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Simulating economic transition processes by genetic programming

Shu-Heng Chen and Chia-Hsuan Yeh

AI-ECON Research Group, Department of Economics, National Chengchi University, Taipei, Taiwan 11623
E-mail: chchen@nccu.edu.tw, g3258501@grad.cc.nccu.edu.tw

Recently, *genetic programming* has been proposed to model agents' adaptive behavior in a complex transition process where uncertainty cannot be formalized within the usual probabilistic framework. However, this approach has not been widely accepted by economists. One of the main reasons is the lack of the *theoretical foundation* of using genetic programming to model transition dynamics. Therefore, the purpose of this paper is two-fold. First, motivated by the recent applications of *algorithmic information theory* in economics, we would like to show the relevance of genetic programming to transition dynamics given this background. Second, we would like to supply two concrete applications to transition dynamics. The first application, which is designed for the pedagogic purpose, shows that genetic programming can simulate the *non-smooth transition*, which is difficult to be captured by conventional toolkits, such as differential equations and difference equations. In the second application, genetic programming is applied to simulate the adaptive behavior of speculators. This simulation shows that genetic programming can generate artificial time series with the statistical properties frequently observed in real financial time series.

Keywords: Kolmogorov complexity, minimum description length principle, genetic programming, bounded rationality, short selling

1. Introduction and motivation

> In Eastern Europe the transition is not like that: people there are confronted with *unprecedented opportunities*, *new and ill-defined rules*, and a daily struggle to determine the "mechanism" that will eventually govern trade and production. Economists who dispense advice about government strategies to enable transitions to a market economy can do so with ample help from "equilibrium theories"..., *but with virtually no theories about the transition itself*. ([31, p. 1]. Italics Added.)

At the current state of economics, *transition dynamics* can be considered one of the most poorly understood areas. The major reason is that we know very little about how to model *learning or adaptive agents* in a complex transition process.[1] Traditional approaches to modeling *adaptive agents* are mainly based on *econometrics*, either the

[1] In fact, the whole book of [31] is devoted to this "wild" area.

classical or the *Bayesian*, and since the celebrated work by [18], econometrics has been founded on probability theory. However, interesting transition dynamics usually involves the notion of *novelty*, which is a very controversial object and may be too difficult to be dealt with by probability theory. For example, [21] stated: “The majority of observed phenomena of randomness in Nature (always excluding games of chance) cannot be and should not be explained by (conventional) probability theory; there is little or no experimental evidence in favor of (conventional) probability but there is massive, accumulating evidence that explanations and descriptions should be sought outside of the conventional framework.” (Ibid, p. 142.)[2] Similarly, [32] asserted “For very complex problems, like predicting earthquakes, or effects of medicine on humans, we have very poor understanding of the phenomena and we do not know what all the relevant information is . . . The big problem is to discover which variables are the relevant predictors.” (Ibid, p. 4.)

Recently, *genetic programming* has been proposed to model agents’ adaptive behavior in a complex transition process whose uncertainty cannot be formalized within the usual probabilistic framework [10,25]. However, this approach has not been widely accepted by economists. One of the main reasons is the lack of the *theoretical foundation* of using genetic programming to model transition dynamics. Therefore, the purpose of this paper is two-fold. First, motivated by the recent applications of *algorithmic information theory* in economics, we would like to show the relevance of genetic programming in transition dynamics given this background. Second, we would like to supply two concrete applications to illustrate that genetic programming can be a promising approach to modeling transition dynamics. The first application, which is designed for the pedagogic purpose, shows that genetic programming can simulate the *non-smooth transition*, which is difficult to be captured by conventional toolkits, such as differential equations and difference equations. In particular, this application displays the process by which market participants adapt to a typical external shock, i.e., *a sudden change in demand*. In the second application, genetic programming is applied to simulate the adaptive behavior of speculators. The simulation shows that genetic programming can generate artificial time series with the statistical properties frequently observed in real financial time series.

The rest of this paper is organized as follows. In section 2, the relevance of genetic programming to transition dynamics is discussed in the light of Kolmogorov complexity. Sections 3 and 4 presents the applications of genetic programming mentioned as above. Section 5 gives the concluding remarks.

2. Genetic programming and Kolmogorov complexity

While a relevant notion of randomness in economics has yet to be established, *algorithmic randomness* or *Kolmogorov complexity* has recently become the focus of the attention of many economists who are reexamining randomness [7,15,16,33]. This

[2] I owe this reference to Prof. K. Velupillai.

modern notion of randomness is proposed by A.N. Kolmogorov in 1965 to quantify the randomness of individual objects in an *objective* and *absolute* manner.[3] The distinguishing features of Kolmogorov complexity are two-fold.

- It defines randomness from the perspective of *individual* objects.
- It defines randomness based on *finite* strings.

In conventional probability theory, randomness cannot be defined without reference to probability theory, and according to Mises, the existence of probability rests on the existence of the limit of relative frequency. Therefore, the classical notion of randomness is a notion based on *infinite* string rather than a *finite* one. Furthermore, the random object, called the random variable in probability theory, refers to a *mapping* from the *sample space* to the real line. Clearly, it is a notion based on a *group* of objects (a sample space) rather than the *individual* object. Therefore, Kolmogorov complexity and probability theory results in two fundamentally different notions of randomness. But which one is more relevant to transition dynamics?

To answer this question, one has to notice that a non-trivial transition process can be characterized by its introduction of *novelties*. Technically speaking, it is a *nonergodic* process. "Whenever economists talk about "structural breaks" or "changes in regime", they are implicitly admitting that the economy is, at least at that point of time, not operating under the ergodic presumption that the past objective probabilities will continue to govern future events." [13, p. 129]. Davidson's observation may be further formalized by the following *jump process*.

A jump process is a continuous-time and discrete-state Markov process. Let Ψ be the *state space* which is a collection of models (regimes), i.e.,

$$\Psi \equiv \{f_1, f_2, \ldots, f_n, \ldots\}. \tag{1}$$

The model (f_i) is corresponding to the state i. The cardinality of Ψ may be infinity. Now, let ω be the *waiting time* for regime switches or structural changes to occur, and ω is randomly distributed with the *waiting time distribution* $\wp(\omega)$. If at time t_1, the "switch" operator is *on*, the state at time $[t_1]$, say state j (f_j), shall switch to a state k (f_k) $(f_k \in \Psi$ and $k \neq j)$ at time $[t_1]+1$ where [.] is the Gauss symbol. The switch from j to k is randomly determined by the *embedded transition matrix* $T(j,k)$ where

$$T(j,k) \equiv \mathrm{Prob}\big((f_k) \mid (f_j)\big), \quad \forall\ j,k \in \Psi. \tag{2}$$

Clearly, the jump process described above is in a generalized direction of the *switching regime Markov process* proposed by [20]. As a consequence, it shares some common features with it. First of all, both take *structural changes* formally into account. For the switching regime Markov process, this is done by the Markov transition process and is discrete-time, while for the jump process, it is done by a waiting time distribution and is hence continuous. Furthermore, through fine-tuning

[3] To control the size of this paper to a reasonable limit, a technical introduction to Kolmogorov complexity is omitted here. The interested reader is referred to [26].

the parameters of either the Markov transition process or the waiting time distribution, both models are good enough to simulate different frequencies of structural changes.

Despite these similarities, the term "structural changes" are not used in the same way in these two models. In switching regime Markov process, there are only a finite number of states (models). Hence, in the long run, structural changes only mean revisiting one of these models and have nothing to do with *novelties*. However, in the jump process, there are infinite number of states; therefore, even in the long run, structural changes can always refer to novelties. As a result, these two models also haves different implications for the significance of adaptation. In the switching regime Markov process, eventually, there is no need to discover the new pattern. Adaptation only means to *recall the relevant portion of memory*. In the jump process, one has to be alert to the patterns never seen before, and adaptation means the capability of continuously detecting structural changes and discovering new patterns.

In general, the jump process is *nonerogodic*. When the stochastic process is nonerogodic, the usual claim to justify the statistical meaning of *a single realization of time series* no longer holds. Furthermore, infrequent structural breaks with novelties introduced may nullify the use of the large-sample dataset as an effective way of learning. Thus, a notion of randomness which refers only to *individual objects* with *finite strings* may be more appropriate to interpret a nonergodic world. This is the reason why we consider *Kolmogorov complexity* a useful notion of uncertainty observed in the real-world transition dynamics.

Unfortunately, Kolmogorov complexity is not computable. Therefore, to put it into application, some approximation techniques are needed and this is done by Rissanen's *stochastic complexity* or the *minimum description length principle* (MDLP) [30]. While MDLP has been extensively used to solve model selection problems in econometrics [23], its relevance to modeling adaptive agents in transition process has not been well noticed. As [24] pointed out, "Rissanen's MDL principle is a criterion for the most economical description of the regularities to be found in "strings" of data. We should not look at Rissanen's work as "only" providing a new statistical foundation to the econometricians; it also offers theorists a way to populate their models with agents that learn by *induction*." [24, p. 6].

In a series of papers [7–9], Chen and Tan used MDL to model the adaptive perception of investors in financial markets. They found that not only can MDL formalize [3]'s notion of the efficient market hypothesis, but it also enables us to observe the transition between sophisticated behavior and myopic behavior. However, as noticed by [29], while MDL is quite effective in the assessment of hypotheses, given some data, it is mute on the question of *how one can best generate hypotheses deemed worth assessing*. The solution proposed by [29] is to use *genetic algorithms* to generate hypotheses. In this paper, we extend their solution by using *genetic programming* to generate hypotheses.[4]

[4] The difference between the use of genetic algorithms and genetic programming in modeling adaptive agents in economics is well documented in [10] and [12].

3. Simulating transition processes with GP: A pedagogic example

The analysis of an economic model is usually composed of three parts, namely, the *static analysis*, the *comparative static analysis* and the *dynamic analysis*. The static analysis predicts the long-run behavior implied by a model provided *other things are equal*. The comparative analysis instructs us how to modify the prediction if exogenous variables change. Instead of the long-term prediction, the dynamic analysis informs us of the *transition process* from the old state to the new one when exogenous variables change. In conventional education of analytical economics, the mainstream techniques employed in the *adjustment process* are *differential* and *difference equations*. Furthermore, associated with the commonly accepted simplification, i.e., the *representative agent*, the whole adjustment process can be dramatically reduced to the study of only a few *difference* or *differential equations*. This setup also trivializes one important aspect of the transition process, i.e., *coordination of beliefs*, including the evolution of beliefs of agents.

Motivated by the study of [31], we use *genetic programming* to provide a *population-based approach*, also known as the *agent-based approach*, to incorporate the *coordinating process of beliefs* into *transition dynamics*. While genetic programming is not the only technique available for modeling transition, economists may consider this approach attractive because it packs with many old and good economic ideas, such as the *principle of survival of the fittest*, the process of *random choice and matching*, the usage of *primitive ideas*, *norms*, *conventions* or *a rule of thumb*, and the implicit restriction of the *complexity* of decision making. In addition, it is a *population* model. It is our belief that all these features are crucial for modeling transition processes. Alternatively speaking, if we are thinking of transition processes within the context of *heterogeneous agents* who can *adapt* and *learn* while subject to *limited cognitive capability* (*bounded rationality*), then *genetic programming* should have great potential to serve this need.

3.1. The market: Demand and supply

The particular economic example which we work with is a *standard demand and supply model*, more precisely, a *cobweb model*. The cobweb model used in this paper is based on [28]. Consider a competitive market composed of n firms which produce the same goods by employing the same technology and which face the same cost function described in equation (3):

$$c_{i,t} = xq_{i,t} + \frac{1}{2}ynq_{i,t}^2, \tag{3}$$

where $q_{i,t}$ is the quantity supplied by firm i at time t, and x and y are the parameters of the cost function. The production of the goods takes one unit of time, i.e., the quantity to be supplied at time t must start to be produced at time $t-1$. Since at time $t-1$, the price of the goods at time t, P_t, is not available, the decision about optimal $q_{i,t}$ must be based on the expectations of P_t, i.e., $P^e_{i,t}$.

Given price expectations $P^e_{i,t}$ and the cost function $c_{i,t}$, the expected profit of firm i at time t can be expressed as follows:

$$\pi^e_{i,t} = P^e_{i,t} q_{i,t} - c_{i,t}. \tag{4}$$

Given $P^e_{i,t}$, $q_{i,t}$ is chosen at the level such that $\pi^e_{i,t}$ can be maximized and, according to the first order condition, is given by

$$q_{i,t} = \frac{1}{yn}\left(P^e_{i,t} - x\right). \tag{5}$$

Once $q_{i,t}$ is decided, the aggregate supply of the goods at time t is fixed and P_t, which sets demand equal to supply, is determined by the demand function:

$$P_t = A - B\sum_{i=1}^{n} q_{i,t}. \tag{6}$$

Given P_t, the actual profit of firm i at time t is

$$\pi_{i,t} = P_t q_{i,t} - c_{i,t}. \tag{7}$$

In a representative-agent model, it can be shown that the *rational expectations equilibrium price* P^* and *quantity* Q^* are [28]

$$P^*_t = \frac{Ay + Bx}{B + y}, \tag{8}$$

$$Q^*_t = \frac{A - x}{B + y}. \tag{9}$$

3.2. Experimental design: Shift in the demand curve

The cobweb model is frequently taught in the introductory course in economics, and economists repeatedly use this model to exemplify the formation of expectations of the price in a decentralized market [28]. In the literature of *bounded rationality*, the cobweb model is also frequently referred as an illustration of learning schemes [27]. In this paper, we consider two different versions of the cobweb model, one with *low demand* and is *inherently stable* and the other with *high demand* and is *inherently unstable*. The term "*inherent stability*" is inspired by the *cobweb theorem* [17] and is defined in terms of the *cobweb ratio* B/y, i.e., the ratio of the slope of the demand curve to that of the supply curve. An economy is inherently stable if its cobweb ratio is lower than 1 and is inherently unstable if it is higher than 1. In this paper, the stable case is called Economy 1 ($B/y = 0.95$) and the unstable case Economy 2 ($B/y = 1.05$). The setup of all other parameters of these two economies are given in table 1.

Given these two economies, our purpose is to simulate the transition process from Economy 1 to Economy 2. This transition process, while very simple, has some economic significance. First of all, the transition of the economy is a permanent transition from low demand ($P^* = 1.12$) to high demand ($P^* = 2.24$). In introductory economics, it corresponds to nothing but a *shift in the demand curve*. However, to

Table 1
Parameter values of the cobweb model.

	Economy 1	Economy 2
A	2.184	4.592
B	0.0152	0.0168
x	0	0
y	0.016	0.016
n	500	500
(B/y)	0.95	1.05
P^*	1.12	2.24

A and B are the parameters of demand curve and x and y are the parameters of supply curve. B/y is the cobweb ratio. n is the number of firms. P^* is the rational expectations equilibrium price.

some extent, the economic analysis of institutional changes can always be reduced to the scenario of either the shift in the demand curve or the shift in the supply curve. Since in the real world, no one knows where these curves exactly lie, to evaluate how well the economy can adapt to these shifts is difficult, if not impossible. In this case, simulations based on a simple and basic transition can be helpful.

Second, the economy before and after transition have different structures. The one before transition is inherently stable, but the one after is unstable. So, what we are really simulating is not just a shift in the demand curve but also a change in the topology around the equilibrium. We believe that the real economy has no difficulty in adapting to this type of change; however, many existing learning schemes, such as the *recursive least squares*, may be in trouble [27]. Therefore, this simulation can give us a further test on whether genetic programming can approximate the adaptation observed in the real economy better than the classical or Bayesian schemes.

First we have Economy 1 be the true model for 500 periods and after that Economy 2 will take over for another 500 periods. Our artificial producers, however, do not know the relevant parameters of the demand curve, and neither do they know when the *structural break* is going to happen. In other words, our artificial producers have to *discover* the theoretical equilibrium price and *rediscover* it by simultaneously detecting the potential *structural change* on their own. Moreover, this is not a representative agent model. In all our simulations, a large number of producers (500 in total) are used. Hence, potentially the dimension of the space for producers to adpat and coordinate can be extremely huge and complex. Without having some features of *self-organization* or *collective intelligence*, it will be hopeless for any producers to undergo such a coordination and adaptation task.

Genetic programming is thus used to model *population learning* in the model defined above. Let GP_t, a population of LISP trees, represent a collection of firms' price forecasting functions at time period t. A firm i, $i = 1, \ldots, n$, makes a decision about its production for time t using a tree, $gp_{i,t}$ ($gp_{i,t} \in GP_t$), a *parse tree* written over the *function set* and *terminal set* which are prespecified. In this study, all simulations

Table 2
Tableau of GP-based learning.

Number of producers (Population size)	500
Number of trees created by the full method	50
Number of trees created by the grow method	50
Function set	$\{+, -, \times, \%, \mathrm{Exp}, \mathrm{Rlog}, \mathrm{Sin}, \mathrm{Cos}\}$
Terminal set	$\{P_{t-1}, P_{t-2}, \ldots, P_{t-10}, R\}$
Number of trees created by reproduction	50
Number of trees created by crossover	350
Number of trees created by mutation	100
Probability of mutation	0.0033
Mutation scheme	Tree mutation
Maximum depth of tree	17
Probability of leaf selection under crossover	0.5
Number of generations	1000
Maximum number in the domain of Exp	1700
Criterion of fitness	Profits

The number of trees created by the full method or grow method is the number of trees initialized in generation 0 with the depth of tree being 2, 3, 4, 5, and 6. For details, see [22].

conducted are based on the terminal set which includes the ephemeral random floating-point constant R ranging over the interval $[-9.99, 9.99]$ and the price lagged up to h periods, i.e., $P_{t-1}, \ldots, P_{t-h}$. Thus, the forecasting functions that firms may use are the linear and nonlinear functions of $P_{t-1}, \ldots, P_{t-h}$. Genetic programming techniques are applied to the population of LISP trees that serve as the forecaster rule for our heterogeneous population of artificial agents (table 2).

3.3. *Simulation results*

Given the experimental design proposed above, ten runs of simulation were implemented. The time series plots of the prices over 1000 periods, i.e., $\{P_t\}_{t=1}^{1000}$ are depicted in figures 1.1–1.10. The prices over the first 500 periods are generated from Economy 1 and those over the second 500 periods are generated from Economy 2. The two horizontal lines corresponding to the two theoretical equilibrium prices are 1.12 and 2.24. From these figures, some observations about the transition processes are made as follows.

First of all, **before the structural change**, i.e., before period 501, the GP-based markets have already converged to the theoretical equilibrium price P^* $(= 1.12)$. While sometimes random mutations may drive the equilibrium price away from P^* and even cause *crashes*,[5] the markets always have a *self-stabilizing feature* which can bring these deviations *quickly* back to a niche of P^*. Also, by taking a look at the $P^e_{i,t}$,

[5] Here *crashes* refer to the sudden and dramatic slumps in price. This phenomenon is largely caused by the *tree mutation* and hence is artificial. If we turn off the mutation operator once the price moves to a niche of P^*, these crashes can be avoided. But, to keep the simulated evolution as *autonomous* as possible and to avoid being *ad hockery*, we decide not to do so.

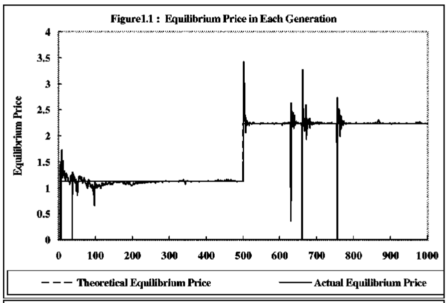
Figure1.1 : Equilibrium Price in Each Generation
Equilibrium Price
Theoretical Equilibrium Price
Actual Equilibrium Price

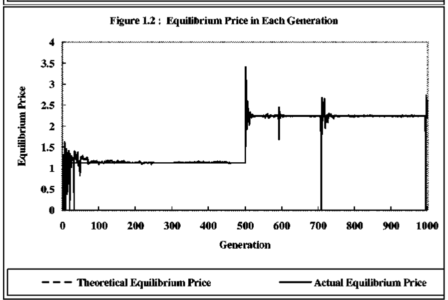
Figure 1.2 : Equilibrium Price in Each Generation
Equilibrium Price
Generation
Theoretical Equilibrium Price
Actual Equilibrium Price

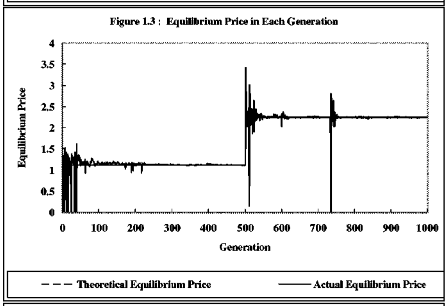
Figure 1.3 : Equilibrium Price in Each Generation
Equilibrium Price
Generation
Theoretical Equilibrium Price
Actual Equilibrium Price

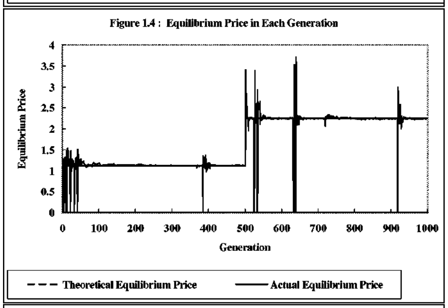
Figure 1.4 : Equilibrium Price in Each Generation
Equilibrium Price
Generation
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Actual Equilibrium Price

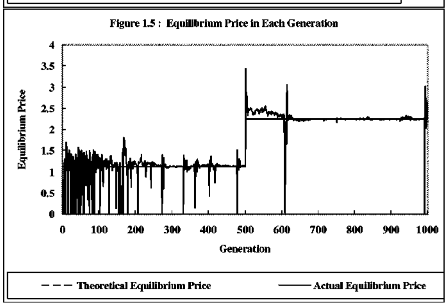


Figure 1.5 : Equilibrium Price in Each Generation
Equilibrium Price
Generation
Theoretical Equilibrium Price
Actual Equilibrium Price

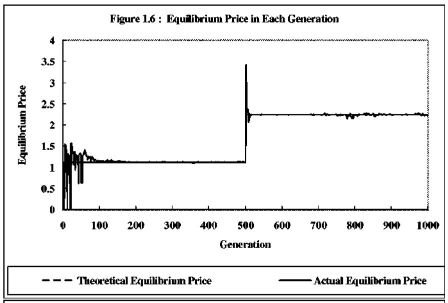


Figure 1.6 : Equilibrium Price in Each Generation
Equilibrium Price
Generation
Theoretical Equilibrium Price
Actual Equilibrium Price

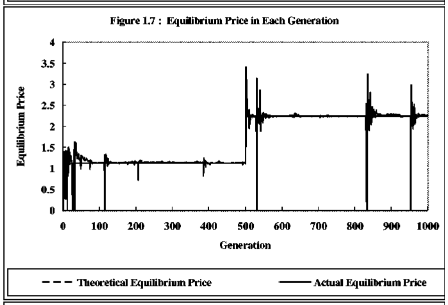


Figure 1.7 : Equilibrium Price in Each Generation
Equilibrium Price
Generation
Theoretical Equilibrium Price
Actual Equilibrium Price

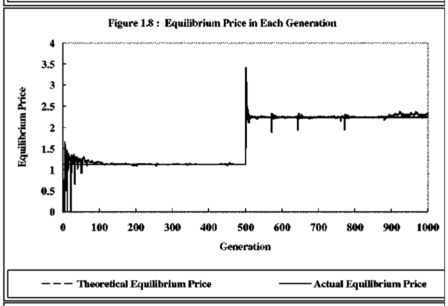


Figure 1.8 : Equilibrium Price in Each Generation
Equilibrium Price
Generation
Theoretical Equilibrium Price
Actual Equilibrium Price

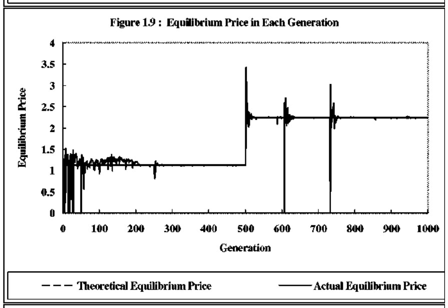


Figure 1.9 : Equilibrium Price in Each Generation
Equilibrium Price
Generation
Theoretical Equilibrium Price
Actual Equilibrium Price

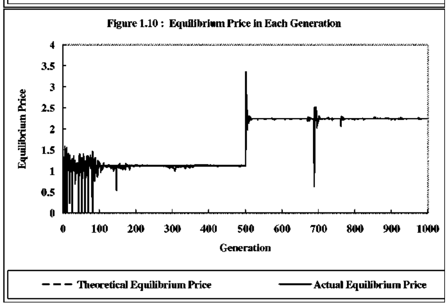


Figure 1.10 : Equilibrium Price in Each Generation
Equilibrium Price
Generation
Theoretical Equilibrium Price
Actual Equilibrium Price

we can see that the only surviving belief is that *tomorrow's price shall be the same as today's price*. Any deviations from this belief caused by mutation cannot sustain.

Second, **at the very beginning of the structural change**, i.e., at period 501, the demand curve shifts up suddenly. Since producers do not anticipate this, the quantities supplied based on the old P^* (1.12) are not sufficient to meet the great demand. As a result, we see that the price jumps dramatically at the beginning of Economy 2 in all runs. To make this jump and the fluctuation after it more visible, P_ts from period 490 to 610 are replotted in figures 2.1–2.10.

Third, **after the jump and during the transition phase**, the price starts to fluctuate. These fluctuations not only characterize the transition phase but also signal a kind of uncertainty to both insiders (producers in the model) and outsiders (model builders). During the transition phase, producers start to question their original belief and wonder "what is going on?". New hypotheses about the world are formed via genetic operators and they are put to the test. The following is a list of samples, written in LISP S-expression, from different runs at different generations.

$$P^e_{i,502} = (\%\ P_{t-5}\ (\%\ P_{t-6}\ (+\ P_{t-2}\ P_{t-5}))), \tag{10}$$
$$P^e_{i,503} = (\%\ P_{t-5}\ (\%P_{t-6}\ (+\ (\text{Log}\ P_{t-2})\ P_{t-5}))), \tag{11}$$
$$P^e_{i,504} = (\%\ (\%\ P_{t-4}\ P_{t-4})\ (\%\ P_{t-1}\ (\%P_{t-4}\ P_{t-9}))), \tag{12}$$
$$P^e_{i,504} = (\%\ P_{t-9}\ (\%\ P_{t-7}\ (\%\ P_{t-2}(\%\ P_{t-1}P_{t-7})))), \tag{13}$$
$$P^e_{i,504} = (\%\ P_{t-4}\ (\%\ P_{t-7}\ P_{t-3})), \tag{14}$$
$$P^e_{i,504} = (\times\ (\times\ P_{t-2}\ P_{t-10})\ P_{t-4}), \tag{15}$$
$$P^e_{i,507} = (\times\ P_{t-7}\ (\times\ P_{t-1}\ P_{t-6})), \tag{16}$$
$$P^e_{i,517} = (\text{Log}\ (\%\ (+\ P_{t-10}\ P_{t-2}))\ (\text{Cos}\ (-\ (\text{Log}\ P_{t-1})\ (\text{Cos}\ P_{t-5})))), \tag{17}$$
$$P^e_{i,518} = (\times\ (+\ P_{t-2}\ P_{t-2})\ P_{t-2}), \tag{18}$$
$$P^e_{i,521} = (\text{Sin}\ (\text{Sin}\ P_{t-2})), \tag{19}$$
$$P^e_{i,528} = (\text{Sin}\ P_{t-10}). \tag{20}$$

From these examples, we can see that during the transition phase, the accepted hypothesis "$P^e_{i,t} = P_{t-1}$" is challenged by the search for more complex forecasting rules. Due to the *random search path* of genetic programming, i.e., the random tree expansion, the accepted hypothesis is more likely to be first challenged by simple nonlinear forecasting rules and then by complex ones, and some of these complex forecasting rules may survive for a while before the price finally converges to the new P^*, i.e., 2.24.

In addition to the description given above, some other features of transition paths are summarized as follows.

- The transition phase is *nonlinear* and *non-smooth*. It is likely to observe *slumps, crashes, and bursts* in the transition phase.
- The *transition speed* can be very fast as is evidenced by simulations 6 and 10, and can be slow as well, as simulations 4 and 5 indicate.

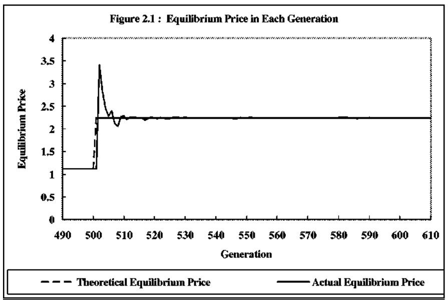


Figure 2.1 : Equilibrium Price in Each Generation

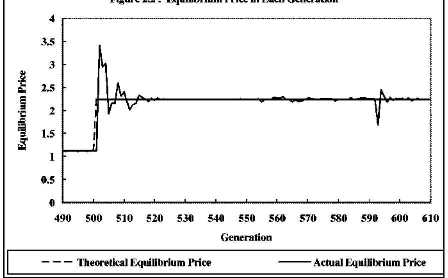


Figure 2.2 : Equilibrium Price in Each Generation

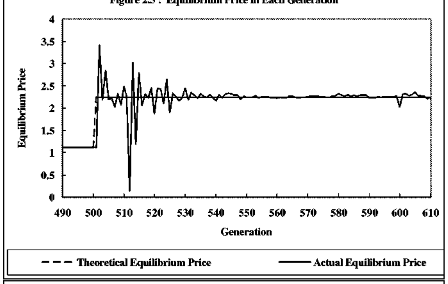


Figure 2.3 : Equilibrium Price in Each Generation

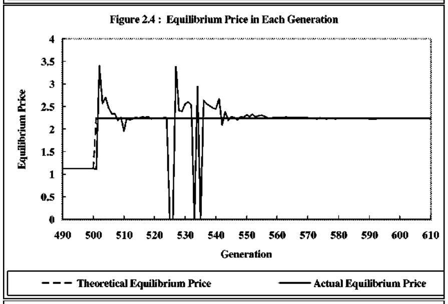


Figure 2.4 : Equilibrium Price in Each Generation

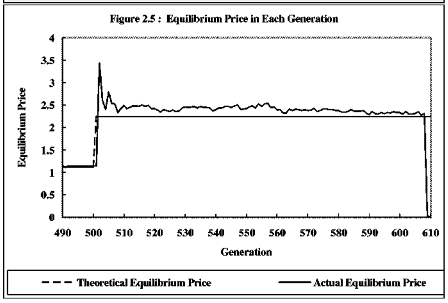


Figure 2.5 : Equilibrium Price in Each Generation

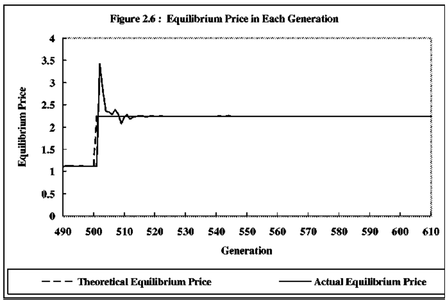


Figure 2.6 : Equilibrium Price in Each Generation

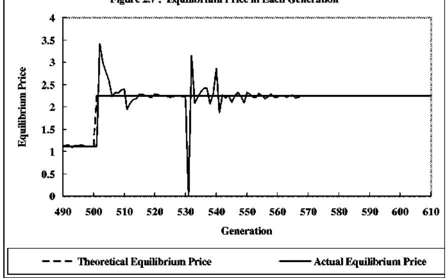


Figure 2.7 : Equilibrium Price in Each Generation

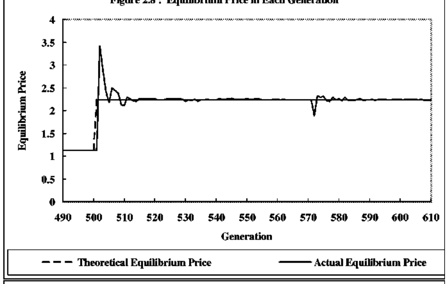


Figure 2.8 : Equilibrium Price in Each Generation

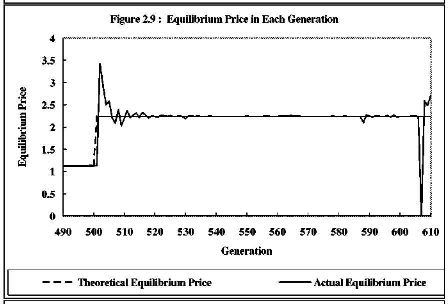


Figure 2.9 : Equilibrium Price in Each Generation

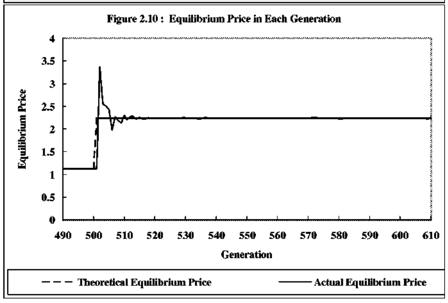


Figure 2.10 : Equilibrium Price in Each Generation

Finally, for all of our simulations, the transition period does not persist for more than 100 periods (figures 1.1–1.10). From figures 1.1 to 1.10, we can see that the new equilibrium price (2.24) is rediscovered after a 100-period evolution. While crashes or bursts can happen occasionally, the self-stabilizing feature still exists.

4. Simulating adaptive speculators with genetic programming

In this section, we shall refer to an artificial database generated from the simulations conducted in [11]. The original purpose of these simulations is to evaluate the impact of the trading restrictions on the role of speculative trades in market efficiency. Through genetic programming, [11] constructed artificial speculative markets and generated the artificial time series data of these markets. They then conducted a social welfare analysis based on this database and compared the market efficiency under different financial regulations. Here, based on the same artificial database, we shall conduct two simple econometric tests. The first is a test for *volatility*, and the second is a test for *structural changes*. In the following, we shall first briefly introduce the analytical framework and the experimental design of [11].

4.1. The analytical framework

In spirit of the earlier works done by [1] and [2,11] simulated the adaptive behavior of speculators via genetic programming in a production economy (Muthian Economy). To extend the model (equations (3)–(7)) so that we can study speculation, the behavior of speculators has to be specified first. Let $I_{j,t}$ represent the inventory of the jth speculator at the end of the tth period; the profit to be realized is

$$\pi_{j,t} = I_{j,t}(P_{t+1} - P_t). \tag{21}$$

Of course, the actual profit $\pi_{j,t}$ is unknown at the moment when the inventory plan is conducted; therefore, like producers, speculators tend to set the inventory up to the level where speculators' expected utility $Eu_{j,t}$ or expected profits $E\pi_{j,t}$ can be maximized. Maximizing $Eu_{j,t}$ and $E\pi_{j,t}$ can be two quite different objectives. Generally speaking, the former will take speculators' risk attitude into account but the latter will not. We shall follow [28] to assume that the objective function for speculators is to maximize the expected utility rather than the expected profit.

Without assuming any specific form of utility function, what [28] did was to approximate the general utility function by taking the second-order Taylor's series expansion about the origin:

$$u_{j,t} \approx \phi(\pi_t) = \phi(0) + \phi'(0)\pi_{j,t} + \frac{1}{2}\phi''(0)\pi_{jt}^2. \tag{22}$$

Based on equation (22), the approximated utility depends on the moments of the probability distribution of π_t, i.e.,

$$Eu_{j,t} \approx \phi(0) + \phi'(0)E\pi_{j,t} + \frac{1}{2}\phi''(0)E\pi_{j,t}^2. \tag{23}$$

Solving the first and the second moment of equation (23), we can rewrite the expected utility function as follows:

$$Eu_{j,t} \approx \phi(0) + \phi'(0)I_{j,t}\big(P_{j,t+1}^e - P_t\big) + \frac{1}{2}\phi''(0)I_{j,t}^2\big[\sigma_{j,t}^2 + \big(P_{j,t+1}^e - P_t\big)^2\big], \tag{24}$$

where $\sigma_{j,t}^2$ is the subjective risk (the second moment) perceived by the jth speculator.

The optimal position of the inventory can then be derived approximately by solving the first order condition and the optimal position of the inventory $I_{j,t}^*$ is given by

$$I_{j,t} = \alpha_j\big(P_{j,t+1}^e - P_t\big), \tag{25}$$

where $\alpha_j = -\phi'(0)/\phi''(0)\sigma_{j,t}^2$. Equation (25) explicitly shows that speculators' optimal decision about the level of inventory depends on their expectations of the price in the next period, $P_{j,t+1}^e$, and their subjective risk, $\sigma_{j,t}^2$.

For a market consisting of n producers and m speculators, the equilibrium condition is given in equation (26),

$$\frac{A}{B} - \frac{1}{B}P_t + \sum_{j=1}^{m}\alpha_j\big(P_{j,t+1}^e - P_t\big) = \sum_{i=1}^{n}\frac{1}{yn}\big(P_{i,t}^e - x\big) + \sum_{j=1}^{m}\alpha_j\big(P_{j,t}^e - P_{t-1}\big). \tag{26}$$

This concludes the construction of our model.

4.2. Experimental design

Since the evolutionary algorithm for producers is the same as the one described in the previous section, we only describe the evolutionary algorithm for speculators here. Unlike its application to modeling producers' adaptive behavior, genetic programming is applied to modeling the *inventory policy* $I_{j,t}$ of speculators rather than their price expectations $P_{j,t}^e$. However, since the inventory policy is a function of price expectations and price expectations are formed based on the history of prices, $I_{j,t}$ can be written as a function of the past prices, namely,

$$I_{j,t} = I_{j,t}(P_{t-1}, P_{t-2}, \ldots). \tag{27}$$

In the following, genetic programming will be employed to model the adaptation of the function form of $I_{j,t}$. Let GP_t^s, a population of LISP trees, represent a collection of speculators' inventory policies $I_{j,t}$. A speculator j, $j = 1, \ldots, m$, makes a decision about its inventory at time t using a tree, $I_{j,t}$ ($I_{j,t} \in GP_t^s$), a *parse tree* written over the *function set* and *terminal set* which are given in table 3. The decoding of a parse tree $I_{j,t}$ gives the policy function used by speculator j at time period t, i.e., $I_{j,t}(\Omega_{t-1})$

Table 3
Tableau of GP-based adaptation.

Number of producers	300
Number of speculators	100
Number of trees created by the full method	30 (P), 10 (S)
Number of trees created by the grow method	30 (P), 10 (S)
Function set	$\{+, -, \text{Sin}, \text{Cos}\}$
Terminal set	$\{P_{t-1}, P_{t-2}, \ldots, P_{t-10}, R\}$
Number of trees created by reproduction	30 (P), 10 (S)
Number of trees created by crossover	210 (P), 70 (S)
Number of trees created by mutation	60 (P), 20 (S)
Probability of mutation	0.2
Maximum depth of tree	17
Probability of leaf selection under crossover	0.5
Number of generations	1000
Maximum number in the domain of Exp	1700
Criterion of fitness	Profit

"P" stands for the producers and "S" stands for the speculators. The number of trees created by the full method or grow method is the number of trees initialized in generation 0 with the depth of tree being 2, 3, 4, 5, and 6. For details, see [22].

where Ω_{t-1} is the information of the past prices up to P_{t-1}. Evaluating $I_{j,t}(\Omega_{t-1})$ at the realization of Ω_{t-1} will give us the inventory of speculator j at time t, i.e., $I_{j,t}$. Without any further restrictions, the range of $I_{j,t}$ is $(-\infty, \infty)$. The case $I_{j,t} < 0$ is called *short selling* in finance. In this section, short selling is permitted for speculators subjected to the corresponding requirement for *short covering*. More precisely, we allow the speculator to sell short but to be constrained by a maximum amount $\underline{s}$. When the speculator sell short up to $\underline{s}$, he is no longer allowed to sell short any more; instead, he has to buy it back. Also, the *short position* cannot be kept for more than d days. In addition to the lower bound of $I_{j,t}$, we also set an upper bound of $I_{j,t}$, i.e., an upper bound of the long position, $\overline{b}$.

The *raw fitness* of a parse tree $I_{j,t}$ is determined by the value of the speculator's payoffs earned at the end of time $t+1$ based on equation (21). To avoid a negative fitness value, each raw fitness value is then adjusted to produce an *adjusted fitness* measure $\mu_{j,t}$ and is given as follows:

$$\mu_{j,t} = \begin{cases} \pi_{j,t} + 50, & \text{if } \pi_{j,t} \geqslant -50, \\ 0, & \text{if } \pi_{j,t} < -50. \end{cases} \tag{28}$$

The choice of "50" as a threshold is due to the similar consideration in [10] and [12]. Each such adjusted fitness value $\mu_{j,t}$ is then normalized. The *normalized fitness* value $p_{j,t}$ is given in equation (29).

$$p_{j,t} = \frac{\mu_{j,t}}{\sum_{j=1}^{n} \mu_{j,t}}. \tag{29}$$

Table 4
Parameter values of the cobweb model.

	Case A	Case B	Case C	Case D
A	2.184	2.296	3.36	4.48
B	0.0152	0.0168	0.032	0.048
x	0	0	0	0
y	0.016	0.016	0.016	0.016
n	300	300	300	300
m	100	100	100	100
(B/y)	0.95	1.05	2	3
P^*	1.12	1.12	1.12	1.12

A and B are the parameters of the demand curve and x and y the parameters of the supply curve. B/y is the cobweb ratio. n is the number of producers, and m is the number of speculators. P^* is the rational expectations equilibrium price derived from equation (26).

Table 5
Codes of simulations.

f.r./c.r.	0.95	1.05	2.00	3
B.M.	A-0	B-0	C-0	D-0
0.005	A-1	B-1	C-1	D-1
0.01	A-2	B-2	C-2	D-2
0.1	A-3	B-3	C-3	D-3
1.0	A-4	B-4	C-4	D-4
10	A-5	B-5	C-5	D-5

The four numbers appearing in the c.r. row are four cobweb ratios. The four ratios are encoded by letters A, B, C, D in the ascending order. The five numbers in the f.r (financial ratio) column are upper limits for the long and short positions $\overline{b}$ and $\underline{s}$. Here, $\overline{b}$ and $\underline{s}$ are set to be identical in all cases. These five limits are encoded by numbers 1, 2, 3, 4, 5 in the ascending order. B.M. refers to the benchmark which is the case without speculators and is encoded by "0". For those cases with speculative trades, the duration for the short position, d, is set to be 20.

Once $p_{j,t}$ is determined, GP^s_{t+1} is generated from GP^s_t by three primary genetic operators, i.e., *reproduction*, *crossover*, and *mutation*. All the control parameters for the Muthian economy are given in table 3.

Given the GP-based adaptive producers and speculators, our computer simulations were implemented by using the stable case with *cobweb ratio* 0.95 and the unstable cases with *cobweb ratios* 1.05, 2, *and* 3 with different financial regulations on the *long* and *short positions*, which are denoted by parameters $\overline{b}$ and $\underline{s}$ (see tables 4 and 5).

From Case x.1 to Case x.5 ($\mathrm{x} = \mathrm{A, B, C, D}$), the trading restrictions on $\overline{b}$ and $\underline{s}$ are gradually relaxed from 0.005 to 10. Since the equilibrium quantity Q^* is 70 and there are one hundred speculators in the market, these settings imply that the proportion of potential speculative trades to Q^* is relaxed from $1/140$ to $100/7$. The larger the $\overline{b}$ and the $\underline{s}$, the larger the possible proportion of "*non-productive activities*" to the economy.

Table 6
Reference table of figures 3.1–3.8.

Figure	Case	GARCH(q,p)	Structural changes
3.1	B-4	GARCH(1,1)	Yes
3.2	B-4	GARCH(1,1)	Yes
3.3	C-2	GARCH(1,1)	Yes
3.4	C-4	GARCH(1,1)	Yes
3.5	C-5	GARCH(1,1)	Yes
3.6	D-0	GARCH(1,1)	Yes
3.7	D-3	GARCH(1,1)	Yes
3.8	D-4	GARCH(1,1)	No

The orders p and q of the GARCH model is determined by the Schwartz information criterion.

Simulations were conducted for all cases, including the case without speculators (the benchmark), in accordance with tables 4 and 5. For each case, multiple runs of simulation were implemented and each run was conducted for one thousand periods (generations). A database was then established after these simulations. In the following, we shall conduct two simple tests based on this database, namely, a test for *volatility* and a test for *structural changes*. For the former, we shall conduct the *GARCH* test [4], and for the latter, the *CUSUMSQ* test [6].

4.3. Two econometric tests on "Speculative Markets"

The database, called the "**Speculative Markets: Version 1**", has 360 time series. Each of them has 1000 observations. A detailed report of all these series is beyond the limit of this paper.[6] Here, for illustrative purposes, only eight selected time series are considered and they are depicted in figures 3.1–3.8.[7] Table 6 lists the respective case of each figure.

The first test is the GARCH test for the time series

$$\Delta P_t = P_t - P_{t-1}. \tag{30}$$

Few approaches are as pervasive in current econometric research as *autoregressive conditional heteroscedasticity*.[8] Yet, important questions remain: *How do GARCH models originate? What are their economic foundations? Why do financial markets show these empirical "signatures"?* [2] conjecture a simple evolutionary explanation.

> Both in real markets and in our artificial market, *agents are constantly exploring and testing new expectations*. Once in a while, randomly, more successful expectations will be discovered. Such expectations will change the market, and trigger further

[6] However, this dataset itself is available upon request.
[7] These eight samples are selected in a manner such that they together can represent most of the patterns observed in other time series.
[8] For a survey article, the reader is referred to [5].

Figure 3.1 : Equilibrium Price in Each Generation

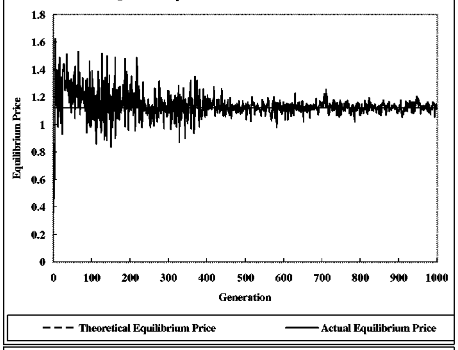


Figure 3.2 : Equilibrium Price in Each Generation

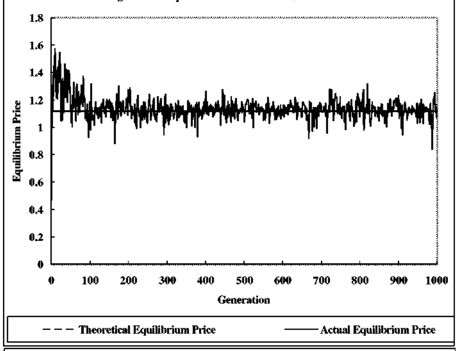


Figure 3.3 : Equilibrium Price in Each Generation

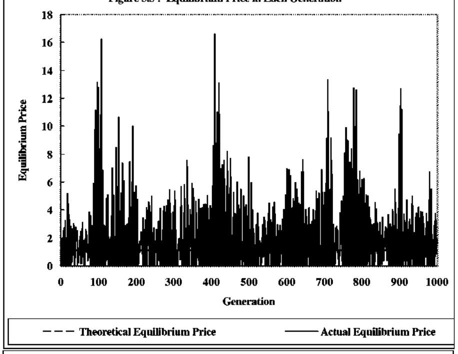


Figure 3.4 : Equilibrium Price in Each Generation

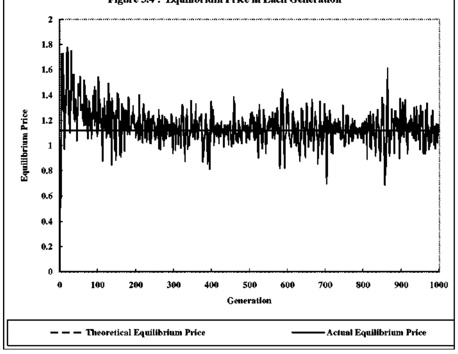


Figure 3.5 : Equilibrium Price in Each Generation

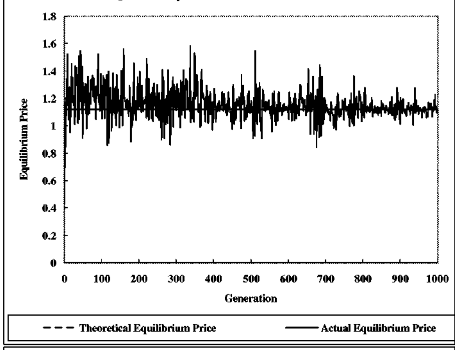


Figure 3.6 : Equilibrium Price in Each Generation

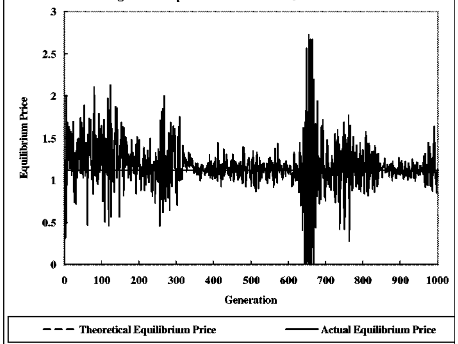


Figure 3.7 : Equilibrium Price in Each Generation

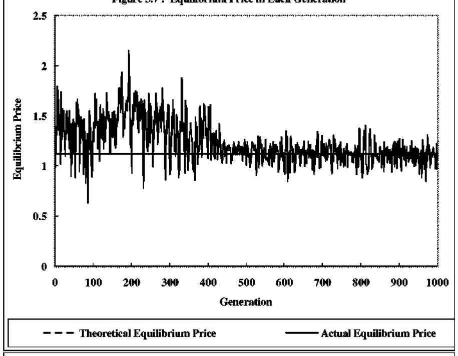
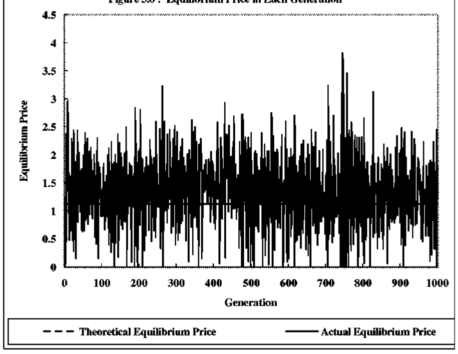


Figure 3.8 : Equilibrium Price in Each Generation



changes in expectations, so that small and large "avalanches" of change will cascade through the system. (Ibid, p. 19. Italics Added.)

Since GP-based speculators also have the feature of *constantly exploring and testing new expectations*, our *GARCH* test is well motivated.

The procedure of the *GARCH* test is briefly described as follows.[9]

- First, the ΔP_t correlations are removed by an AR filter with order chosen by the Schwartz information criterion (SIC).
- Second, the GARCH-type filters are then applied to the filtered series ε_t, and the GARCH(p,q) model is chosen based on SIC.

The resulting GARCH model for each time series ΔP_t is presented in table 6. From table 6, we can see that the GARCH property generally holds for all selected time series.[10] This result confirms the earlier conjecture by [2] and has further suggested the possibility that *interactive heterogeneous agents driven by genetic programming may provide us with an economic foundation of ARCH models*. Needless to say, more works are needed in this direction.

The second test is the *CUSUM of squares* (*CUSUMSQ*) test for the constancy over time of the coefficients of the AR model of ΔP_t. This test is motivated by [14] which shows that *infrequent structural changes* may result in the nonlinear dependence of stock returns which may be mistaken as the GARCH effect. The result of this test is exhibited in figures 4.1–4.8. In these figures, between the two parallel dash lines is the 95% confidence interval for the CUSUMSQ statistic.[11] If this statistic goes beyond this interval, then a structural change is detected. From figures 4.1–4.8, all cases evidence structural changes except the last one. What is interesting about this result is that, in our simulation, there is no external shock or regime change imposed. Therefore, if a structural change is detected, it is *endogenous*, and is caused by the complex interacting adaptive behavior of producers and speculators.

5. Concluding remarks

Using genetic programming, this study proposed a population-based evolutionary approach to modeling transition dynamics. Since genetic programming can generate hypotheses automatically, it can fit quite well the characteristics of adaptive agents. In particular, in a constantly changing environment, such as the *jump process*, adaptive agents have to *constantly explore and test new expectations in order to survive*, and genetic programming can be a very useful toolkit to capture this attribute. In the pedagogic example, we have shown that GP-based adaptive agents can automatically detect the shift in the demand curve and adapt to it by trying new hypotheses. The transition

[9] For details, the interested reader is referred to [19, chapter 21].

[10] In fact, the GARCH property turns out to be the most popular property in the database "Speculative Markets: Version 1".

[11] The *CUSUMSQ* statistic under the null hypothesis follows a *beta* distribution.

Figure 4.1

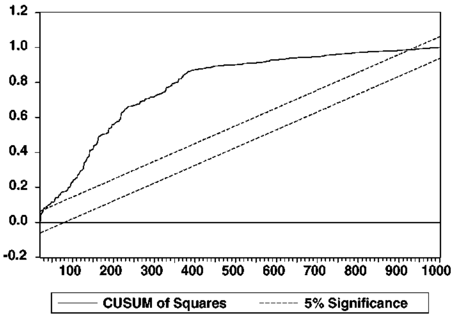


Figure 4.2

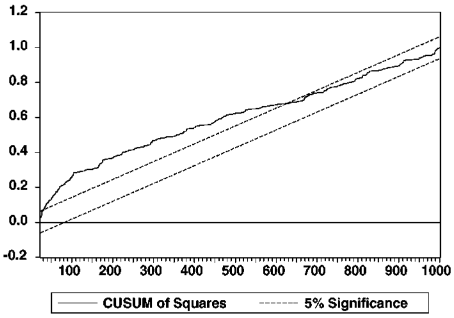


Figure 4.3

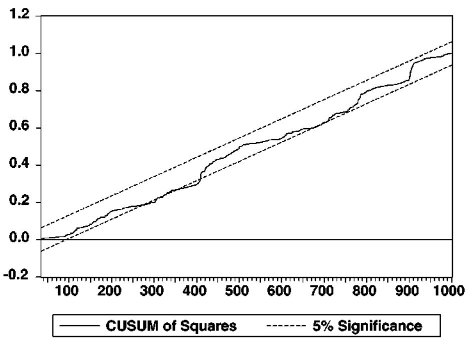


Figure 4.4

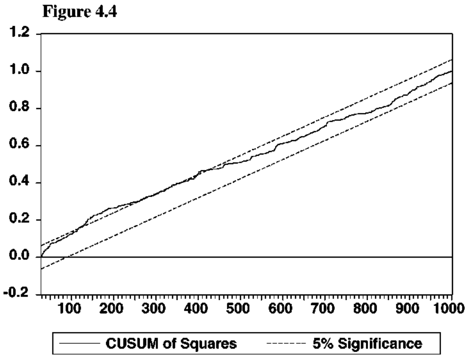


Figure 4.5

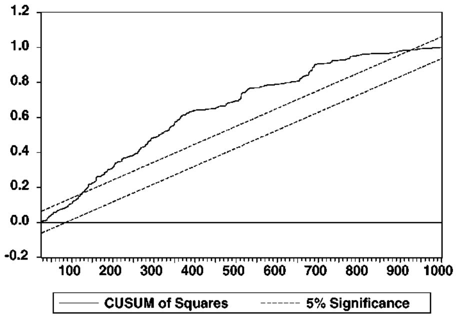


Figure 4.6

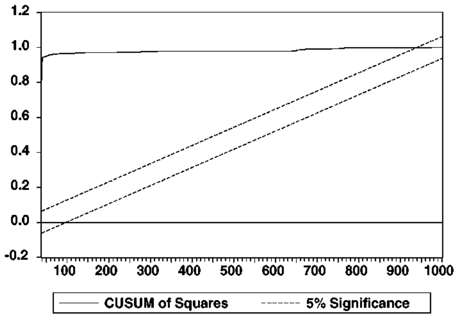


Figure 4.7

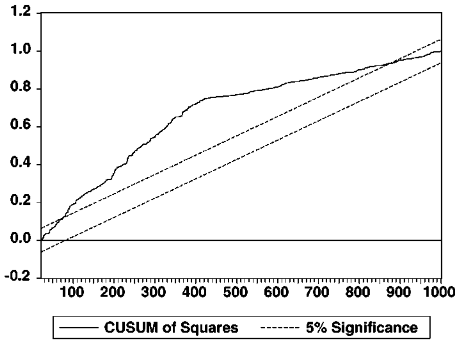
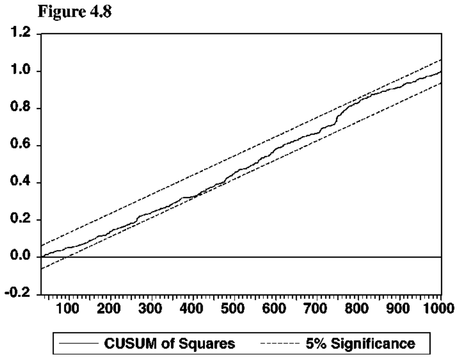


Figure 4.8



process also exhibits the belief-coordinating process which is usually assumed away in the conventional mathematics of transition dynamics, such as the difference equations and differential equations.

In the second example, we show that GP-based speculators can generate nonlinear financial time series characterized by the *GARCH-like* phenomenon. To our best knowledge, this is the first report to show that some important features of financial time series analysis can be generated from the GP-based market. Furthermore, if we perform the *CUSUMSQ* test on it, we find that the complex interaction process of these GP-based speculators can even generate *endogenous* structural changes. This interesting feature confirms the earlier finding by [1].

> We find no evidence that market behavior ever settles down; the population of predictors continually coevolves. One way to test this is to take agents out of the system and inject them in again later on. *If market behavior is stationary they should be able to do as well in the future as they are doing today*. But we find that when we "freeze" a successful agent's predictors early on and inject the agent into the system much later, the formerly successful agent is now a *dinosaur*. His predictions are *unadapted* and perform poorly. *The system has changed*. ([1, p. 24]. Italics Added.)

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References

[1] B. Arthur, On learning and adaptation in the economy, Santa Fe Institute Economic Research Program, 92-07-038 (1992).

[2] B. Arthur, J. Holland, B. LeBaron, R. Palmer and P. Tayler, Asset pricing under endogenous expectations in an artificial stock market, Santa Fe Institute Working Paper Series (1996).

[3] F. Black, Noise, Journal of Finance 41 (1986) 529–543.

[4] T. Bollerslev, Generalized auroregressive conditional heteroskedasticity, Journal of Econometrics 31 (1986) 307–327.

[5] T. Bollerslev, R. Chou and K. Kroner, ARCH modeling in finance: A review of theory and empirical evidence, Journal of Econometrics 52 (1992) 5–59.

[6] R.L. Brown, J. Durbin and J.M. Evans, Techniques for testing the constancy of regression relationship over time, Journal of Royal Statistical Society, B 37 (1975) 149–163.

[7] S.-H. Chen and C.-W. Tan, Measuring randomness by Rissanen's stochastic complexity: Applications to the financial data, in: *ISIS: Information, Statistics and Induction in Science*, eds. D.L. Dowe et al. (World Scientific, Singapore, 1996) pp. 200–211.

[8] S.-H. Chen and C.-W. Tan, Measuring stock market efficiency by Rissanen's predictive stochastic complexity, in: *Managing International Business in the Twenty First Century: Decision Making, Market Development, Information and Technology*, Vol. 1, eds. H. Steele and N. Antonio, CIBS (Lingnan College and Macau Foundation Press 1996) pp. 159–172.

[9] S.-H. Chen and C.-W. Tan, Modeling the inductive learning agents in financial markets with the adaptive MDL, Presented at *The 1996 Chinese Society of Statistics Annual Meeting*, June 1, 1996, National Tsing Hwa University, Hsinchu, Taiwan, R.O.C.

[10] S.-H. Chen and C.-H. Yeh, Genetic programming learning and the cobweb model, in: *Advances in Genetic Programming*, Vol. 2, ed. P. Angeline (MIT Press, Cambridge, MA, 1996) pp. 443–466.

[11] S.-H Chen and C.-H. Yeh, Modeling speculators with genetic programming, in: *Evolutionary Programming VI*, eds. P. Angeline et al., Lecture Notes in Computer Science, Vol. 1213 (Spewringer-Verlag, Berlin, 1997) pp. 137–147.

[12] S.-H. Chen, J. Duffy and C.-H. Yeh, Genetic programming in the coordination game with a chaotic best-response function, in: *Proceedings of the Fifth Annual Conference on Evolutionary Programming* (EP'96), eds. P. Angeline et al. (MIT Press, Cambridge, MA, 1996) pp. 277–286.

[13] P. Davidson, Is probability theory relevant for uncertainty? A post Keynesian perspective, Journal of Economic Perspectives 5(1) (1991) 129–143.

[14] P.J.F. de Lima, Nonlinearities and nonstationarities in stock returns, Working Paper, Department of Economics (Johns Hopkins University, 1997).

[15] A. De Vany, Information, bounded rationality, and the complexity of economic organization, Taiwan Journal of Political Economy 2(1) (1966) 21–44.

[16] D.L. Dowe and K.B. Korb, Conceptual difficulties with the efficient market hypothesis: Towards a naturalized economics, in: *ISIS: Information, Statistics and Induction in Science*, eds. D.L. Dowe et al. (World Scientific, Singapore, 1996) pp. 200–211.

[17] M. Ezekiel, The cobweb theorem, Quarterly Journal of Economics 52 (1938) 255–280.

[18] T. Haavelmo, The probability approach in econometrics, Econometrica 12 (1944) 1–18, supplement.

[19] J.D. Hamilton, *Time Series Analysis* (Princeton University Press, 1994).

[20] J.D. Hamilton and R. Susmel, Autoregressive conditional heteroskedasticity and changes in regime, Journal of Econometrics 64 (1994) 307–333.

[21] R.E. Kalman, Randomness reexamined, Modeling, Identification, and Control 15(3) (1994) 141–151.

[22] J.R. Koza, *Genetic Programming: On the Programming of Computers by Means of Natural Selection* (MIT Press, 1992).

[23] C.-M. Kuan and T. Liu, Forecasting exchange rates using feedforward and recurrent neural networks, Journal of Applied Econometrics 10 (1995) 347–364.

[24] A. Leijonfhufvud, Towards a not-too-rational macroeconomics, Southern Economic Journal 60(1) (1993) 1–13.

[25] T. Lensberg, A genetic programming experiment on investment behavior under knightian uncertainty, in: *Genetic Programming 1997*, Proceedings of the Second Annual Conference, July 13–16, 1997, Stanford University, eds. J. Koza et al. (Morgan Kaufmann, San Francisco, CA, 1997) pp. 231–239.

[26] M. Li and P. Vitanyi, *An Introduction to Kolmogorov Complexity and its Applications* (Springer, 1993).

[27] A. Marcet and T. Sargent, Convergence of least squares learning mechanisms in self referential linear stochastic model, Journal of Economic Theory 48 (1989) 337–368.

[28] J.F. Muth, Rational expectations and the theory of price movements, Econometrica 29 (1961) 315–335.

[29] J.R. Neil and K.B. Korb, The MML evolution of classification graphs, in: *ISIS: Information, Statistics and Induction in Science*, eds. D.L. Dowe et al. (World Scientific, 1996) pp. 78–89.

[30] J. Rissanen, *Stochastic Complexity and Statistical Inquiry* (World Scientific, 1989).

[31] T. Sargent, *Bounded Rationality in Macroeconomics* (Oxford, 1993).

[32] R. Solomonoff, Does algorithmic probability solve the problem of induction, Invited paper given at the *Conference on Information, Statistics and Induction in Science*, Melbourne, Australia, August 20–23 (1996).

[33] K. Velupillai, The computable approach to economics, Taiwan Journal of Political Economy 1(1) (1995) 53–110.