Extending Genetic Programming with Recombinative Guidance

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  - Correlation-based Guidance for Symbolic Regression
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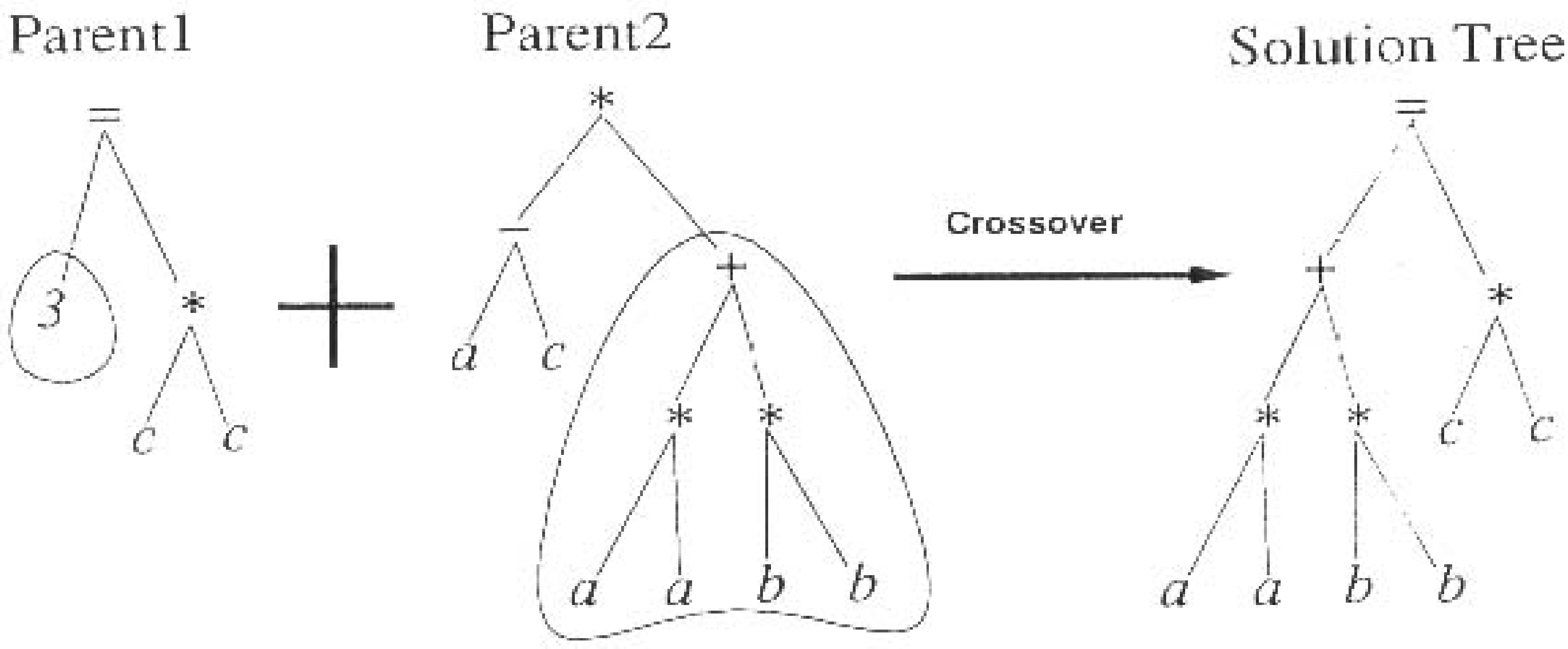
Motivation

- In traditional GP, recombination causes disruption of beneficial building-blocks, and mutation causes abrupt changes in the semantics. Example.
- Propose a "recombinative guidance" mechanism for GP so as to realize an "adaptive recombination".
- In GA
- In GP

Example

$a^2+b^2=c^2$



In GA

- Schaffer(1987)
  - Use a string feature called “punctuation”, which decides the crossover points of multi-point crossover.
- Back(1991, 1992, 1993)
  - Propose an adaptive control scheme for a mutation operator. Use a bit string to represent a mutation rate for each allele.

In GP

- Koza(1990, ch.4. 12.2)
  - Use a constrained crossover operator to evolve Neural Networks.
- D’haeseleer(1994)
  - The crossover operators attempt to preserve the context in which subtrees appeared in the parent tree.
- Haynes(1995)
  - Propose a strongly typed GP.
- Whigham(1995)
  - Use a context free grammar to control crossover and mutation operators for GP.

Recombinative Guidance for GP

- S-values(subtree values)
  - Denote：SV(T), for a subtree T
  - Performance-based S-value
  - Correlation-based S-value
  - MDL-based S-value
- Use S-values to decide which subtree will be chosen.

S-value

- S-values are sorted in ascending order $(\overline{\mathrm{SV}})$
  - The larger S-value, the better subtree
- S-values are sorted in descending order $(\underline{\mathrm{SV}})$
  - The smaller S-value, the better subtree
- For subtrees T and T', T $\preceq$ T' denoted that T' is a better building-block candidate than T.
- If the S-value of T' is better than that of T, then

$$\mathrm{T} \preceq \mathrm{T'} \Leftrightarrow \overline{\mathrm{SV}}(\mathrm{T}) \leq \overline{\mathrm{SV}}(\mathrm{T'})$$

or

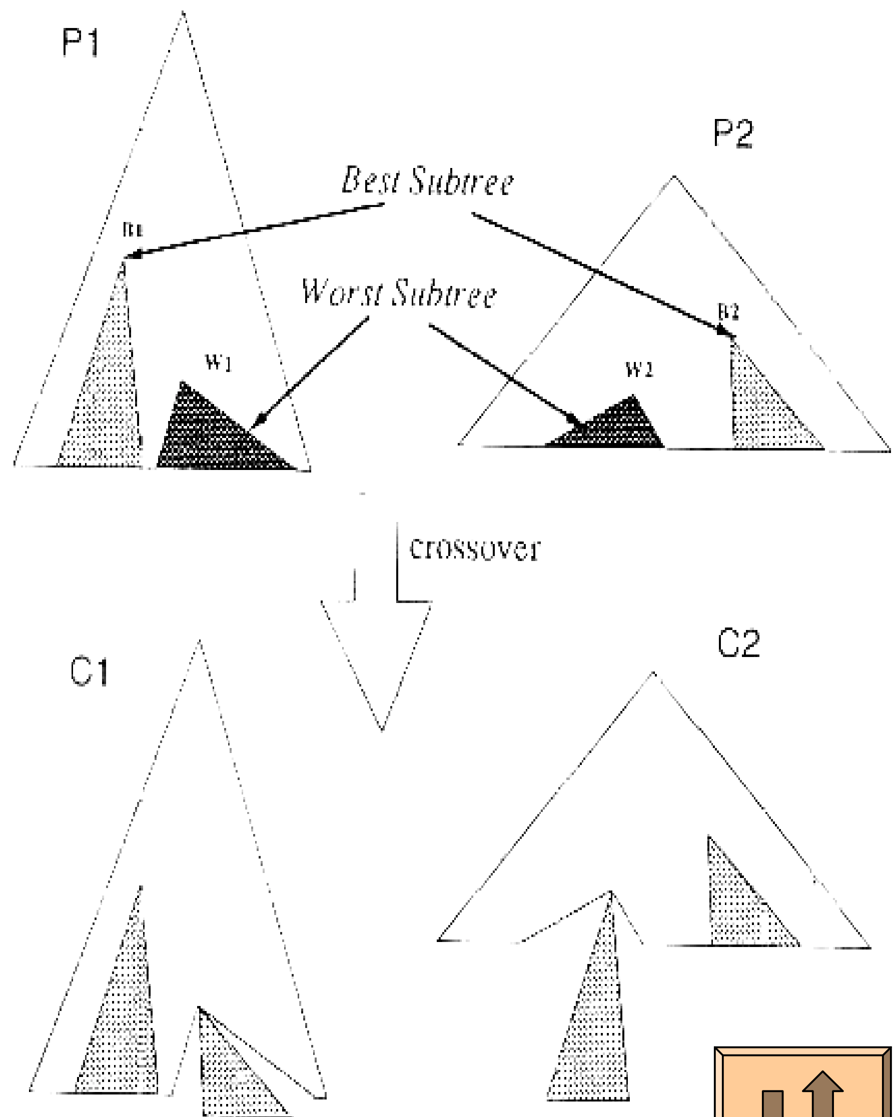
$$\mathrm{T} \preceq \mathrm{T'} \Leftrightarrow \underline{\mathrm{SV}}(\mathrm{T}) \geq \underline{\mathrm{SV}}(\mathrm{T'})$$

Process

- Apply a mutation operator to a subtree whose S-value is *worse*.
- Apply a crossover operator to a subtree whose S-value is *worse*, and get a subtree whose S-value is better from another parent.
- Example

Example

- Let W_1 and W_2 be the subtrees with the worst S-values of P_1 and P_2.
- Let B_1 and B_2 be the subtrees with the best S-values of P_1 and P_2.
- A new child C_1 is a copy of P_1, in which W_1 is replaced by B_2.
- A new child C_2 is a copy of P_2, in which W_2 is replaced by B_1.



Performance-based Guidance

- Experiment 1
  - The Lawnmower Problem
  - (Koza 1994, ch.8)
- Experiment 2
  - Artificial Ant on the San Mateo Trail
  - (Koza 1994, ch.12)

The Lawnmower Problem

- Goal：To find a program for controlling the movement of a lawnmower so that it cuts all the grass in the yard.
- Discrete 8x8 toroidal square area
- Fitness & Termination criterion
- Parameters
- S-value
- Result

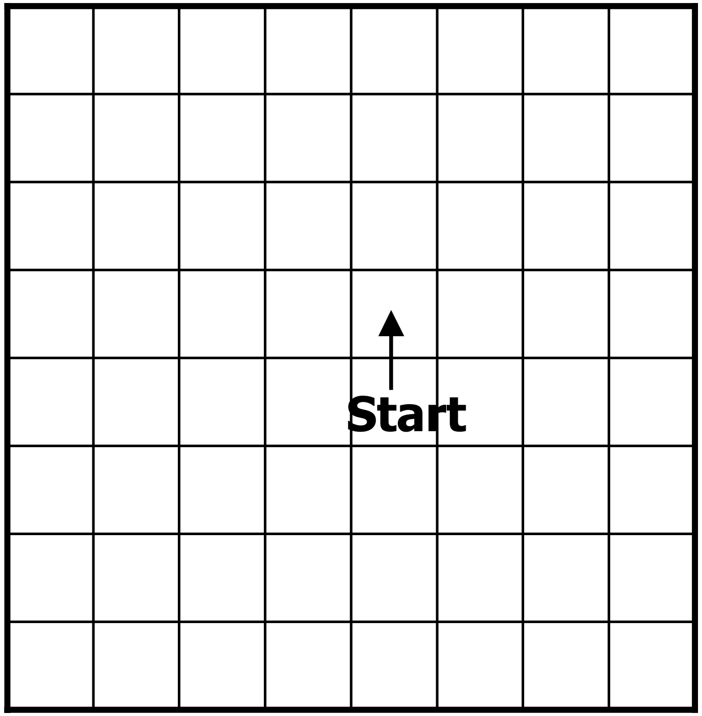
Area

- LEFT

  Turn the orientation of the lawnmower counterclockwise by 90 degrees.

- MOW

  Move the lawnmower in the direction it is currently facing and mows the grass.



Fitness & Termination criterion

- Fitness
  - Raw fitness：the amount of grass mowed
  - Standard fitness：64－(Raw fitness)
- Termination criterion：

  Either the lawnmower has executed a total of 100 LEFT turns or 100 MOVEs.

Parameters

Terminal Set	{MOW, LEFT}
Function Set	{PROG2, PROG3}
Population Size	120
Crossover Prob.	0.7
Mutation Prob.	0.033

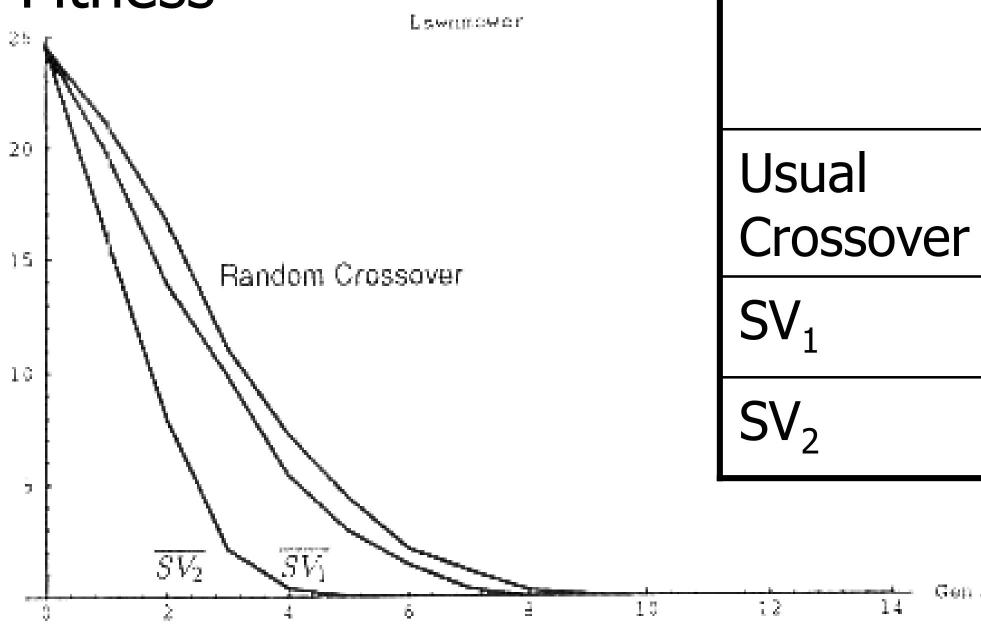
S-value

- $$\overline{SV_1}(T) = \frac{\text{raw fitness}(T)}{\text{raw fitness}(ROOT)}$$
  - It is not defined for the root node.
  - It does not reflect the complexity of the program.
- Complexity-based S-value

$$\overline{SV_2}(T) = \frac{\text{raw fitness}(T)}{\text{raw fitness}(ROOT)} + \frac{\#N(ROOT) - \#N(T)}{\#N(ROOT)}$$

Result

Std. Fitness



Generation

Method	Number of nodes	
	Average	Variance
Usual Crossover	603.00	95.76
SV_1	655.53	120.27
SV_2	496.27	50.99

Artificial Ant on the San Mateo Trail

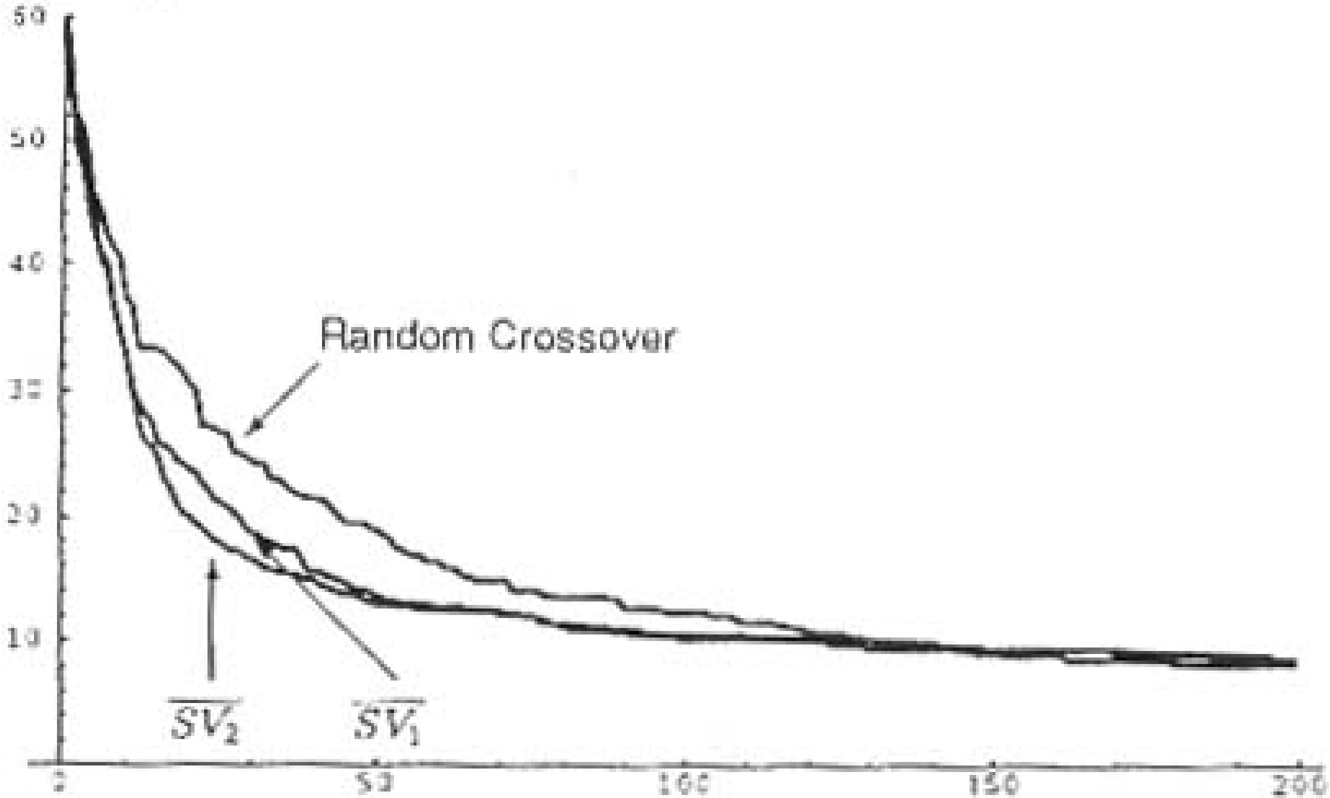
- Goal
  - To find a program for controlling the movement of an artificial ant so as to find all of the food lying along an irregular trail on a two-dimensional toroidal grid.
- Area：13x13 grid
- Raw fitness：the sum of food pieces eaten
- Parameters & Result

Parameters

Terminal Set	{RIGHT, LEFT, MOVE}
Function Set	{IF_FOOD_AHEAD, PROGN}
Population Size	1200
Crossover Prob.	0.7
Mutation Prob.	0.033

Result

Std. Fitness



Gen.

Correlation-based Guidance for Symbolic Regression

- S-value
- Experiment 3
  - A simple symbolic regression
- Experiment 4
  - Discovery of trigonometric identities

S-value

- $\underline{SV_3}(T) = \frac{1}{N}\sum_{i=1}^{N}(y_i - t_i)^2$
- $\overline{SV_4}(T) = |r(T)|$

$$r(T) = \frac{S_{yT}}{\sqrt{S_{TT} \cdot S_{yy}}}$$

$$S_{yy} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})^2$$

$$S_{TT} = \frac{1}{N}\sum_{i=1}^{N}(t_i - \bar{t})^2$$

$$S_{yT} = \frac{1}{N}\sum_{i=1}^{N}(t_i - \bar{t})(y_i - \bar{y})$$

- $-1 \le r(T) \le 1$
- A value of r(T) near zero indicates that the variable y_i and t_i are uncorrelated.

A simple symbolic regression

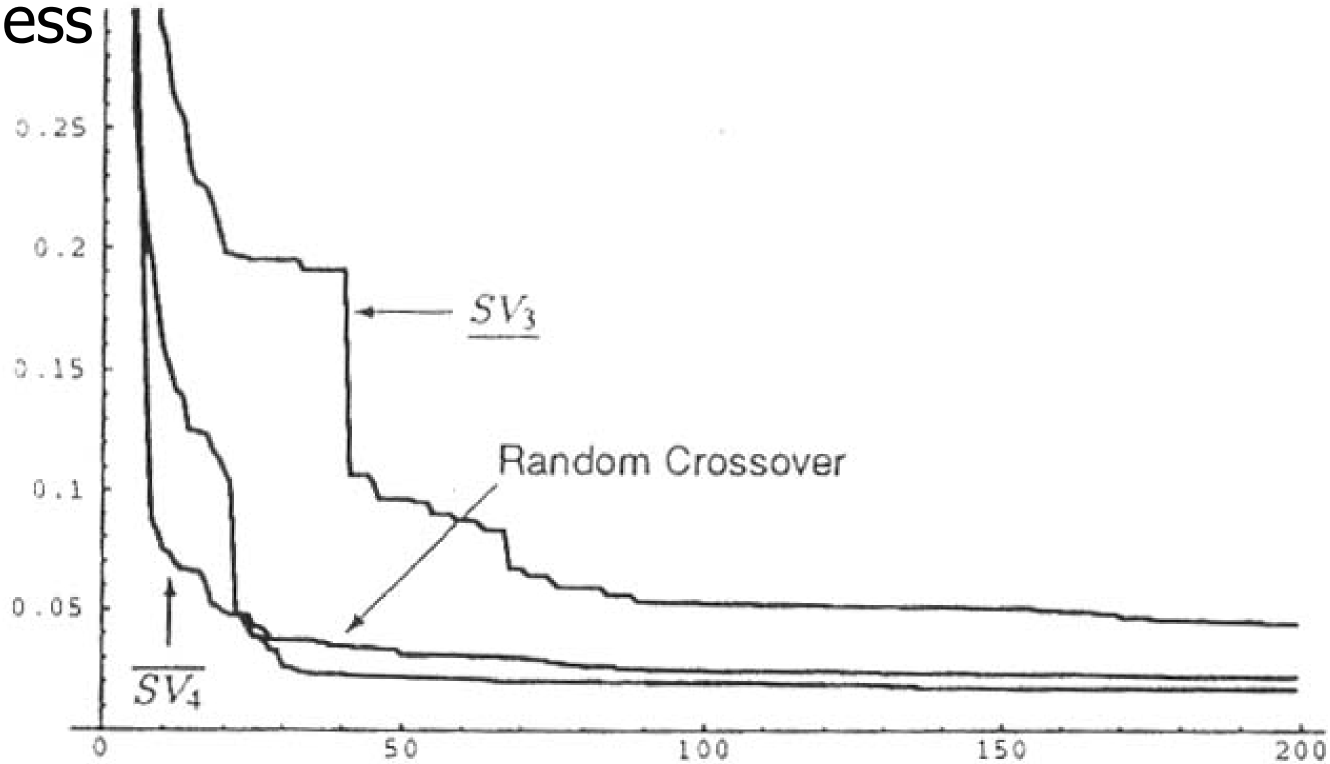
- $y=1/2\ x^2$
- Parameters
- Standard fitness vs. Generation
- Average Number of Success Generations (the number of success for 20 runs within a maximum of 200 generations)

Parameters

# of Fitness Cases	20
Terminal Set	{X}
Function Set	{+,−,x,÷}
Population Size	40
Crossover Prob.	0.7
Mutation Prob.	0.033
Generation	200

Standard fitness vs. Generation

Std. Fitness



Gen.

Average Number of Success Generations

Method	# of Success	Average Gen.
Usual Recombination	10	18.88
SV_3	9	25.5
SV_4	14	5.93

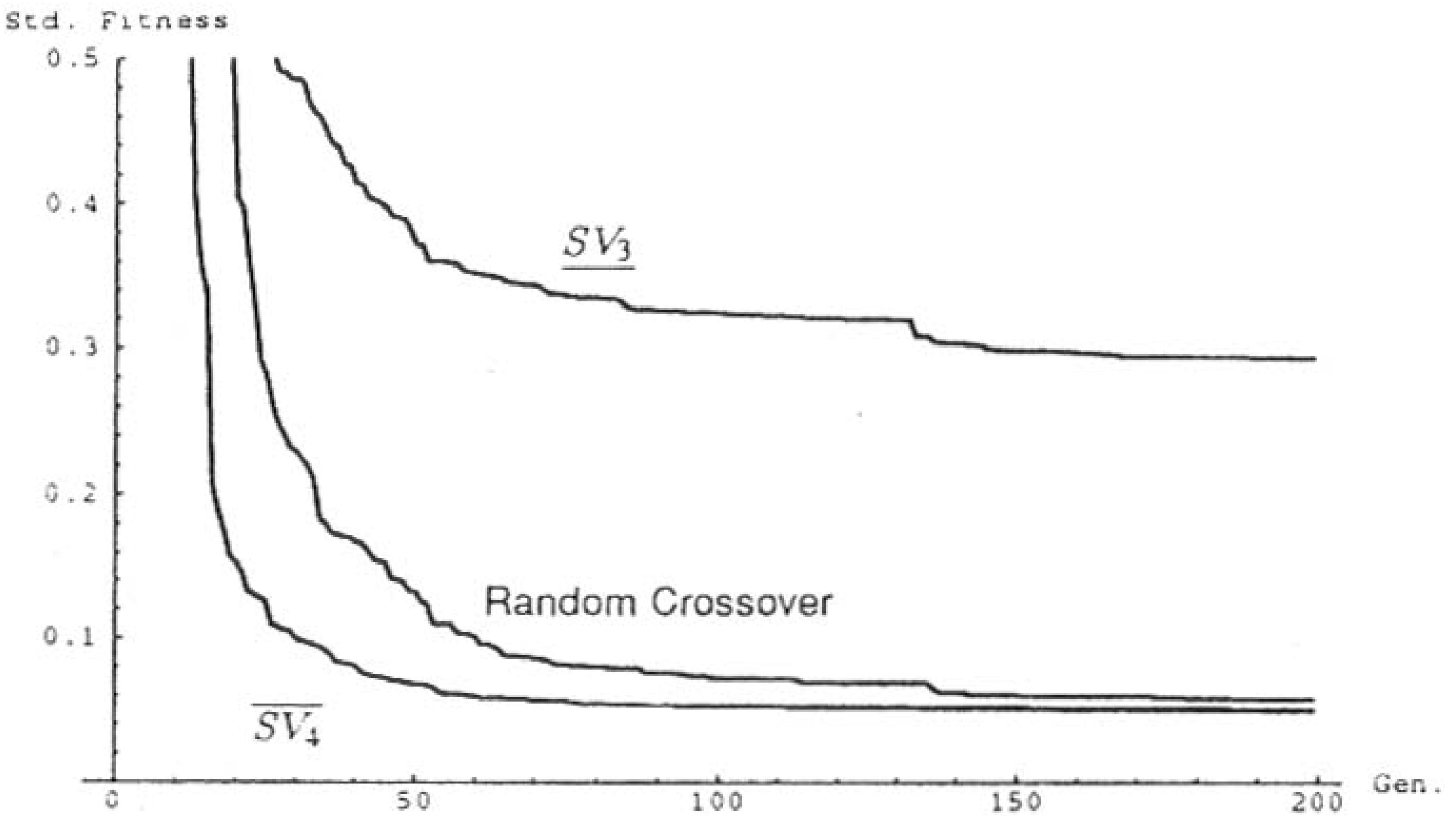
Discovery of trigonometric identities

- $\cos^2 x = 1 - \sin^2 x$
  - 20 pairs (x_i, y_i), $x_i \in [0, 2\pi]$
- Parameters
- Std. Fitness vs. Gen.

Parameters

# of Fitness Cases	20
Terminal Set	{x, 1.0}
Function Set	{+, -, ×, ÷, sin}
Population Size	500
Crossover Prob.	0.7
Mutation Prob.	0.033

Std. Fitness vs. Gen.



MDL-based Guidance for Numerical GP

- Advantages of STROGANOFF
- MDL fitness definition
- Chose the MDL value as the S-value for each subtree T : $SV_{MDL}(T)=MDL(T)$
- Predict the Mackey-Glass time series
- STROGANOFF parameters
- Results

Advantages of STROGANOFF

- Analog(polynomial) expressions complemented the digital(symbolic) semantics.The representational problem of standard GP does not arise for STROGANOFF.
- MDL-based fitness evaluation works well for tree structures in STROGANOFF, which controls GP-based tree search.
- Multiple-regressions tuned the node coefficients so as to guide GP recombination effectively.

MDL fitness definition

- $MDL = 0.5N\log S_N^2 + 0.5k\log N$
  - N：the number of data pairs
  - S_N^2：the mean square error
  - $S_N^2 = \frac{1}{N}\sum_{i=1}^{N}\left|\bar{y}_i - y_i\right|^2$
  - k：the number of parameters of the tree

The Mackey-Glass time series

$$\frac{dx(t)}{dt} = \frac{ax(t-\tau)}{1 + x^{10}(t-\tau)} - bx(t)$$

- a=0.2, b=0.1 and τ=17
- The trajectory is chaotic and lies on an approximately 2.1 dimensional strange attractor.

STROGANOFF parameters

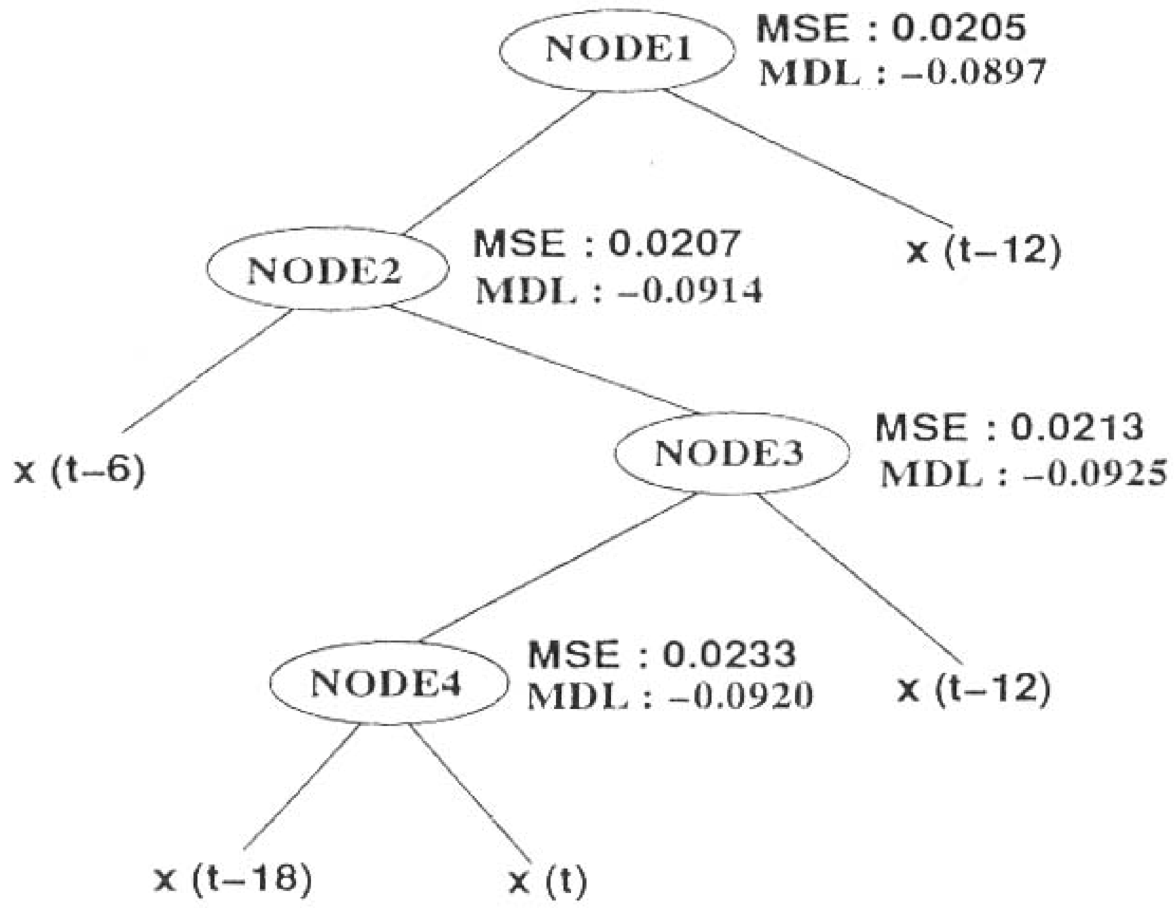
Population Size	120
Prob. of Crossover	60%
Prob. of Mutation	3.3%
Selection Method	Tournament
Non-terminal Nodes	$\{a_0+a_1x_1+a_2x_2+a_3x_1x_2+a_4x_1^2+a_5x_2^2\}$
Terminal Nodes	{x(t), x(t-6), x(t-12), x(t-18)}
Target Variable	x(t+85)
# of Training Data	500
# of Testing Data	500

Following Hartman(1991)

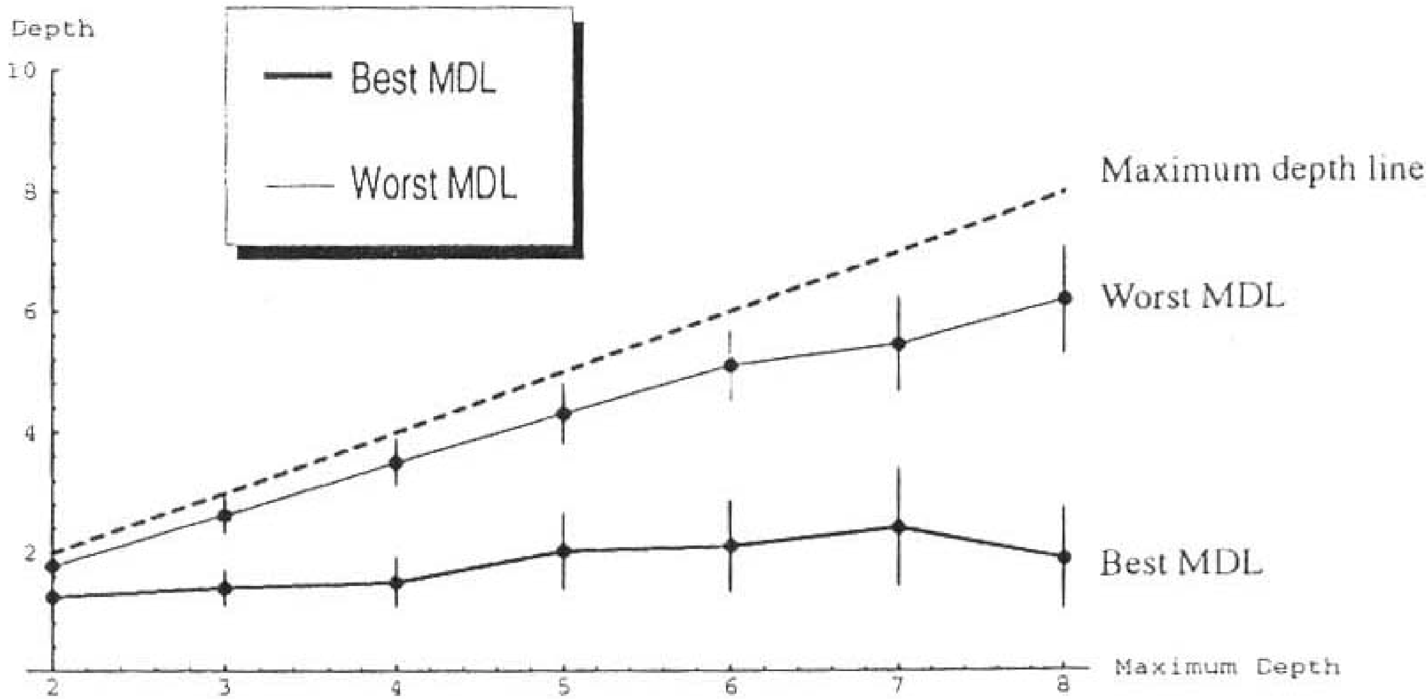
Results (MDL-based)

- An Exemplar Tree
- Average Depths of Node of Best and Worst MDL values
- Prediction of Mackey-Glass Equation

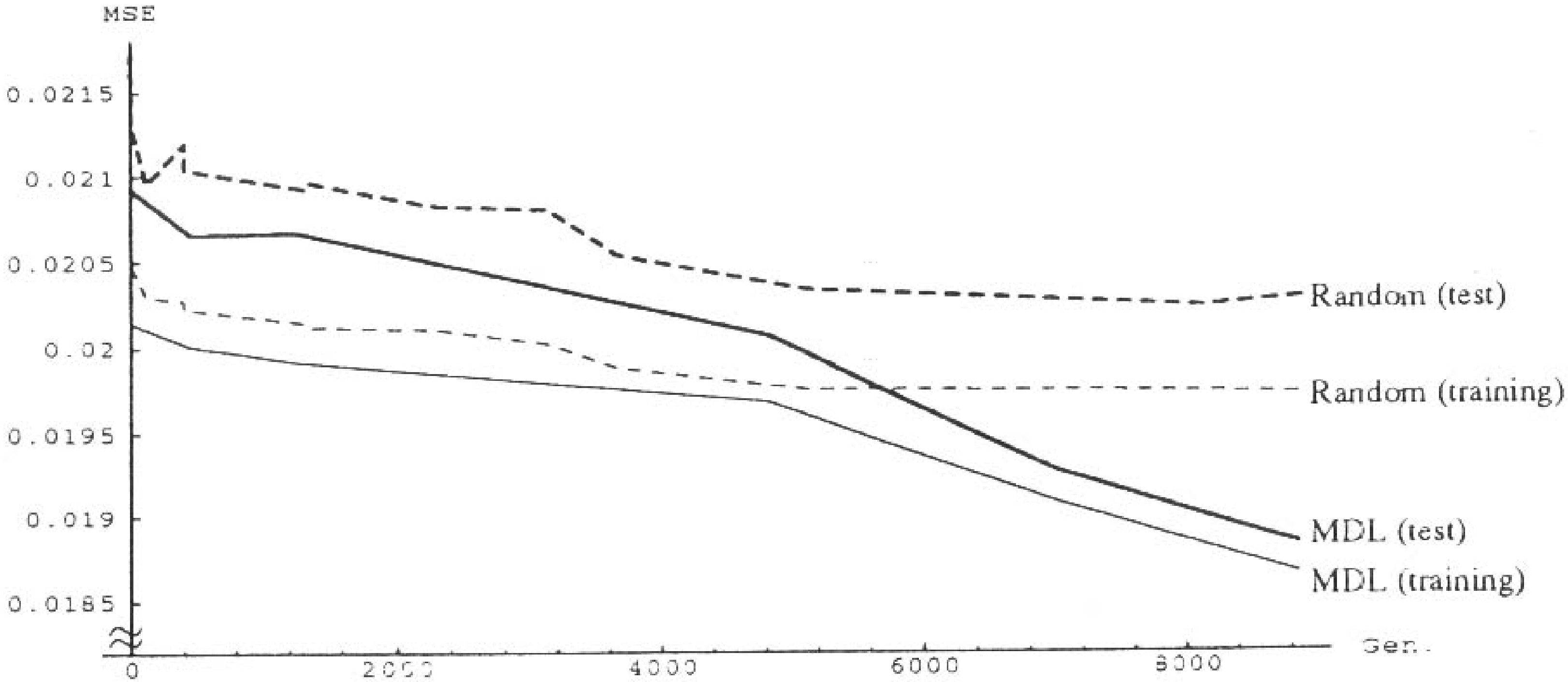
An Exemplar Tree



Average Depths of Node of Best and Worst MDL values



Prediction of Mackey-Glass Equation



Discussion

Problem	Reference	Result
Lawnmower	Koza 1994, ch.8	$R \preceq \overline{SV_1} \preceq \overline{SV_2}$
Ant Trail	Koza 1994, ch.12	$R \preceq \overline{SV_1} \preceq \overline{SV_2}$
Regression 1	Koza 1992, p.163	$\underline{SV_3} \preceq R \preceq \overline{SV_4}$
Regression 2	Koza 1992, ch.10.1	$\underline{SV_3} \preceq R \preceq \overline{SV_4}$
Time Series Pred.	Iba 1993, 1994	$R \preceq \underline{SV_{MDL}}$

The Success of methods

- Although there is no theoretical background for SV_2 definition, SV_2 is similar to MDL-based evaluation.
  - We may regard the first term of SV_2 as the inverse of the Exception_Coding_Length and the second term as the inverse of the Tree_Coding_Length.
- SV_2 and <u>SV</u>$_{MDL}$ evaluate the trade-off between performance and tree descriptions.

The Success of methods

- The MSE gives very little information as to whether a building-block will be useful.
- But, a correlation coefficient is a rather poor statistic for deciding whether an observed correlation is statistically significant.
- SV_4 is introduced as a way to evaluate the S-value heuristically, not as an absolute index.

Related Works

- Rosca (1994)
  - Use Block fitness function to discover a useful building block by his system AR-GP.
- Angeline (1996)
  - Selective self-adaptive crossover and self-adaptive multi-crossover.
- Tackett (1995a)
  - Greedy Recombination operator.
- Teller (1996)
  - The co-evolution of intelligent recombination operators.
- Nordin (1996)
  - The introns regulate the crossover probabilities for subtrees.

Future Works

- Is it possible to design effective S-values for various problems?
- The approach can be combined with ADF.
- Formalize S-values in more mathematical ways.
- Eliminate the constraint of the "linear order" for the S-values.

Formalize S-values

- A characteristic of S-values：if a subtree is a good building-block, it has a good S-value. That is

$$T \preceq T' \Leftrightarrow \Pr ob(T) < \Pr ob(T')$$

Where Prob(T) is the probability that a subtree T is part of a solution tree.