Using Genetic Algorithms to Simulate the Evolution of an Oligopoly Game*

Shu-Heng Chen[1] and Chih-Chi Ni[2]

[1] AI-ECON Research Group
Department of Economics
National Chengchi University
Taipei, Taiwan 11623
E-mail: `chchen@cc.nccu.edu.tw`
[2] AI-ECON Research Group
Department of Economics
National Chengchi University
Taipei, Taiwan 11623
E-mail: `g2258503@grad.cc.nccu.edu.tw`

Abstract. This paper extends the N-person IPD game into a more interesting game in economics, namely, the *oligopoly game*. Due to its market share dynamics, the oligopoly game is more complicated and is in general not an exact N-person IPD game. Using genetic algroithms, we simulated the oligopoly games under various settings. It is found that, even in the case of a three-oligopolist (three-player) game, collusive pricing (cooperation) is not the dominating result.
Keywords: Oligopoly, Cartels, Price Wars, Genetic Algorithms, State-Dependent Markov Chain, Coevolution.

1 Motivation and Introduction

In the past, the *prisoner's dilemma* was frequently applied to the study of *collusive pricing* or *predatory pricing*. However, this application is largely restricted to the *duopoly* industry because most economists are only familar with the 2-person *Iterated Prisoner's Dilemma* (IPD) game. In terms of the *oligopoly* industry, the more relevant one should be the *n-person IPD game*, which economists are less familiar with. Recently, the n-person IPD game was studied in Yao and Darwen (1994). Using *genetic algorithms* (GAs), they showed that *cooperation can still be evolved in a large group, but that it is more difficult to evolve cooperation as the group size increases.* Considering this result as a guideline for the oligopoly pricing probelm, then what the n-person IPD game tells us is that *when the number of oligopolists is small, say 3, it is very likely to see the emergence of collusive*

* This is a revised version of a paper presented at *The Second Asia-Pacific Conference on Simulated Evolution and Learning* in Canberra, Australia, 24-27 November, 1998. The authors thanks two anonymous referees for helpful comments. Research support from NSC grant NSC. 86-2415-H-004-022 is also gratefully acknowledged.

X. Yao et al. (Eds.): SEAL'98, LNCS 1585, pp. 293–300, 1999.

pricing (cooperation). However, real data usually shows that, even in a three-oligopolist industry, the observed pricing pattern is not that simple. (Midgely, Marks and Cooper, 1996)

- First, while collusive pricing is frequently observed, it is continually interrupte by the occurence of predatory pricing (price wars).
- Second, it is not always true that oligopolists are either *collectively* charging high prices (collusive pricing) or low prices (price wars). In fact, a dispersion of prices can persistently exist, i.e., some firms are charging a higher price, whilst others are charging a lower price.
- Third, the firms who charge a high price may switch to a low price in a later stage, and vice versa.

These features seem to be difficult to be displayed in 3-person IPD games (See Yao and Darwen,ibid, Figure 5). Therefore, one may reasonably conject that *the oligopoly game is not an exact n-person IPD game.* While they share some common features, there are other essential elements which distinguish these two games.

In this paper, we consider the payoff matrix determined by *the market share dynamics* as such an essential element. In Section 2, we propose a very simple oligopoly game with 3 oligopolists. We then in Section 3 show that this setup disqualify the oligopoly game from being an n-person IPD game. Due to the non-equivalence of these two games, we use genetic algroithms to simulate the evolution of oligopoly games in Sections 4 and 5. The simulation results are given in Section 5, followed by concluding remarks in Section 6.

2 The Analytical Model

For simplicity, an oligopoly industry is assumed to consist of three firms, say $i = 1, 2, 3$. At each period, a firm can either charge a high price P_h or a low price P_l. Let a_i^t be the action taken by firm i at time t. $a_i^t = 1$ if the firm i charges P_h and $a_i^t = 0$ if it charges P_l. To simplify notations, let S_t denote the row vector (a_1^t, a_2^t, a_3^t). To characterize the price competition among firms, the *market share dynamics* of these three firms are summarized by the following *time-variant state-dependent Markov transition matrix*,

$$M_t = \begin{bmatrix} m_{11}^t & m_{12}^t & m_{13}^t \\ m_{21}^t & m_{22}^t & m_{23}^t \\ m_{31}^t & m_{32}^t & m_{33}^t \end{bmatrix} \tag{1}$$

where m_{ij}^t, the transition probability from state i to state j, denotes the proportion of the customers of firm i switching to firm j at time period t. Let n_i^t (i=1,2,3) be the number of customers of firm i at time period t, and N_t the row vector $[n_1^t, n_2^t, n_3^t]$. Without loss of generality, we assume that each consumer will purchase only one unit of the commodity. In this case, N_t is also the vector of quantities consumed. With N_t and M_t, the customers of each firm at period $t+1$ can be updated by:

$$N_{t+1} = N_t M_t \tag{2}$$

To see the effect of price competition on the market share dynamics, the transition probabilities m_{ij}^t are assumed to be dependent on the pricing strategy vector S_t. If three firms charge the same price, then M_t is an *identity matrix*. Furthermore, if firm i charges P_h, then it will lose $\frac{\alpha}{2}$ percent of its consumers each to firms j and k, who charge P_l. Furthermore, if firms i and j charge P_h, then they each will lose α percent of their consumers to firm k, who charges P_l.

Given these state-dependent transition matrices, Equation (2) can be rewritten as:

$$N_{t+1} = N_t M_t(S_t), \tag{3}$$

where $S_t = (a_1^t, a_2^t, a_3^t)$ and $a_i^t \in \{0, 1\}$.

Equation (3) summarizes the *intra*-industry competition given a number of customers $n_t = \sum_{i=1}^{3} n_t^i$. The next step of our modeling is to endogenize n_t by setting n_{t+1} as a function of S_t. More precisely,

$$n_{t+1} = n_t(1+\beta), \quad \beta = \beta(S_t) \tag{4}$$

The $\beta(.)$ function explicitly shows how the market share of the industry can be affected by its pricing strategies S_t. The simple $\beta(.)$ function considered in this paper is as follows.

$$\beta = \begin{cases} \delta_W, \text{ if } & \sum_{i=1}^{3} a_i = 0, \\ \delta_w, \text{ if } & \sum_{i=1}^{3} a_i = 1, \\ \delta_c, \text{ if } & \sum_{i=1}^{3} a_i = 2, \\ \delta_C, \text{ if } & \sum_{i=1}^{3} a_i = 3. \end{cases} \tag{5}$$

where $\delta_W \geq \delta_w \geq \delta_c \geq \delta_C$.

Given Equations (3)-(5), the objective of oligopolists is to maximize their profits or the present value of the firm, and the profits for a single period is given by Equation (6).

$$\pi_i^s = (P_i^s - C)n_i^s \tag{6}$$

where P_i^s is the price charged by firm i at period s, n_i^s number of customers, and C fixed unit cost. n_i^s can be obtained from Equations (3) - (6).

3 The Oligopoly Game: an N-Perosn IPD Game?

Before proceeding further, let us consider the relevance of the n-person IPD games to the oligopoly game. *Is an oligopoly game necessarily an n-person IPD game?* If not, what is their relation? For simplicity, let us consider the first r periods of an oligopoly game. Here, "cooperate" (C) means "charging high prices for all r periods" and "defect" (D) means "charging low prices for all r periods". We first work out the *payoff matrix* defined by Yao and Darwen (1994). In our

Table 1. Parameters and Payoffs

Set	P_H	P_L	C	α	r	D_2	D_1	D_0	C_2	C_1	C_0
1	1.4	1.2	1	0.2	8	3.47	2.07	1.6	3.2	1.33	1.33
2	1.4	1.2	1	0.2	25	13.40	7.10	5	10	1.60	1.60
3	2	1.2	1	0.2	8	3.47	2.07	1.6	8	3.33	3.33
4	2	1.2	1	0.2	25	13.40	7.10	5	25	3.98	3.98

case (3 oligopolists), there are six elements in the payoff matrix, namely C_i and D_i ($i = 0, 1, 2$). Here, C_i (D_i) denotes the payoff for a specific player who plays C (D) when there are i players acting cooperatively. From Equations (3)-(6), C_i and D_i can be derived. Without losing generality, let us assume that $\beta = 0$ and $n_1 = n_2 = n_3 = 1$, then the explicit solutions obtained are:

$$\begin{bmatrix} D_2 \; C_1 \; C_1 \end{bmatrix}' = \begin{bmatrix} (P_L - C)[3r - 2\frac{(1-\alpha)-(1-\alpha)^{r+1}}{\alpha}] \\ (P_H - C)[\frac{(1-\alpha)-(1-\alpha)^{r+1}}{\alpha}] \\ (P_H - C)[\frac{(1-\alpha)-(1-\alpha)^{r+1}}{\alpha}] \end{bmatrix}$$

$$\begin{bmatrix} D_1 \; D_1 \; C_0 \end{bmatrix}' = \begin{bmatrix} (P_L - C)[r + \frac{1}{2}r - \frac{1}{2}\sum_{j=1}^{r}(1-\alpha)^j] \\ (P_L - C)[r + \frac{1}{2}r - \frac{1}{2}\sum_{j=1}^{r}(1-\alpha)^j] \\ (P_H - C)(\sum_{t=1}^{r}(1-\alpha)^t) \end{bmatrix}$$

$$\begin{bmatrix} C_2 \; C_2 \; C_2 \end{bmatrix}' = \begin{bmatrix} (P_H - C)r \\ (P_H - C)r \\ (P_H - C)r \end{bmatrix}, \begin{bmatrix} D_0 \; D_0 \; D_0 \end{bmatrix}' = \begin{bmatrix} (P_L - C)r \\ (P_L - C)r \\ (P_L - C)r \end{bmatrix}.$$

Whether the oligopoly game is an n-person IPD game depends on the following criteria (Yao and Darwen, 1994):

- (1) $D_2 > C_2$, (2) $D_1 > C_1$, and (3) $D_0 > C_0$.
- (4) $D_2 > D_1 > D_0$, and (5) $C_2 > C_1 > C_0$.
- (6) $C_2 > \frac{D_2+C_1}{2}$, and (7) $C_1 > \frac{D_1+C_0}{2}$.

It is not difficult to see that not all of these conditions can be satisfied. For example, in Table 1, four sets of parameters and their associated payoffs are given. The conditions which can be satisfied by these four sets of parameters are summarized in Table 2.

Given the analysis above, we may consider the oligopoly game is a perturbation or a generalization of an n-person IPD game, and it is interesting to see whether the evolution process of the n-person, in particular, the 3-person, IPD game documented by Yao and Darwen (1994) still applies.

Table 2. Parameter Sets and Testing Results

Inequality	Set 1	Set 2	Set 3	Set 4
1. $D_2 > C_2$	$>$	$>$	$<$	$<$
2. $D_1 > C_1$	$>$	$>$	$<$	$>$
3. $D_0 > C_0$	$>$	$>$	$<$	$>$
4. $D_2 > D_1 > D_0$	$>,>$	$>,>$	$>,>$	$>,>$
5. $C_2 > C_1 > C_0$	$>,=$	$>,=$	$>,=$	$>,=$
6. $C_2 > 0.5(D_2 + C_1)$	$>$	$>$	$>$	$>$
7. $C_1 > 0.5(D_1 + C_0)$	$<$	$<$	$>$	$<$

The sign $>$ in columns 2-5 means the condition is satisifed. Other signs means the condition is weakly violated ($=$) or strongerly violated ($<$).

4 Modeling the Adaptive Behavior of Oligopolists with GAs

The main idea of genetic algorithms is to encode the variable one wants to optimize as a binary string and work with it. Following, Midgley et al (1996), we consider the following special class of pricing strategy ψ,

$$\psi : \Omega_k \longrightarrow \{0, 1\}, \tag{7}$$

where Ω_k is the collection of all $\{S_{t-j}\}_{j=1}^{k}$. By this simplification, the oligopolist's memory is assumed to be *finite*.

While, potentially, different choices of k may lead to quite different sets of strategies (Beaufils et al., 1998), the issue concerns us is the smallest value of k which can reasonably replicate the price dynamics of the oligopoly industry, and as we shall see later, setting k to equal 1 is good enough to achieve this goal.

5 Experimental Designs and Results

For all the experiments conducted in this study, P_h is set at "2", P_l "1.2" and C "1". Other control parameters of GAs are set according to Tables 3 and 4.

The first experiment is to test whether GA-based oligopolists can achieve a reasonable level of adaptation. For this purpose, we design the experiment "*absolute-loyalty-with-no-external-effects*". In terms of notations, *absolute loyalty* means $\alpha = 0$, and *the absence of external effects* means $\beta = 0$. When $\alpha = \beta = 0$, the most profitable pricing strategy for firm i is obviously an unconditional high-price strategy, i.e.,

$$\psi_i = 1, \quad \forall S_t \in \Omega_1, \tag{8}$$

since a lower price will not help the firm to gain any advantages over its competitors or other industries. So, we expect that the GA-based oligopoly industry should converge to a state of a collusive price, i.e., the state $(1, 1, 1)$.

In order to test whether GAs can find out this simple solution, we ran experiment 1 for 1000 periods (125 generations) with the prespecified parameters

Table 3. The Parameters of the GA-based Oligopoly Game

Memory size (k)	1
Number of oligopolists	3
Population size (l)	30
Number of periods in a single play (r)	8 (25)
Selection Scheme	Roulette-wheel selection
fitness function	Profits (π)
Number of generations evolved (Gen)	125 (126)
Number of periods (T)	1000 (3150)
Crossover Style	One-Point Crossover
Crossover rate	0.8
Mutation rate	0.0001
Immigration rate	0.001

Table 4. Experimental Designs and Results

Experiment	r	# of Simulations	α	δ_W	δ_w	δ_c	δ_C	Results
Pilot	8	5	0	0	0	0	0	C(5)
1	25	5	0.2	0	0	0	0	C(2), c(1), NC(2)
2	8	5	0.2	0	0	0	0	C(5)

given in Tables 3 and 4. To facilitate the report of simulations, we need a few more notations. Let "W" refer to the state "*price war*" (0,0,0), "C" the state "collusive price" (1,1,1), "w" the states which are closer to "W" and "c" the states closer to "C". "Closer" is defined in terms of *Hamming distance*. Thus, "w" includes states (0,0,1), (0,1,0) and (1,0,0), and "c" includes (1,1,0), (1,0,1), (0,1,1). Since there are 30 pairs of oligopolists in each period of the evolution, to summarize simulation results of S_t in terms of its *distribution*, let p^t_W, p^t_w, p^t_c, and p^t_C denote respectively the percentage of the pairs who, in period t, are in the states labeled with "W", "w","c", and "C" respectively. Figures 1.1-1.5 display the time series plot of the distribution of S_t. From Figures 1.1-1.5, we can see that the industry converges to the state "C" (1,1,1) very quickly.

In Experiment 1, α is set to be 0.2. In the meantime, we still assume the absence of external effects, i.e., β remains to be zero. In this situation, it is not difficult to see that the best solution is to *form a cartel* and to *jointly charge a high price*. To see how well our GA-based adaptive oligopolists evolve in this scenario, we ran Experiment 1 for 3150 periods (126 generations), and the time series of the distribution of S_t is shown in Figures 2.1-2.5. From Figures 2.2 and 2.5, we can see that, like the Pilot Experiment, p^t_C gradually increases and eventually converges to 1. However, as compared with Figure 1.1-1.5, it can be seen that the convergence speed is *much slower*.

The interesting patterns observed in this experiments are shown in Figures 2.3 and 2.4. In these two simulations, we experience an oscillation between the

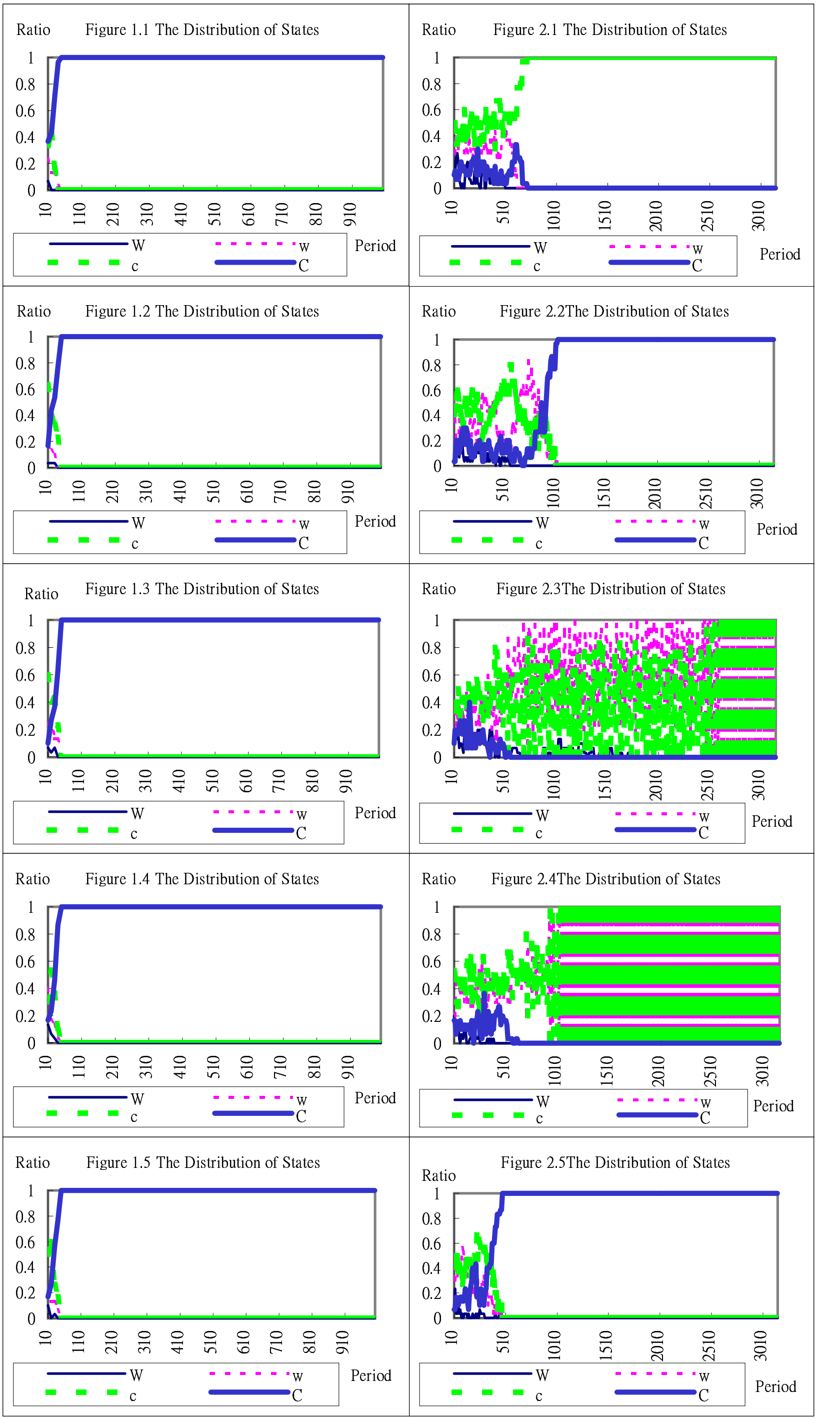
Figure 1.1 The Distribution of States
Figure 2.1 The Distribution of States
Figure 1.2 The Distribution of States
Figure 2.2The Distribution of States
Figure 1.3 The Distribution of States
Figure 2.3The Distribution of States
Figure 1.4 The Distribution of States
Figure 2.4The Distribution of States
Figure 1.5 The Distribution of States
Figure 2.5The Distribution of States
Ratio
Period
1
0.8
0.6
0.4
0.2
0
10
110
210
310
410
510
610
710
810
910
1010
1510
2010
2510
3010
W
w
c
C

states "w" and "c", i.e., *three firms are continuously charging different prices.* This is the second and the third stylized facts of oligopoly industries summarized in Section 1. The emergence of persistenly heterogeneous pricing may be caused by the inconsistency between "$D_2 < C_2$" and "$D_1 > C_1$" for the first r periods (Table 2). This inconsistency may encourage an early defection, and once that happens, by the path-dependent property, the oligopoly game is further perturbated away from a standard n-person IPD game and may support its own complex dynamics. To see whether or not this conjecture is correct, we design the experiment 2 as shown in Table 4.

The only difference between Experiment 1 and Experiment 2 lies in the choice of the parameter r. The setting has been changed from 25 to 8. By Table 2, this makes the first three inequalities all consistent, i.e., $D_i < C_i, i = 0, 1, 2$. This structure shall punish early defection, and keep the payoff structure unchanged. Then the whole process can be reinforced (an aspect of the path-dependent property). The simulation results, as we have conjected, all converge to the state of collusive pricing.

6 Concluding Remarks

The message revealed in this paper is simple: *the oligopoly game in general is not an n-person IPD game and, in effect, is more complicated than that.* Therefore, the simulated results can be quite rich in even a 3-person oligopoly game. But, that also bridges the gap between the complexity of the oligopolists' pricing behaviour and the the simplicity of the insight gained from the n-person IPD games. In a word, we think that the oligoply game is a meaningful generalization of the n-person IPD game, and a formal mathematical treatment of it is definitely a direction for future research.

References

1. Beaufils, B. J.-P. Delahaye and P. Mathieu (1998), "Complete Classes of Strategies for the Classical Iterated Prisoner's Dilemma," in V. W. Porto, N. Saravanan, D. Waggen and A. E. Eiben (eds.), *Evolutionary Programming VII*, pp. 32-41.
2. Midgley, D. F., R. E. Marks, and L. G. Cooper (1996), "Breeding Competitive Strategies," forthcoming in *Management Sciences*.
3. Yao, X. and P. J. Darwen (1994), "An Experimental Study of N-Person Iterated Prisoner's Dilemma Games," *Inoformatica*, Vol. 18, pp. 435-450.