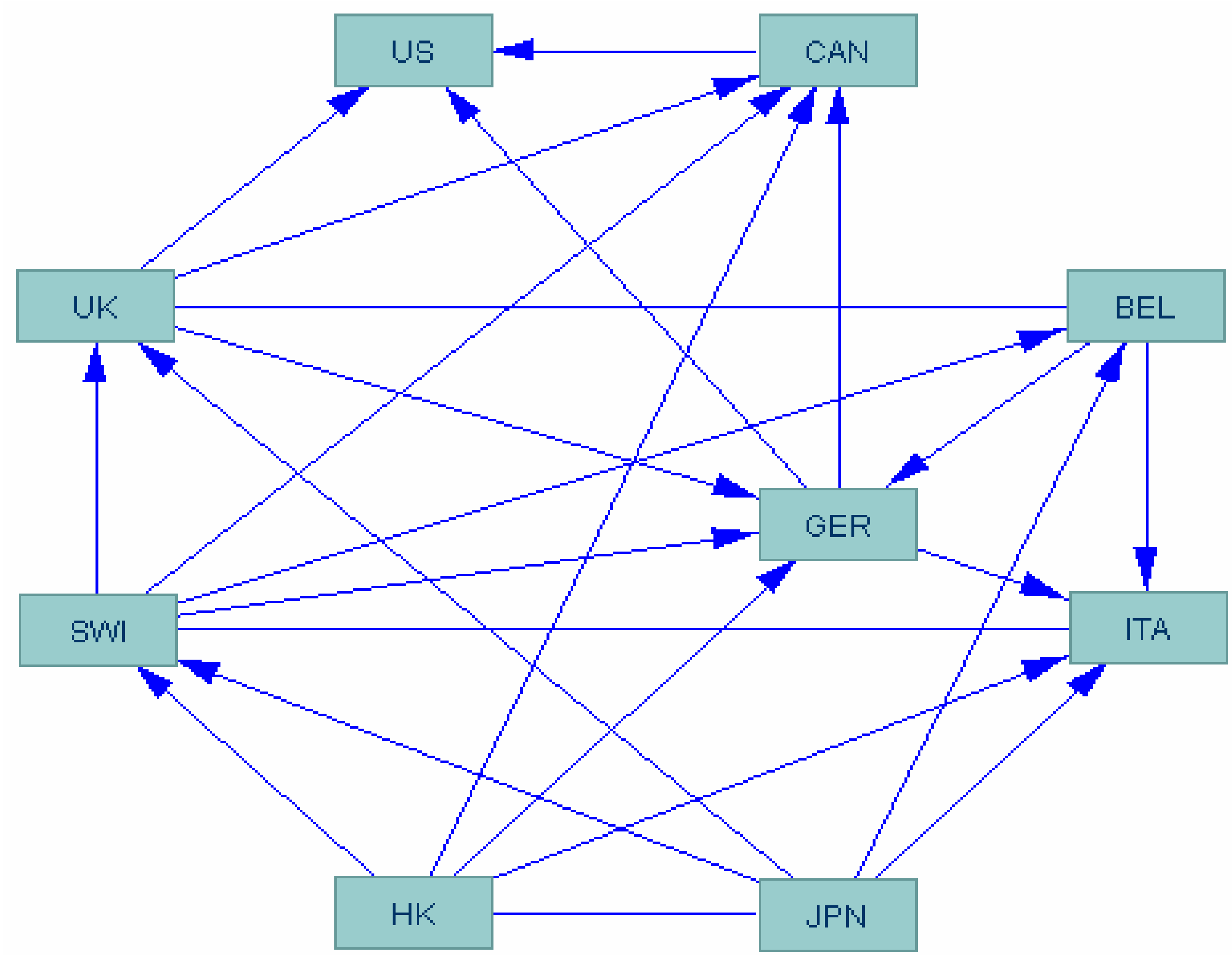
Direct the edges between triples: if Y is not in the sepset of *X* and *Z*. If , *Y* and *Z* are adjacent, *X* and *Z* are not adjacent, and there is no arrowhead at *Y*, then orient as X → Y ← Z

If there is a directed path from *X* to *Y*, and an edge between *Y* and *Z*, then direct (Y— Z) as: Y→ Z .

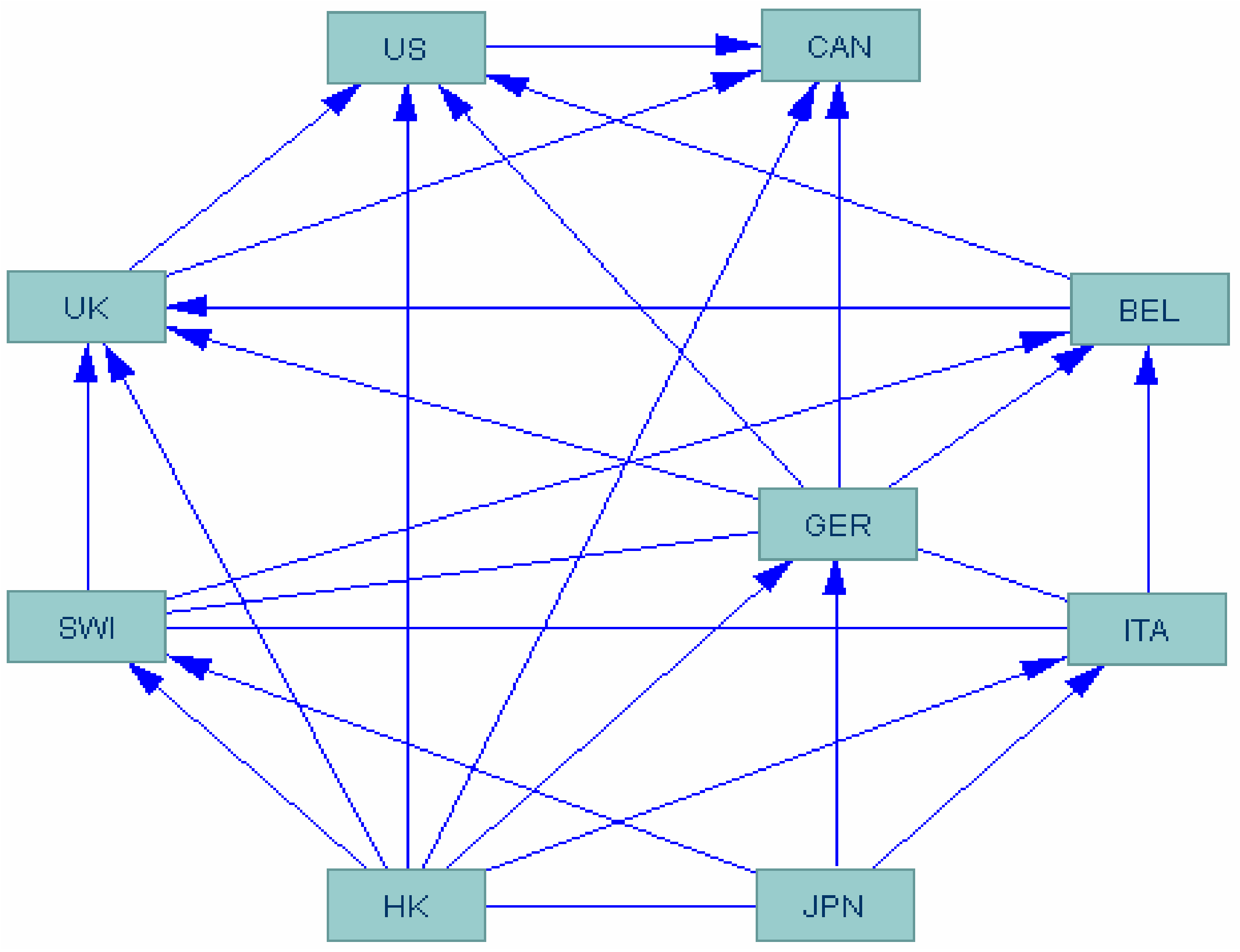
DAG Based on SAC



DAG Based on US Dollars



DAG in MCI



Summary

- This paper investigates the dynamic interdependence structure of nine national equity markets during the post-Bretton Woods era from January 1, 1973 to February 28, 2003.

- Unlike previous studies on the linkages among equity markets, we seek to examine the effects of base currency choice on market interrelationships.

Summary Continued

- Based on the standard deviations of equity indexes, we demonstrate that movements in home currencies can understate or overstate the market variations depending upon whether a home currency is appreciating or depreciating over time.
- By converting equity index prices denominated in local currencies to different basket currencies, the exchange rate risk associated with currency fluctuations can be substantially minimized.