Risk preference, forecasting accuracy and survival dynamics: Simulations based on a multi-asset agent-based artificial stock market

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Abstract

The relevance of risk preference and forecasting accuracy to the survival of investors is an issue that has recently attracted a number of theoretical studies. By using agent-based computational modeling, this paper extends the existing studies to an economy where adaptive behaviors are autonomous and complex heterogeneous. Specifically, a computational multi-asset artificial stock market corresponding to Blume and Easley [Blume, L., Easley, D., 1992. Evolution and market behavior. Journal of Economic Theory 58, 9–40] and Sandroni [Sandroni, A., 2000. Do markets favor agents able to make accurate predictions? Econometrica 68, 1303–1341] is constructed and studied. Through simulation, we present results that contradict the market selection hypothesis. © 2007 Elsevier B.V. All rights reserved.

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1. Introduction

Agent-based computational economic (hereafter ACE) modeling is distinguished from the conventional economic modeling by its great flexibility in terms of agents’ heterogeneity and the associated population dynamics. This advantage may be very helpful in studying the survivability of different types of agents, specifically when they are placed in a complex interactive environment. In this paper, the ACE approach is applied to address a debate that can be related to the market selection hypothesis, according to which markets favor rational traders over irrational traders (Alchian, 1950; Friedman, 1953).

The debate, if we trace its origin, started as a result of the establishment of what become known as the Kelly criterion (Kelly, 1956), which basically says that a rational long run investor should maximize the expected growth rate of his wealth share and, therefore, should behave as if he were endowed with a logarithmic-utility function. In other words,
the Kelly criterion implicitly suggests that there is an optimal preference (rational preference) that a competitive market will select and that is logarithmic-utility. The debate on the Kelly criterion has a long history, so not surprisingly, there is a long list of both pros and cons with regard to it as the literature develops.\footnote{See Sciubba (2006) for a quite extensive review.}

A possible implication of the Kelly criterion is that an agent who maximizes his expected utility under the correct belief may be driven out by an agent who maximizes his expected utility under an incorrect belief, simply because the former does not maximize a logarithmic-utility function, whereas the latter does. Blume and Easley (1992) were the first to show this implication of the Kelly criterion in a competitive asset market. In their seminal study, they questioned the survivability of rational investors. In a nutshell, they showed that rational investors who are characterized by their selection of a portfolio that maximizes their expected utility with respect to the correct belief may not be good enough to survive. To enhance their survivability, their preference over risk (utility function) must also be “optimal.” If not, an even more striking result is that these rational agents may be driven out of the market by those agents who base their decisions on incorrect beliefs, but have a “nearly optimal” preference.\footnote{Other similar findings can also be found in Sciubba (2006).}

The market selection hypothesis, therefore, fails because agents with accurate beliefs are not necessarily selected. A consequence of this failure is that asset prices may not eventually reflect the true value of the asset and may fail to converge to the rational expectations equilibrium.

Nonetheless, a series of recent studies indicates that the early analysis of Blume and Easley (1992) is not complete. Sandroni (2000) shows that, if the saving behavior is endogenously determined, then the market selection hypothesis is rescued, and in the long-run, only those optimizing investors with correct beliefs survive. The surviving agents do not have to be log-utility maximizers, and they can have diverse preferences over risk. Sandroni’s analysis is further confirmed by Blume and Easley (2006) in a connection of the market selection hypothesis to the first theorem of welfare economics. They show that in a dynamic complete market Pareto optimality is the key to understanding selection for or against traders with correct beliefs: in any optimal allocation the survival or disappearance of a trader is determined entirely by beliefs and not by risk preference.

Sandroni (2000)’s and Blume and Easley (2006)’s studies are largely analytical. They both take a Pareto optimal allocation as a starting point to work with. The dynamic process converging to a Pareto optimal allocation itself is, nonetheless, beyond the scope of their analysis. Issues related to the dynamic process are twofold. First, there is individual dynamic optimization. A Pareto optimal allocation rests upon the optimization of all individuals. In this specific context, this requires that all agents are able to solve the infinite-time stochastic dynamic optimization problem facing them, regardless of their preferences over risk or utility functions. However, analytical solutions known to us are severely restricted to certain classes of preferences. In general, one has to rely on numerical approximation, which means that Pareto optimality may not always be attainable.

What makes this problem even more complex is, however, the second issue: trading at an equilibrium consistent with price expectations. Notice that what we study here is not a simple representative-agent optimization problem, but a market composed of heterogeneous agents. Each one of them, upon maximizing his expected utility, has to know the prices of assets in the future. These prices are, nonetheless, endogenously generated by agents’ own perceptions. As a result, a typical fixed-point problem occurs. The market, as a distributed decentralized processor, may fail to coordinate its participants to such a fixed point. In general, it will depend on agents’ forecasting rules and the associated learning schemes, and it is likely that agents will trade at prices that are inconsistent with their ex ante expectations of the prices. In this case, Pareto optimality is also not attainable.

Both of the two issues discussed above are directly related to the attainability of Pareto optimality. However, Pareto optimality per se was only taken by Sandroni and Blume and Easley as a convenient starting point for their analytical work. To facilitate their further analysis, the learning dynamics concerned with the updating of agents’ beliefs are also needed to be simplified. Sandroni, for example, did not deal with learning dynamics directly; instead, he assumed that there will be a day when some agents can eventually make accurate predictions or eventually make accurate next period predictions and started his major analytical work from there. Nevertheless, a plausible process to show the appearance of these sages was absent. It is, therefore, not entirely clear whether these types of agents will ever emerge. What happens when no trader has correct beliefs?\footnote{Sandroni does consider the case when no one has correct beliefs. His Proposition 3 basically compares two kinds of agents: one persistently forecasts more accurately than the other, while neither has correct beliefs. He then shows that the former...}
Blume and Easley (2006) do recognize that the market selection hypothesis would be of little interest if it were to address only selection for traders with correct beliefs. Their delicate analysis of learning leads to two major findings as to the superiority of Bayesians. First, a Bayesian almost surely survives for almost all possible truths in the support of her prior. Second, in the presence of a Bayesian trader, any traders who survive are not too different from Bayesians. We admire the beauty of the analysis of the Bayesians, but are not entirely easy about traders being simply Bayesians. The experimental evidences are certainly not always in favor of Bayesian learning. Therefore, this consideration does not stop us from asking: what happens when no traders are Bayesians?

This review and discussion of the early literature now seems to indicate clearly where we are moving. Needless to say, the above-mentioned analytical work on the market selection hypothesis has already provided us with an interesting benchmark to reflect upon, namely, the irrelevance of risk preference. However, since the conclusive statement is very interesting, it would be useful to see how strongly we can put it by relaxing some tight constraints. In this paper, we do not assume Pareto optimality, the emergence of the sages, or the Bayesians. This relaxation allows for a more extensive class of bounded-rational behaviors, and we examine whether the irrelevance of risk preference still holds with this enlargement. Concretely speaking, what is proposed here is a computational model, namely, an agent-based computational version of Blume–Easley–Sandroni’s model.

The rest of the chapter is organized as follows. Section 2 briefly reviews the Blume–Easley–Sandroni model. An agent-based computational version of the model, provided in Section 3, can be regarded as an extension of the single-asset artificial stock market to its multi-asset version. The debate then proceeds with the experimental designs given in Section 4. The simulation results and analysis are provided in Section 5, followed by the concluding remarks in Section 6.

2. The Blume–Easley–Sandroni model

Our agent-based artificial stock market is built upon the analytical model that was first initiated by Blume and Easley (1992) and was later extended by Sandroni. The Blume–Easley–Sandroni (hereafter, the BES) model is briefly reviewed in this section.

Consider a complete securities market. Time is discrete and indexed by \( t = 0, 1, 2, \ldots \). There are \( M \) states of the world indexed by \( m = 1, 2, \ldots, M \), one of which will occur at each date. States follow a stochastic process. Asset \( m \) pays dividends \( w_m > 0 \) when state \( m \) occurs, and 0 otherwise. At each date \( t \), the outstanding volume of each asset is exogenously fixed at one unit so that the total wealth in the economy at date \( t \), \( W_t \), will simply be the dividends paid at date \( t \) (i.e. \( W_t = w_m \)). The wealth will be distributed among the investors proportionately according to their owned share of asset \( m \). The distribution received by each agent, \( W_{t,1} \), can be used to consume and re-invest. Following the discussion of Sciubba, we assume that there is aggregate uncertainty so that \( w_m \neq w_v \), for \( m \neq v \).

There is a finite number of agents with heterogeneous temporal preferences in this economy, indexed by \( i \in \{1, 2, \ldots, I \} \). Each agent \( i \) has his subjective beliefs about the future sequence of the states. Each of these subjective beliefs is characterized by a probabilistic model, denoted by \( B^i \). Since \( B^i \) may change over time, the time index \( t \) is added as \( B^i_t \) to make such a distinction. The agent’s objective is to maximize his lifetime expected utility, and there are two decisions that are involved in this optimization problem. First, he has to choose a sequence of saving rates starting from now to infinity, and second a sequence of portfolios to distribute his saving over \( M \) assets. Let us denote these two sequences of decisions by

\[
\{\{\delta^i_{t+r} \}_{r=0}^{\infty} \}, \quad \{\alpha^i_{t+r} \}_{r=0}^{\infty}\}
\]

where \( \delta^i_t \) is the saving rate at time \( t \), and \( \alpha^i = (\alpha^i_{1,t}, \alpha^i_{2,t}, \ldots, \alpha^i_{M,t}) \) is the portfolio comprising the \( M \) assets. The two sequences of decisions will be optimal and are denoted by \( \{\delta^i_{t+r} \}_{r=0}^{\infty} \) and \( \{\alpha^i_{t+r} \}_{r=0}^{\infty} \), if they are the solutions to the
following optimization problem:

\[
\max_{\{[\delta^i_{t+r}]_{r=0}^\infty, \{\alpha^i_{t+r}\}_{r=0}^\infty\}} \mathbb{E}\left\{ \sum_{r=0}^{\infty} (\beta^i)^r u^i(c^i_{t+r}) | B^i_t \right\}
\]

subject to

\[
c^i_{t+r} + \sum_{m=1}^{M} \alpha^i_{m,t+r} \delta^i_{t+r} W^i_{t+r-1} \leq W^i_{t+r-1}, \quad \forall r \geq 0,
\]

\[
\sum_{m=1}^{M} \alpha^i_{m,t+r} = 1, \quad \alpha^i_{m,t+r} \geq 0, \quad \forall r \geq 0.
\]

In Eq. (1), \( u^i \) is agent \( i \)'s temporal utility function, and \( \beta^i \), also called the discount factor, reveals agent \( i \)'s time preference. \( c^i_t \) denotes consumption. The expectation \( \mathbb{E}(\cdot) \) is taken with respect to the most recent belief \( B^i_t \). Eqs. (2) and (3) are the budget constraints. By combining constraint (3), constraint (2) can also be written as (4),

\[
c^i_{t+r} \leq (1 - \delta^i_{t+r}) W^i_{t+r-1}.
\]

These budget constraints do not allow agents to consume or invest by borrowing.

Given the saving rate \( \delta^i_{t+r} \), agent \( i \) will invest a total of \( \delta^i_{t} W^i_{t-1} \) in the \( M \) assets according to the portfolio \( \alpha^i_{t} \). In other words, the investment put into each asset \( m \) is \( \alpha^i_{m,t} \delta^i_{t} W^i_{t-1} \). By dividing this investment by the market price of asset \( m \) at date \( t \), \( \rho_{m,t} \), one derives the share held by agent \( i \) of that asset, \( q^i_{m,t} \):

\[
q^i_{m,t} = \frac{\alpha^i_{m,t} \delta^i_{t} W^i_{t-1}}{\rho_{m,t}}, \quad m = 1, 2, \ldots, M.
\]

The equilibrium price \( \rho_{m,t} \) is determined by equating the demand for asset \( m \) to the supply of asset \( m \):

\[
\sum_{i=1}^{I} \frac{\alpha^i_{m,t} \delta^i_{t} W^i_{t-1}}{\rho_{m,t}} = 1, \quad m = 1, 2, \ldots, M.
\]

Rearranging Eq. (6), one obtains the market equilibrium price of asset \( m \):

\[
\rho_{m,t} = \sum_{i=1}^{I} \alpha^i_{m,t} \delta^i_{t} W^i_{t-1}.
\]

Agents’ shares of assets will be determined accordingly by Eq. (5). Afterwards, state \( m \) happens and is made known to all agents at date \( t \). The dividends \( w^i_{m} \) will be distributed among all stockholders of asset \( m \) in proportion to their shares, and their wealth will be determined accordingly as \( W^i_{t} = q^i_{m,t} w^i_{m} \). The date moves to \( t + 1 \), and the process then repeats itself as shown in Fig. 1.

3. The agent-based multi-asset artificial stock market

3.1. Agents’ cognition

Like all agent-based computational economic models, we shall first begin with a description of a typical agent, including his cognition and adaptive behavior. Let us first start with the problem presented to our agents. Agents in our model behave like normal investors who try to maximize their lifetime discounted expected utility by appropriately choosing their investment strategy. The investment strategy is mainly composed of two parts, namely, saving and portfolio.

At each point in time, say, \( t \), investor \( i \) observes a time series (history) of the realization of the states, namely, \( S_{t-1} \equiv \{m_{s}\}_{s=0}^{t-1} (m_s \in \{1, 2, \ldots, M\}) \). Based on this realization \( S_{t-1} \), he makes his decisions on a sequence of investment strategies: \( \{[\delta^i_{t+r}]_{r=0}^\infty, \{\alpha^i_{t+r}\}_{r=0}^\infty\} \). Given investor \( i \)'s temporal utility function \( u^i \), it is hoped that this sequence of
investment strategies is rational in the sense that his lifetime discounted expected utility can be maximized (see Eqs. (1)–(3)).

The discrete-time stochastic optimization problem defined by Eqs. (1)–(3) may be analytically solvable when one considers some specific types of utility functions with some other necessary simplifications. However, since the purpose of this paper is to examine the relevance of risk preference to survivability, we place little in the way of restrictions on the types of risk preference. In fact, in this paper we even allow for the risk preference to be randomly generated, as long as it is well-behaved. This causes the optimization problem, when generally posed, to be difficult to solve analytically, not to mention the further complications arising from their beliefs or the conditional expectations. Therefore, we assume that all agents in our model are computational. They cope with the optimization problem with a numerical approximation method, and the specific numerical method used in this paper is the genetic algorithm.

Over the last few years, the genetic algorithm has been the most active tool in agent-based computational economics. It is mainly used to deal with either the cognitive limit of optimizing or the cognitive limit of forecasting. Very few studies use the GA to conduct multi-level evolution. In this paper, we use the genetic algorithm to evolve both agents’ investment strategies and beliefs simultaneously. The two-level evolution proceeds as follows:

- At a fixed time horizon, investors update (evolve) their beliefs of the states coming in the future.
- They then evolve their investment strategies based on their beliefs.

The two-level evolution allows agents to solve a boundedly rational version of the optimization problem (1). First, the cognitive limit of investors and the resultant adaptive behavior free them from an infinite-horizon stochastic optimization problem, as in Eq. (1). Instead, due to their limited perception of the future, the problem effectively posed to them is the following:

$$\max_{\{d_{t+h}^i\}_{h=0}^{H-1}, \{\alpha_{t+h}^i\}_{h=0}^{H-1}} \mathbb{E}\left\{\sum_{h=0}^{H-1} (\beta^i)^h u'(c_{t+h}^i)B_t^i \right\}. \quad (8)$$

Here, we replace the infinite-horizon perception with a finite-horizon perception of length $H$, and the filtration ($\sigma$-algebra) induced by $S_{t-1}$ with $B_t^i$, where $B_t^i$ is investor $i$’s belief at date $t$. In a simple case where $m_t$ is independent (but not necessarily stationary), and this is known to the investor, then $B_t^i$ can be just the subjective probability function, $B_t^i = (b_{1,t}^i, \ldots, b_{M,t}^i)$, where $b_{m,t}^i$ is investor $i$’s subjective probability of the occurrence of the state $m$ in any of the next $H$ periods. In a more general setting, $B_t^i$ can be a high-order Markov process. With the replacement (8), we assume that investors have only a vague perception of the future, but will continuously adapt when approaching it. As we shall see in the second level of evolution, $B_t^i$ is adaptive.

Furthermore, we assume that investors will continuously adapt their investment strategies according to the sliding window shown in Fig. 2. At each point in time, the investor has a perception of a time horizon of length $H$. All his investment strategies are evaluated within this reference period. He then makes his decision based on what he considers to be the best strategy. While the plan comes out and covers the next $H$ periods, only the first period, $\{d_t^{i,*}, \alpha_t^{i,*}\}$, will
be actually implemented. The next period, \( \{ \delta^*_t, \alpha^*_t \} \), may not be implemented because it may no longer be the best plan when the investor receives the new information and revises his beliefs.

With this sliding-window adaptation scheme, one can have two further simplifications of the optimization problem (1)–(3). The first one is that the future price of the asset \( m, \rho_{m,t+h} \) remains unchanged for each experimentation horizon, namely, at time \( t, \rho^i_{m,t+h} = \rho_{m,t-1}, \forall h \in \{0, H - 1\} \), where \( \rho^i_{m,t+h} \) is \( i \)'s subjective perception of the \( h \)-step-ahead price of asset \( m \). Second, the investment strategies to be evaluated are also time-invariant under each experimentation horizon (i.e. \( \delta^i_t = \delta^i_{t+1} = \delta^i_{t+2} = \cdots = \delta^i_{t+H-1} \), and \( \alpha^i_t = \alpha^i_{t+1} = \alpha^i_{t+2} = \cdots = \alpha^i_{t+H-1} \)).

With these two simplifications, we replace the original optimization problem, (1)–(3), that is presented to the infinitely smart investor, with a modified version which is suitable for a boundedly rational investor:

\[
\max_{\{\delta^i_t, \alpha^i_t\}} \mathbb{E} \left\{ \sum_{h=0}^{H-1} (\beta^i)^h u^i(\epsilon^i_{t+h} | B^i_t) \right\}
\]  

subject to

\[
\epsilon^i_{t+h} + \sum_{m=1}^{M} \alpha^i_{m,t} \delta^i_{t+h-1} W^i_{t+h-1} \leq W^i_{t+h-1}, \quad \forall h \in \{0, H - 1\},
\]

\[
\sum_{m=1}^{M} \alpha^i_{m,t} = 1, \quad \alpha^i_{m,t} > 0, \quad \forall m,
\]

\[
\epsilon^i_{t+h} = (1 - \delta^i_t) W^i_{t+h-1}, \quad \forall h \in \{0, H - 1\}.
\]

3.2. Autonomous agents

In this paper, we follow what was initiated in Holland and Miller (1991) and equip our agents with the genetic algorithm to cope with the finite-horizon stochastic dynamic optimization problem, (9)–(12). The GA is applied here at two different levels, a high level (learning level) and a low level (optimization level). First, at the high level, it is applied as a belief-updating scheme. This is about the \( B^i_t \) appearing in Eq. (9). Agents start with some initial beliefs of state uncertainty that are basically characterized by parametric models, say, Markov processes. However, agents do not necessarily confine themselves to just stationary Markov processes. Actually, they can never be sure whether the underlying process will change over time, so they stay alert to that possibility and keep on trying different Markov processes with different time frames (time horizons). Specifically, each belief can be described as “a \( k \)th order Markov process that appeared over the last \( d \) days and may continue.” These two parameters can be represented by a binary string and a canonical GA is applied to evolve a population of these two parameters with a set of standard genetic operators. Details are given in Section A.2 of the Supplementary material (appendixes are available on the JEBO website).

Once the belief is determined, the low-level GA is applied to solve the stochastic dynamic optimization problem defined in Eqs. (9)–(12). Basically, we use Monte Carlo simulation to generate many possible ensembles consistent with the given belief and use them to evaluate a population of investment plans composed of a saving rate and a portfolio. GA is then applied to evolve this population of candidates. Details are given in Section A.1 of the Supplementary material.
In sum, the high-level GA finds an appropriate belief, and under that belief the low-level GA searches for the best decisions in relation to savings and portfolios. These two levels of GA do not repeat with the same frequency. As a matter of fact, the belief-updating scheme is somewhat slow, whereas the numerical optimization scheme is more frequent. Intuitively, changing our belief of the meta-level of the world tends to be slower and less frequent than just fine-tuning or updating some parameters associated with a given structure. In this sense, the idea of incremental learning is also applied to our design of autonomous agents.

3.3. The behavior of CAPM believers

Investors whose behavior is governed by the procedure described in Sections A.1 and A.2 of the Supplementary material above are autonomous, which may not come to any stereotype familiar to us. To make sense of the evolution going on in the artificial markets, it would be useful to include some familiar types of investors as well, for example, those investors, no matter what happens, who just follow a guideline that is exogenously given. These investors are not autonomous. We shall call them formula investors. The GA procedure will not be applied to these formula investors. Instead, their behavior is prespecified by a formula. As an illustration of the idea of formula investors, we follow Sciubba to introduce CAPM believers into the market.

The CAPM believers are investors who base their portfolio rule upon the well-known capital asset pricing model (CAPM). In the spirit of the CAPM, they first find out the market portfolio and the risk-free portfolio. Then, according to their risk preference (degree of risk aversion), they choose a weighted combination of the two. We index CAPM traders by means of $\kappa$, and let $\gamma^\kappa$ be the associated risk aversion coefficient, which is randomly determined by the uniform distribution $U(0,1)$. At date $t$, each investor $\kappa$ invests in asset $m$ a portion $\alpha_{m,t}^{\text{CAPM}(\kappa)}$ of his savings such that

$$\alpha_{m,t}^{\text{CAPM}(\kappa)} = \gamma^\kappa \alpha_{m,t}^F + (1 - \gamma^\kappa) \alpha_{m,t}^M, \quad m = 1, \ldots, M,$$

where $\alpha_{m,t}^F \equiv (\hat{\rho}_{m,t}/w_m)/(\sum_{m=1}^M \hat{\rho}_{m,t}/w_m)$ and $\alpha_{m,t}^M \equiv (\hat{\rho}_{m,t}/\sum_{m=1}^M \hat{\rho}_{m,t})$.

This way of defining the CAPM investment strategy is proposed by Sciubba. It is straightforward to show that the vector $\alpha_t^M$ is in effect the weighted average of all market participants’ portfolios in the previous period. Thus, it is intuitively consistent with the idea of the market portfolio. Furthermore, it can be shown that the vector $\alpha_t^F$ is a weighted average of the price-earnings ratio, and with this portfolio the agent can expect to earn the same rate of return regardless of the occurrence of the state. As a result, it can be taken to approximate the idea of the risk-free rate.

Similarly, one can come up with a respective saving rate for the CAPM investor as follows:

$$\delta_{t}^{\text{CAPM}(\kappa)} = \gamma^\kappa \delta_{t}^F + (1 - \gamma^\kappa) \delta_{t}^M,$$

where $\delta_{t}^F = 0$ and $\delta_{m,t}^M = \sum_{i=1}^I (W_{i-2}/\sum_{i=1}^I W_{i-2}) \delta_{i-1}^M$.

The CAPM believers’ adherence to the CAPM portfolio rule (13) is not affected during the entire course of evolution.

Fig. 3 is a summary of the agent-based artificial stock market.

4. Experimental designs

Two experiments are conducted in this agent-based artificial stock market. In the first experiment, the autonomous agents are identical in all respects except for their preferences over risk. The purpose of this design is to see whether the survivability of investors has anything to do with their preferences. In the second experiment, we further distinguish each type of agent by their belief formation processes, which, under some circumstances, are pertinent to forecasting accuracy. Obviously, the second design is to examine the relevance of forecasting accuracy to survivability. We believe that these two designs together will contribute to the resolution of the debate of the irrelevance of risk preference or the dominance of forecasting accuracy in a rich empirical context.

5 For simplicity, we may assume that they have static expectations as other autonomous agents do, such as $\hat{\rho}_{m,t} = \rho_{m,t-1}, \forall m = 1, \ldots, M$. 


4.1. Market and participants

In both experiments, the market is composed of 40 agents \((I = 40)\). Five out of the 40 are CAPM believers. The \(\gamma\) values exogenously given to five CAPM believers are 0.1, 0.2, 0.3, 0.4 and 0.5, respectively. Since their portfolio and saving decisions are determined by Eqs. (13) and (14) and are irrelevant to their preferences over risk, their preferences will not be specified here. The remaining 35 agents are all autonomous. Their behavior is mainly driven by what has been detailed in Sections A.1 and A.2 in the Supplementary material. For these agents, the specification of their preferences is required, and they are detailed in Table 1.

In total, we consider seven types of autonomous agents, and each type is assigned to five agents. Type 1 has the logarithmic-utility function. We are very much interested in knowing whether this type of agent has any advantage over others in terms of the long-run wealth share. As to types 2–6, they are also frequently used in economic analysis. Among them, type 4 is the well-known constant absolute risk aversion (CARA) utility function. In addition to these six familiar types of utility functions, we also consider any arbitrary utility function. By using Taylor expansion, an arbitrary analytical utility function can be approximated by a finite-order polynomial function. Here, we consider the approximation only up to the sixth order.

Notice that types 3–7 refer to a class of parametric utility functions. Parameters of these types of utility functions, namely, \(\alpha_1, \ldots, \alpha_4, \beta_1, \ldots, \beta_3, \gamma_3\) and \(a_0, a_1, \ldots, a_6\), can in principle be randomly or manually generated as long

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**Fig. 3.** A summary of agent-based artificial stock markets.
as they satisfy the regular first- and second-order conditions: \( u' > 0 \) and \( u'' < 0 \). Since each type of utility function is assigned to five agents, parameter values are generated for each agent of each type separately, so agents of type 3 may have different values of \( (\alpha_1, \beta_1) \), agents of type 4 may have different values of \( (\alpha_2, \beta_2) \), and so on and so forth.

There are five assets available in the market \( (M = 5) \), corresponding to five states. Asset \( m \) pays dividends \( 6 - m \) \( (m = 1, 2, \ldots, 5) \). Two stochastic processes are considered in the experiments, namely, \( iid \) and the first-order Markov. Each is employed for one half of the total number of runs. Parameters of these two stochastic processes are also randomly generated in such a way that the axioms of the probability function are satisfied.

### 4.2. Parameters related to autonomous agents

At each point in time, agents have a perception of a time horizon with length \( H = 25 \). To solve the optimization problem (8), agents simulate five 25-horizon ensembles \( (L = 5) \) of the states based on their belief in order to evaluate the fitness of their investment strategies (Section A.3 of the Supplementary material).

The design for the agents’ adaptation is composed of two parts. For the low-level evolution, the architecture is a population GA. The population size \( N \) (number of investment strategies) for each agent is 100. The genetic parameters applied to evolving this population are as follows. The crossover rate \( (p_{\text{cross}}^j) \) is set to be 1, while the mutation rate \( (p_{\text{mutate}}^h) \) is set to be 0.03. Tournament selection with a tournament size of 4 is applied. The number of generations that the low-level GA runs in one period, \( G \), is set to be 50.

As to the high-level evolution, the architecture is also a population GA, and the population size \( J \) (number of beliefs) maintained by each agent is 100. The crossover rate and the mutation rate are the same as those of the low-level evolution, as is the tournament size used for the tournament selection. The belief set will be renewed after every two periods \( (\Delta = 2) \).

Table 2 provides a summary of the design. Two experiments are conducted based on this common design. The two experiments mainly differ in the parameter \( v \), which is the size of the data used to validate the model. In Experiment 1, our focus is on the role of the utility function. As a result, the \( v \) is fixed for all autonomous agents, which is 100. Nonetheless, since the stochastic process simulated in this market is stationary, it is expected that a larger \( v \) will help to validate the model and enhance the forecasting accuracy. Therefore, to see how significant the forecasting accuracy can be (Sandroni’s main argument), in Experiment 2 we let autonomous agents have different values of \( v \), starting from the very small one, 10, increasing it to 15, 25, 50, and finally to the largest one 100. To avoid mixing the results of the two experiments, we let these five values of \( v \) be evenly distributed among each type of agent (i.e. one for each agent of each type). With this distinction, the two experiments together can help us see better which factor is more important for the survivability of agents, namely, the preference or forecasting accuracy. Both experiments are run 100 times, and each run lasts for 100 market periods \( (T = 100) \). The simulation is conducted through the software AIE-ASM Version 5.0. The software is written with Delphi, Version 6.0.
Table 2
Experimental design

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<tbody>
<tr>
<td>Number of market participants (I)</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Number of types of agents</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Number of each type of agent</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Number of assets (states) (M)</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Dividends paid by asset m</td>
<td>6 − m</td>
<td></td>
</tr>
<tr>
<td>Stochastic processes</td>
<td>iid or first-order Markov</td>
<td></td>
</tr>
<tr>
<td>Number of market periods (T)</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Discount rate (β)</td>
<td>0.45</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Autonomous agents</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Agents’ perception of the time horizon (H)</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Number of ensembles (L)</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Population size (number of strategies) (N)</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Number of generations (G)</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Population size (number of beliefs) (J)</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Frequency of running GA on the belief set (Δ)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Crossover rate (p_c, p_h)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Mutation rate (p_m)</td>
<td>0.05, 0.03</td>
<td></td>
</tr>
<tr>
<td>Tournament size</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Number of bits for beliefs (τ_1 + τ_2)</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

The eight types refer to the seven types of autonomous agents whose utility functions are specified in Table 1 plus the CAPM believers.

5. Experimental results

In each single run, we generate a series of artificial data. At the micro-level, it includes the dynamics of agents’ beliefs, investment behavior, and the associated wealth \{B_{i,t}, \delta_i, \alpha_i, W_i\}_{t=1}^{100}, i = 1, \ldots, 40. At the aggregate level, we observe the asset price dynamics \{\rho_{m,t}\}_{t=1}^{100}, m = 1, \ldots, 5. Since the main concern of this paper is with which types of agent survive, our focus is on the wealth share dynamics. In addition, in order to understand what makes surviving agents survive, agents’ belief dynamics and investment behavior also attract our attention. Thus, the result presented in this section will be pretty much based on the micro-level data. Little will be said of the price dynamics.\(^6\)

5.1. Experiment 1

5.1.1. Wealth share dynamics

Fig. 4 shows the wealth share dynamics of the eight types of investors. Notice that each line is based on the average of 100 simulations. The results clearly demonstrate the strong dominance of the type-1 investors.\(^7\) It is also interesting to notice that the type-1 investor has a constant relative risk aversion coefficient that is one. Therefore, our findings can lend support to Blume and Easley’s main argument: the market selects those investors whose coefficient of relative risk aversion is nearly one.\(^8\)

In fact, if we consider the family of the CARA utility functions, namely,

\[
u(c) = \begin{cases} \frac{e^\rho}{\rho}, & \text{if } -\infty < \rho < \infty \text{ and } \rho \neq 0, \\ \ln c, & \text{if } \rho = 0, \end{cases}
\] (15)

\(^6\) Some preliminary time series analysis of price dynamics can be found in Chen and Huang (2005).

\(^7\) One may suspect that if the number of iterations (T) is long enough, say T = 500, then the type-1 agents are the only type of survivors. Actually, in a separate experiment, we have found that this is indeed the case (Chen and Huang).

\(^8\) See Blume and Easley (1992, Theorem 5.4, pp. 23–24). The italics shown in the main text are not the exact quotation of that theorem, which was originally based on controlling saving rates. Since saving rates are treated endogenously in our paper, our finding suggests that the theorem can still be true even though the assumption regarding saving rates is relaxed. Theorem 5.4 also rests upon forecasting accuracy, which we will address later.
Table 3

<table>
<thead>
<tr>
<th>Performance measurements</th>
<th>Type 1</th>
<th>Type 2</th>
<th>Type 3</th>
<th>Type 4</th>
<th>Type 5</th>
<th>Type 6</th>
<th>Type 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecasting accuracy (K–S statistics)</td>
<td>0.05219</td>
<td>0.05220</td>
<td>0.05233</td>
<td>0.05228</td>
<td>0.05222</td>
<td>0.05226</td>
<td>0.05220</td>
</tr>
<tr>
<td>Portfolio performance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean rate of return</td>
<td>2.38024</td>
<td>2.83639</td>
<td>2.80554</td>
<td>2.79910</td>
<td>2.80125</td>
<td>2.92166</td>
<td>2.78132</td>
</tr>
<tr>
<td>Variance of return</td>
<td>53.14375</td>
<td>65.75532</td>
<td>69.48951</td>
<td>79.44515</td>
<td>91.41895</td>
<td>119.17748</td>
<td>66.07364</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.32651</td>
<td>0.34978</td>
<td>0.33656</td>
<td>0.31404</td>
<td>0.29298</td>
<td>0.26763</td>
<td>0.34217</td>
</tr>
</tbody>
</table>

Type-2 investors \( u(c) = \sqrt{c} = c^{(1/2)} \) also have a constant RRA coefficient of 0.5 \((1 - \rho)\), but it is not close enough to 1.9

5.1.2. Forecasting accuracy

Sandroni, however, considered forecasting accuracy to be the sole important factor in determining who may survive. In this section, we shall take a closer look at the forecasting performance of different types of agents. The Kolmogorov–Smirnov statistic (K–S statistic hereafter) is chosen to measure the forecasting accuracy of agents. The K–S statistic is a metric, or more precisely the sup norm, for two distribution functions. Formally, let \( F \) and \( G \) be two distribution functions

\[
K–S(F, G) = \sup_x |F(x) - G(x)|. \tag{16}
\]

Using the K–S statistic, we can measure the forecasting error by the difference between the true distribution function and the subjective distribution perceived by agents. The results are shown in Table 3 (the top panel). The statistical test cannot reject the null hypothesis that the forecasting accuracy among different types of autonomous traders is equally good. This test result is not surprising given the fact that all autonomous agents are supplied with the same adaptive scheme to update their beliefs. Therefore, forecasting accuracy, at least, is not the sole important factor in the determination of survivability.

5.1.3. Saving rates

Since forecasting accuracy cannot necessarily guarantee agents’ survival, there are only two decision variables left for us to see the uniqueness of type-1 agents, namely saving and portfolio. Notice that in Blume and Easley (1992),

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9 In a separate study (Chen and Huang), we simulated the cases with different \( \rho \) s from 0 to 0.9 with an increment of 0.1. It was found that the agents’ share of wealth is positively related to their relative risk coefficients, \( 1 - \rho \). The wealth share dynamics of agents with a \( \rho \) of 0.1 is very close to that of type-1 agents considered here. Both survive to the end of the 100-period simulations.
the saving rate is exogenously given, and it is found that the saving rate can play an important role in determining who survives. However, its impact becomes *implicit* when the saving decision is endogenized (Sandroni). It is therefore useful to examine the saving behavior of different types of agents if one desires to know the survival of type-1 agents.

Fig. 5 is a box–whisker plot of the saving rates among the seven types of agents. Each plot shows the lifetime distribution of the saving rate $\delta_t$ associated with a specific type of agent. To generate each plot, we first take an average of the saving rate of the five agents of the same type. This is done period by period. Then we have a time series of the saving rate $\delta_t$ ($t = 1, 2, \ldots, 100$) for each type of agent. To have an idea of the distribution (dispersion) of the saving behavior, the minimum, the first quartile, the medium, the third quartile, and the maximum of $\{\delta_t\}$ are recorded for each type of agent. Furthermore, we then derive sample statistics for these order statistics by taking an average over the entire 100 simulation runs.

The line appearing in the middle of the box indicates the median saving rates for a specific type of agent. While higher saving rates, as Blume and Easley suggested, will place agents in an advantageous position to survive, we find that the saving rate of type-1 agents is in fact the lowest among all seven types of agents. This is evidenced by the lowest median of the type-1 agents. What, however, makes type-1 agents unique is their very stable saving behavior. This is revealed by comparing the boxes and whiskers of the plots. Not only is it stable, but it is also stable around 0.45, which is exactly the discount rate $\beta$ set in Table 2.

From an analytical viewpoint, Blume and Easley (1992) have already showed that $\delta_t = \beta$ for log-utility agents. In other words, the optimal saving rate for type-1 agents is just a constant and is independent of beliefs, wealth and asset prices. Our genetic algorithm (the low-level GA) just confirms this property *numerically*. We think that this property has an important implication for the survival of type-1 agents, and our reasons for this follow.

In general, the saving decision is made jointly with the portfolio decision, which means that when agents are equipped with a GA, the low-level GA actually works with the high-level GA. While an agent uses the low-level GA to make the saving and portfolio decision, the quality of that decision also depends on the belief (forecasting accuracy) supplied by the operation of the high-level GA. Hence, any imperfection in the high-level GA may compound the imperfection of the low-level GA in the usual sense of error propagation. Nevertheless, since for type-1 agents the saving decision is independent of their beliefs and hence forecasting accuracy, it separates the performance of the low-level GA from that of the high-level GA. It behaves like this. The low-level GA first learns that $\beta$ is the only relevant factor for the saving decision and simply works with $\beta$. Furthermore, since $\beta$ is a noiseless constant, the saving decision associated with the low-level GA is well grounded. In the end, it helps type-1 agents come up with a quality decision on the saving rate that is almost exact.

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10 For example, see Blume and Easley (1992, Proposition 3.2, p. 16).
11 In his illustrating example on p. 1311, Sandroni compared two agents with different preferences: the one with the square-root utility function had the correct prediction, whereas the other with the log-utility function did not have the correct prediction. On p. 1313, he then showed that the former with a higher saving rate drove out the latter with a lower saving rate, and the difference in their saving rates was endogenously generated as a part of the equilibrium.
The same story above unfortunately does not apply to other types of agents. In general, their saving decisions are not independent of their beliefs. Even though these agents are equipped with the same kind of GA used by type-1 agents, their low-level GA is operated under the belief determined by the high-level GA, which may suffer from some degree of inaccuracy all the time. This will in turn have an adverse effect on the quality of the saving decision. From what we have seen in Fig. 5, this happens in terms of the appearance of the large fluctuation in saving rates. In addition to the unstable saving behavior, types 3–7 agents have suffered further from the extremely low down-side saving rates, which may contribute to the fast decline in their wealth share. Type-2 agents do not share this feature of down-side saving rates. Obviously they perform better, but their unstable saving behavior eventually causes a threat to them.

It is worth noting that the significance of a stable saving behavior was not a focus of Blume and Easley’s original analysis when they treated the saving rate as an exogenously given constant. It is also hard to address this in Sandroni’s dynamic equilibrium framework where the learning dynamics is absent.

5.1.4. Portfolio performance

The second decision is regarding portfolio. The log-utility (type-1) agents, as described by Blume and Easley (1992), are the kind of agents who maximize their expected growth rate of wealth. This observation naturally raises a question: would type-1 agents survive because their portfolio performance is superior to that of other types of agents? To answer this question, we can start with two different performance measurements; one is ex ante, and the other is ex post. The former evaluates the agents’ investment based on its (probabilistic) expected value, while the latter evaluates it on the basis of its realized value. In this paper, we consider the ex post approach to be more pertinent. There are two reasons for us to think so.

First, agents in the ACE model are not optimizing agents; instead, they are adaptive agents. The adaptation scheme, be it GA or not, is generally driven only by realized returns rather than by the expected returns. It is well known that such a discrepancy can deviate agents’ behavior from that of expected-utility maximizers (Lettau, 1997). This leads us to the second point: in the real world, what is used to evaluate or rank the performance of mutual fund managers certainly is the historical returns and not the hypothetical expected returns. Having said that, we suggest calculating the rate of return as follows. We first calculate \( r^i_{m,t} \), the realized rate of return of agent \( i \) on the investment in the asset \( m \) at time \( t \), as

\[
r^i_{m,t} = \alpha^{i}_{m,t} \delta^{i}_W_{t-1} \frac{\delta^i W^i_{t-1} \alpha^{i}_{m,t}}{\rho^{i}_{m,t}} = \frac{w^i_m}{\rho^{i}_{m,t}} - 1.
\]

(17)

Weighting \( r^i_{m,t} \) over all assets \( (m = 1, \ldots, M) \) by its associated portfolio, one then derives the rate of return of agent \( i \) for period \( t \);

\[
r^i_t = \sum_{m=1}^{M} \alpha^{i}_{m,t} r^i_{m,t}. \]

(18)

Let \( \tilde{r}^i \) and \( \text{var}(r)^i \) denote the sample mean and the respective sample variance of the series \( \{r^i_t\} \). Table 3 shows these two statistics, which are averaged over the entire 100 simulation runs and are further averaged over the five agents of the identical type.

From Table 3, we learn that the type-1 agents do not survive because of their superior performance in the expected rate of return. As a matter of fact, among all the seven types of agents, it is they that have the lowest rate of return (2.38), which is different from what one may expect from the Kelly criterion. Nonetheless, the column “variance of return” indicates that these agents are under different exposure to risk, and it is the type-1 agents who are exposed to the lowest risk. This result is not totally unanticipated given the fact that the type-1 agents are the most risk-averse.

Motivated by this finding, we go further to examine the risk-adjusted return, also known as the Sharpe ratio, and we find that type-1 agents do not perform particularly well in terms of the Sharpe ratio.

\[\text{Stable saving behavior is not new in economic analysis. It has already drawn the attention of economists in the recent studies on the lock-up savings (Laibson, 1998).}\]

\[\text{However, one important observation in Sandroni’s analysis, when he endogenized the saving decision, is that the consequence of forecasting accuracy should not just be limited to the portfolio decision, but also the saving decision. This is by and large consistent with our analysis above.}\]
Especially, one may suppose that every investor whose performance is situated at the efficient frontier has an equal chance to survive.\(^{14}\) Therefore, we see no particular reason to attribute the survival of the type-1 agents to their portfolio performance (Fig. 6).

5.2. Experiment 2

In Section 5.1.2, we already saw that agents with identical capabilities of forecasting do not survive equally well. This result already evidenced that forecasting accuracy is not the primary force in the determination of survivability. Instead, preference plays a dominating role. However, since all agents forecast equally well (Table 3), the result obtained in Experiment 1 does not lend strong support to the evidence that shows that forecasting accuracy does not matter at all. Therefore, to see whether we can consolidate the argument that forecasting accuracy does not matter, a different experimental design is proposed in this section.

Experiment 2 assigns different values of \(v\) (i.e. different horizons of the validation sample; see Eq. (A.9)). Since the underlying dividends series is stationary, doing this will normally lead those agents using a long validation horizon to outperform those agents using a short validation horizon.\(^{15}\) Now, given the difference in their forecasting accuracy, it is time to ask whether agents, regardless of their preference types, with superior forecasting accuracy can survive better as opposed to those agents with inferior forecasting accuracy, which is also the main issue addressed in Sandroni’s proposition 3. What is particularly relevant to our concern is whether agents of types 2–7 who have long validation horizons can drive out those type-1 agents who have short validation horizons.

In Fig. 7, we classify agents with different preferences into two groups, namely, agents of type 1 alone, and agents of types 2–7 together, and plot the time series of the wealth share dynamics of these two different groups of agents. Additional restrictions are added to these two groups. For type-1 agents, only those whose validation horizons are short (to be specific, \(v = 10, 15\)) are included, and the respective time series plot is denoted by diamonds. For the remaining two groups of agents, we only consider those with long horizons (\(v = 50, 100\)), and the time series plots are denoted by squares.

From Fig. 7, we can see that, even though the validation horizon is lengthened to enhance the forecasting accuracy of agents of types 2–7, this group of agents does not perform well; their wealth share per capita continues to decline toward nil. Therefore, it is clear that forecasting accuracy does not help them much to survive. Nonetheless, if we restrict our attention to agents with different validation horizons, then the significance of forecasting accuracy is revealed. Fig. 8 shows the wealth dynamics per capita by polling agents with the same validation horizon together. While the wealth dynamics fluctuates quite severely, the general tendency indicates that, regardless of their risk preference,

\(^{14}\) To see this, the risk-return plot is drawn in Fig. 6. The continuous frontier line is constructed by smoothly connecting the three points on the frontier. The three points on the frontier correspond to type-1, type-2 and type-6 agents. While the other four types of agents do not lie exactly on the frontier, they are not far away from it.

\(^{15}\) This result is confirmed by the K-S statistic. The K-S statistic of the five validation schemes, \(v = 10, 15, 25, 50 \text{ and } 100\), are 0.0757, 0.0695, 0.0623, 0.0551, and 0.0497, respectively. The ANOVA analysis shows that they are significantly different with an \(F\) statistic of 26.41.
agents with better forecasting accuracy tend to have higher wealth shares than agents whose forecasting accuracy is worse. Therefore, forecasting accuracy, as our conventional wisdom may suggest, does matter for the prosperity of agents. It is significant when agents share the same risk preference. Nevertheless, when agents have heterogeneous risk preference, its importance is only secondary, and the effect of risk preference may dominate.

6. Concluding remarks

Earlier theoretical studies have already shown the irrelevance of risk preference to survivability. They have also shown that what matters is forecasting accuracy. After introducing bounded-rational behavior, we have almost the opposite: risk preference matters, and it is even more important than forecasting accuracy. Why does risk preference

---

\(^{16}\) The result that agents with better forecasting accuracy would not necessarily survive may not be so surprising for the agent-based financial models in which agents have expectations about future prices rather than future dividends. As Hommes (2006) has made clear, in this more popular way of modeling expectations, a positive feedback mechanism can exist that will help agents with “wrong” beliefs to survive. We completely agree with his argument. In other words, if we replace future dividends with future prices in our model, we may enhance the survivability of agents with “wrong” beliefs. Nonetheless, there is an important reason to study the survivability issue in the spirit of Blume–Easley–Sandroni, namely, a well-defined notion of forecasting accuracy. When expectations are made on future prices, the self-fulfilling property makes it more difficult to identify agents with wrong beliefs. Hence, the issue as to the survivability of agents with “poor forecasting accuracy” becomes less clear and less motivated. Maybe, in this situation, the right question to ask is the survivability of fundamentalists or chartists, as we frequently see in the literature.
matter? The paper examines three interdependent possibilities: forecasting accuracy, portfolio, and saving among different types of bounded-rational agents. While being bounded rational, all of these agents are at least potentially equally smart in the sense that they are all equipped with the same adaptive search scheme, namely, the genetic algorithm. The neutrality of GA, in that GA does not make one type of agent smarter than the others, is also supported in Sections (5.1.2) and (5.1.4). Therefore, the forecasting accuracy and portfolio performance are excluded, and what is left is only the saving behavior. Therefore, the significance of risk preference is manifested by the saving behavior.

While earlier studies recognized the importance of saving behavior to this issue, the attention has been restricted to the rational-equilibrium path. Various moments of saving behavior are not a concern as long as they are on the rational-equilibrium path. However, when they are not, this paper shows that such moments have strong implications for survivability. In addition to the first moment, which has already been explicitly noticed by Blume–Easley–Sandroni, we also find the significance of other high-order moments. Downside saving rates as well as the dispersion of saving rates can both be important.

Since the saving decision in general is dependent upon the belief, errors in forecasting accuracy can propagate through the saving decision and manifest themselves in moments of saving. This observation generally applies to all types of agents, except type-1 agents, the log-utility agents, whose saving decisions are independent of their beliefs and are only determined by the exogenously given discount rate. That is what makes the log-utility agent so different from other agents. The conventional wisdom characterized by the Kelly criterion also works on this, but has not successfully established its validity, in particular, in the general equilibrium context. This paper shows that incorporating learning dynamics is one way of demonstrating its validity.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.jebo.2006.11.006.

References