Agent-Based Artificial Markets in the AI-ECON Research Center: A Retrospect from 1995 to the Present

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ABSTRACT

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1 Motivation and Introduction

AI-ECON Research Center, formerly the AI-ECON Research Group, was established in 1995 at the College of Social Sciences, National Chengchi University. While interdisciplinary innovation may have triggered the idea, it is the growth of computer power that has given momentum to such efforts. Our goal of the center is to acknowledge the significance of the growth of computer power for the future of economics: it facilitates the integration of economics into the multidisciplinary research stream and benefits economists from the cross-fertilization of ideas with different disciplinary backgrounds.

In this article we shall review some of our contributions made to *agent-based artificial markets* (**AAMs**) since 1995. The review mainly covers the following three active research areas in AAMs, namely, the *cobweb models*, *overlapping generations models*, and *asset pricing models*.

2 The Cobweb Model

The *cobweb model* is probably the first neo-classical prototype to which an agent-based computational approach was applied. ([1]). Attentions were drawn to this prototype partially because this model is the motherland of the rational expectations revolution. In addition, it is also one of the simplest models from which price instability can be induced. Nonetheless, the neo-classical analysis simplifies the cobweb model by assuming the homogeneity of market participants. It is therefore not entirely clear whether those stability or instability conditions would apply to *a real market process* with participants who are obviously heterogeneous.

Early findings based on [1] are very optimistic about the inherent stabilization force in the market. However, this finding was soon challenged and enriched by a series of follow-up studies ([27], [28], [30]).

AI-ECON's contributions to the cobweb model started from [18]. This study distinguishes itself from other studies in *the tool* chosen to model adaptive firms. The latter applied *genetic algorithms* (**GAs**), whereas the former is perhaps the first application of *genetic programming* (**GP**) to modeling adaptive firms. It is a little surprising that few economists have ever addressed the distinction between the GA and GP in their applications to modeling *autonomous agents*.[1] However, [18] did make this distinction and justified the choice of GP.

The main attraction of using GP to modeling adaptive economic agents is the *parse-tree representations* and the associated *expression power*. Expression power is crucial to agent-based economic modeling, because agents' behavior in general can be too complex to be embedded in a finite-dimensional space. In symbolic regression, genetic programming proves to be a useful tool for non-parametric and non-linear modeling.[2] Therefore, it is natural to assume that when agents are adapted to the potential infinitely complex world, they are not confining themselves to any parametric model, but are trying instead to maximize their flexibility with genetic program-

[1] Generally speaking, most researchers on agent-based artificial markets do not justify well the choice of their tools; be it GA or other machining learning tools. A good exception can be found in [29], which showed that the behavior rules of artificial agents can be suitably modeled on the basis of prior evidence from human subjects experiments. Based on observations from fieldwork, [29] did not even consider the necessity of genetic algorithms in modeling artificial agent behavior.

[2] Earlier applications of genetic programming to econometrics can be found in [44], [46], and [47]. [11] provides a review of this early-stage development.

ming.

By suitably choosing the *terminal set* and *function set*, GP can represent a very large class of behavior rules. As also stated by [34], "one of the as yet unrealized strengths of genetic programming is that it is general enough to integrate many of the different techniques and approaches used in other styles of evolutionary computation into a more powerful whole."[3] This superb expression power would leave the determination of behavior rules *endogenously* to the market, instead of exogenously to researchers' intervention.[4]

Chen and Yeh ([18]) compared the learning performance of GP-based learning agents with that of GA-based learning agents. They found that, like GA-based learning agents, GP-based learning agents also can learn the homogeneous rational expectations equilibrium price under both the stable and unstable cobweb case. However, the phenomenon of *price euphoria*, which did not happen in [1], did show up quite often at the early stages of the GP experiments. This is mainly because agents in their setup were initially endowed with very limited information as compared to [1]. Nevertheless, GP-based learning can quickly coordinate agents' beliefs so that *the emergence of price euphoria is only temporary*. Furthermore, unlike [1], [18] *did not use* the *election operator*. Without the election operator, the rational expectations equilibrium is exposed to potentially persistent perturbations due to agents' adoption of the *new*, but *untested*, rules. However, what shows up in [18] is that the market can still bring any deviation back to equilibrium. Therefore, the self-stabilizing feature of the market, known as the *invisible hand*, is more powerfully replicated in their GP-based artificial market.

The self-stabilizing feature of the market demonstrated in [18] was furthered tested with two complications. In the first case, [19] introduced a population of speculators to the market and examined the effect of speculations on market stability. In the second case, the market was perturbed with a structural change characterized by a shift in the demand curve, and [21] then tested whether the market could restore the rational expectations equilibrium. The answer for the first experiment is generally *negative*, i.e., speculators do not enhance the stability of the market. On the contrary, they do destabilize the market. Only in the special cases when *trading regulations*, such as the *transaction cost* and *position limit*, were tightly imposed could speculators enhance the market stability. The answer for the second experiment is, however, *positive*. [21] showed that GP-based adaptive agents could detect the shift in the demand curve and adapt to it. Nonetheless, the transition phase was *nonlinear* and *non-smooth*; one can observe slumps, crashes, and bursts in the transition phase. In addition, the transition speed is uncertain. It could be fast, but could be slow as well.

This series of studies on the cobweb model enriches our understanding of the self-stabilizing feature of the market. The market has its limit, beyond which it can become unstable with crazy fluctuations. However, imposing trading regulations may relax the limit and enhance market stability. One is still curious to know where the self-stabilizing capability comes from in the first place. Economists have known for a long time that it comes from the *free competition principle*, or the *survival-of-the-fittest principle*. In the GA or GP, this principle is implemented through *selection pressure*. [10] studied the role of selection pressure by replacing the usual proportionate selection scheme with the one based on the approximate uniform distribution, showing that if selection pressure is removed or alleviated, then the self-stabilizing feature is lost. In a word, selection pressure plays the role of the *invisible hand* in economics. By nullifying the implementation of the survival-of-the-fittest principle, the market mechanism is paralyzed.

It is interesting to know whether the time series data generated by the artificial market can replicate some dynamic properties observed in the real market. [14] and [21] started the analysis of the time series data generated from the artificial market. The time series data employed was generated by simulating the agent-based cobweb model with the presence of speculators. It was found that many stylized features well documented in financial econometrics can in principle be replicated from the GP-based artificial markets, which include *leptokutosis*, *non-IIDness* and *volatility clustering*. Furthermore, [21] performed a CUSUMSQ test, a statistical test for structural change, on the data. The test indicated the presence of structural changes in the data, which suggested that that the complex interaction process of these GP-based producers and speculators can even generate *endogenous* structural changes.

3 Overlapping Generations Models

While there are several approaches to introducing *dynamic general equilibrium structures* to economics, the *overlapping generations model* (hereafter, OLG), may be regarded as the most popular one in current macroeconomics. Over the last two decades, the OLG model has been extensively applied to studies of savings, bequests, demand for assets, prices of assets, inflation, business cycles, economic growth, and the effects of taxes, social se-

[3] This advantage is particular important for the purpose of modeling adaptive economic agents. Today, we experience various models of adaptive economic agents based on different machine learning tools, e.g., decision trees, nearest neighborhood, reinforcement learning, neural networks, and auto-regressive models. It would be useful to have a general representation such that these different modeling techniques can be integrated. In other words, they can all be endogenously generated without first in-graining them. However, the powerful integration which may be possibly brought by GP is yet to be seen.

[4] An in-depth discussion can be found in [11].

curity and budget deficits. Despite its popularity, the OLG models are well known for their *multiplicity of equilibria*. "When there are multiple equilibria it means that the physical description of the economy together with the notion of equilibrium are not sufficient to pin down a unique predicted outcome.([43], p.26)" Things can be even more intriguing if these equilibria have different welfare implications.

To see whether decentralized agents are able to coordinate intelligently to single out a Pareto-superior equilibrium rather than be trapped in a Pareto-inferior equilibrium, Arifovic ([2]) proposed the first agent-based modification of an OLG model of inflation ([52]). She applied *genetic algorithms* (GAs) to modeling the learning and adaptive behavior of agents in [52]. There are two stationary equilibria in [52], and they differ in the inflation rate. The one with a higher inflation rate is the *Pareto-inferior equilibrium*, whereas the one with a lower inflation rate is the *Pareto-superior equilibrium*. In her study, GA-based agents were shown to be able to select the Pareto-superior equilibrium. She further compared the simulation results based on GAs with those from laboratories with human subjects, and concluded that GAs were superior to other learning schemes, such as the *recursive least squares*.

This line of research was further carried out in [6], [7], [8], and [5]. [8] made the distinction between two implementations of GA learning: depending on what to encode, GA learning can be implemented two different ways, namely, *learning how to optimize* ([2] and *learning how to forecast* ([8]). It was found that these two implementations lead to the same result: agents can indeed learn the Parato superior equilibrium.[5] Nevertheless, a robust analysis showed that coordination was more difficult when the number of inflation values considered by agents was higher, when the two stationary equilibria of the model were closer together, and when agents entertained inflation rate forecasts outside the bounds of possible stationary equilibria.

[6] extends [2]'s two-period OLG model to an n-period one. It was found that for a relatively low value of n, the system is more likely to achieve coordination on the low inflation stationary perfect foresight equilibrium, which is consistent with the findings of many earlier analyses of adaptive learning behavior in two period OLG economies. However, as n increases we see that persistent currency collapse outcomes become increasingly likely. [7] studied a more complicated version of the two-period OLG model ([31]).[6] It was found that the stationary equilibria on which agents coordinate are always *relatively simple* - either a *steady state* or a *low-order cycle*. It is difficult, however, for an economy comprised of optimizing agents with initially heterogeneous beliefs to coordinate on especially complicated stationary equilibria.

[5] provided perhaps the most extensive coverage of robustness checks ever seen in agent-based artificial markets. Their work covers two different levels of GA designs: one is *genetic operators*, and the other is *architecture*. For the former, they consider different implementations of the four main GA operators, i.e., selection, crossover, mutation, and election. For the latter, they consider a *single-population GA* (*social learning*) vs. a *multi-population GA* (*individual learning*). They find that Bullard and Duffy's results are sensitive to two main factors: the *election operator* and *architecture*. Their experimental results in fact lend support to some early findings, e.g., the significance of the election operator ([1]), and the different consequences of social learning and individual learning ([51]). What is particularly interesting is that *individual learning reduces the rate of convergence to the same belief*. This is certainly an important finding, because most studies on the convergence of GAs to Pareto optimality are based on the social learning version.[7]

In [20], the AI-ECON Research Center generalized [8]'s learning how to forecast version of GA learning with GP. In [8], what learning agents learn is just a *number* of the inflation rate rather than *a regularity about the motion of the inflation rate*, which is a *function*. We consider it too restrictive to learn just a number. By [31], if the equilibrium of an OLG is characterized by *limit cycles* or *strange attractors* rather than by fixed points, then what agents need to learn is not just a number, but a functional relationship, such as $x_t = f(x_{t-1}, x_{t-2}, ...)$. [20], therefore, generalized [8]'s evolution of "beliefs" from a sequence of populations of *numbers* to a sequence of populations of *functions*. Genetic programming serves as a convenient tool to make this extension.

The basic result observed in [20] is largely consistent with [1] and [8], namely, agents being able to coordinate their actions to achieve the Pareto-superior equilibrium. Furthermore, their experiments showed that the convergence is not sensitive to the initial rates of inflation. Hence, the Pareto-superior equilibrium has a large domain of attraction. A test on a structural change (a change in deficit regime) was also conducted. It was found that GP-based agents were capable of converging very fast to the new low-inflationary stationary equilibrium after the new deficit regime was imposed. However, the basic result was not insensitive to the dropping of the survival-of-the-fittest principle. When that golden principle was not enforced, we experienced dramatic fluctuations of in-

[5]The only difference is the speed of convergence. The learning how to forecast version of genetic algorithm learning converges faster than the learning how to optimize implementation studied by [2].

[6]Under time-separable preferences and provided that the value of the coefficient of relative risk aversion for the old agent is high enough and that of the young agents is low enough, [31] showed that stationary perfect-foresight equilibria also may exist in which the equilibrium dynamics are characterized either as periodic or chaotic trajectories for real money balances, and these complicated stationary equilibria are also Pareto optimal.

[7]For more discussion on the distinction between individual learning and social learning, see [12].

flation, and occasionally the appearance of super inflation. The agents were generally worse off.

4 Artificial Stock Markets

Among all agent-based artificial markets built in the *AI-ECON Research Center*, the most exciting one is the *artificial stock market*. By all standards, the stock market is qualified to be a complex adaptive system. However, conventional financial models are not capable of demonstrating this feature. On the contrary, the famous *no-trade theorem* shows in the equilibrium how inactive this market can be [50]. It was therefore invigorating when John Holland and Brian Arthur established an economics program at the Santa Fe Institute in 1988 and chose *artificial stock markets* as their initial research project.[8] What one can possibly learn from this novel approach was well summarized in [42], which is in fact the first journal publication on an agent-based artificial stock market. A series of follow-up studies materialized the content of this new fascinating frontier in finance.

Agent-based artificial stock markets have two main stays: *agent engineering* and *institution (trading mechanism) designs*. Agent engineering mainly concerns *the construction of the financial agents*. [49] showed how to use *genetic algorithms* to encode trading strategies of traders. A *genetic fuzzy* approach to modeling trader's behavior was shown in [48], whereas the *genetic neural* approach was taken by [37]. In [9] and [53], we see a perfect example to bring different learning schemes into the model. The learning schemes incorporated into [9] include an empirical Bayesian trader, a momentum trader, and a nearest-neighbor trader, where those included in [53] are *neural networks* traders and momentum traders. [36] gave a more thorough and general discussion of the construction of artificial financial agents. In addition to *models*, *data* is another dimension of *agent engineering*. What can be addressed here is the issue of *stationarity* that the series traders are looking at. Is the entire time series representative of the same dynamic process, or have things changed in the recent past? [37] studied traders who are initially heterogeneous in a *time horizon parameter*, which characterizes their interpretation of how much of the past is relevant to the current decision making.

The second component of agent-based stock markets is the institutional design. A institutional design should answer the following five questions: who can trade, when and how can orders be submitted, who may see or handle the orders, how are orders processed, and how are prices eventually set. Trading institutional designs in the conventional SFI artificial stock market either follow the *Walrasian tatonnement scheme* or the *rationing scheme*. [9] and [53], however, consider a *double auction* mechanism. This design narrows the gap between artificial markets and the real market, and hence makes it possible to compare the simulation results with the behavior of real data, e.g., tick-by-tick data.[9]

Based on agent engineering and trading mechanism designs, agent-based artificial stock markets can generate various markets dynamics, including *price*, *trading volumes*, *the heterogeneity and complexity of traders' behavior*, and *wealth distribution*. Among them, *price dynamics* is the one under the most intensive study. This is not surprising, because ever since the 1960s price dynamics has been the focus of the studies of random walks, the efficient market hypothesis, and market rationality (the rational expectations hypothesis). With the advancement of econometrics, it further became the focus of the study of non-linear dynamics in the 1980s.[10]

Agent-based artificial stock markets make two important contributions to our understanding of the behavior of stock prices. First, it enables us to understand what may cause the price to deviate from *rational equilibrium price* or the so-called *fundamental value*. [42] made the following observation.

> In sufficiently simple cases — with *few agents*, or *few rules per agent*, or *a low-variance dividend stream* — the agents converge to an equilibrium in which price tracks fundamental value (Eq. 15), volume stays low, and there are no appreciable anomalies such as bubbles or crashes .... On the other hand, in a richer environment, there is no evidence of equilibrium. Although the price frequently stays close to fundamental value, it also displays major upward and downward deviations which may be called bubbles and crashes. ([42], p. 272. Italics added.)

Both [53] and [9] discussed the effect of *momentum traders* on price deviation. [53] found that the presence of momentum traders can drive the market price away from the homogeneous rational equilibrium price. [9] had a similar finding: adding momentum traders to a population of empirical Bayesian has an adverse impact on market performance, although price deviation decreased as time went on. [37] inquired whether *long horizon agents* can learn to effectively use their information to generate a relatively stable trading environment. The experimental results indicated that while the simple model structure with *fixed long horizon agents* replicates the usual efficient market results, the route to evolving a population of short horizon agents to long horizons may be difficult. [3] and [38] found that when *the speed of learning* (*the length of a genetic updating cycle*) was reduced (which

[8] The SFI artificial stock market is built upon the standard asset pricing model ([32], [33]).

[9] Furthermore, since stock market experiments with human subjects were also conducted within the double auction framework ([45]), this also facilitates the conversation between the experimental stock market and the agent-based artificial stock market.

[10] See [41] and [39] for a nice review of the field.

forces agents to look at longer horizon features), the market approached the REE.

As to the second contribution, agent-based artificial stock markets also enhance our understanding of several stylized features well documented in financial econometrics, such as *fat tails*, *volatility clusters*[11], and *non-linear dependence*. [38] showed that the appearance of the *ARCH* effect and the non-linear dependence can be related to *the speed of learning*. [53] found that the inclusion of momentum traders generates a lot of stylized features, such as excess volatility, excess kurtosis (leptokurtotic), lack of serial independence of return, and high trading volume.

To simulate the agent-based artificial stock market based on the standard asset pricing model, the AI-ECON Research Center developed software known as the AI-ECON artificial stock market (**AIE-ASM**). The AIE artificial stock market differs from the SFI stock market in the computational tool that is employed. The former applies genetic programming, while the latter has genetic algorithms. In **AIE-ASM**, genetic programming is used to model agents' expectations of the price and dividends. A menu-like introduction to **AIE-ASM** *Ver. 2* can be found in [26].

[24] contributed to agent engineering by proposing a modified version of social learning. The idea is to include a mechanism, called the *business school*. Knowledge in the business school is *open for everyone*. Traders can visit the business school when they are under great survival pressure. The social learning version of genetic programming is applied to model the evolution of the business school rather than directly on traders. Doing it this way one can avoid making an implausible assumption that trading strategies, as business secrets, are directly imitable.[12] [54] further combined this *modified* social learning scheme with the conventional individual learning scheme in an integrated model. In this integrated model a more realistic description of traders' learning behavior is accomplished: the traders can choose to visit the business school (learning socially), to learn exclusively from their experience (learning individually), or both. In their experiments, based on the effectiveness of learning, traders will switch between social learning and individual learning. Allowing such a competition between these two learning styles, their experiment showed that it is the *individual learning style* which won the trust of the majority. To the best of our knowledge, this is the only study which leaves the choice of the two learning styles to be endogenously determined.

[55] examined another important aspect of agent engineering, i.e., *market size* (*number of market participants*). Few studies have addressed the significance of market size on the performance of agent-based artificial markets.[13] Instead, the number of market participants is usually determined in an arbitrary way, mainly constrained by the computational load.[14] Related to *market size* is *population size*. In the case of social learning (single-population GA or GP), market size is the same as population size. However, in the case of individual learning (multi-population GA or GP), population size refers to something different, namely, the number of solution candidates each trader has. Like market size, population size is also arbitrarily determined in practice.

[55] studied the effect of market size and population size upon market efficiency and market diversity under social and individual learning styles. Their experimental results obtained can be summarized as *two effects* on *market efficiency* (price predictability), namely, the *size effect* and the *learning effect*. The size effect says that the market will become efficient when the number of traders (market size) and/or the number of models (GP trees) processed by each trader (population size) increases. The learning effect says that the price will become more efficient if traders' adaptive behavior become more independent and private. Coming to market diversity, we observe very similar effects except population size: market diversity does not go up with population size. These findings motivate us to search for a linkage between *market diversity* and *market efficiency*. A "theorem" may go as follows: *a larger market size and a more independent learning style will increase the diversity of traders' expectations, which in turn make the market become more active (high trading volume), and hence more efficient (less predictable).* Their simulation results on trading volumes also supported this "theorem". They further applied this "theorem" to explain why the U.S stock market behaves more efficient than Taiwan's stock market. Other aspects of agent engineering studied include search intensity, psychological pressure, and prudence. ([22], [23])

[15] is a study devoted to *price deviation*. They examined how well a population of financial agents can track the equilibrium price in the **AIE-ASM**. By simulating the artificial stock market with different dividend processes, interest rates, risk attitudes, and market sizes, they found that the market price is not an unbiased estimator of the equilibrium price. Except in a few extremely

[11] As [40] described, *large changes tend to be followed by large changes - of either sign - and small changes by small changes*. In financial econometrics, this phenomenon is formalized as the GARCH process, where "GARCH" stands for *Generalized AutoRegressive Conditional Heteroskedasticity*.

[12] This is known as Harrald's criticism. For details, see [24].

[13] One good exception is [4], whose simulation results showed that the simple tradable emission permit scheme (an auction scheme) can be the most effective means for pollution control when the number of participants is small. However, as the number of participants increases, its performance declines dramatically and becomes inferior to that of the uniform tax scheme. Another exception is [8].

[14] [1], however, justified the number of participants from the viewpoint of search efficiency. She mentioned that the minimal number of strings (agents) for an effective search is usually taken to be 30 according to the artificial intelligence literature. Nonetheless, agent-based artificial markets have different purposes and concerns.

bad cases, the market price deviates from the equilibrium price moderately from minus four percent to sixteen percent. The pricing errors are in fact not *patternless.* They are actually negatively related to *market sizes*: a thinner market size tends to have a larger pricing error, and a thicker market tends to have a smaller one. For the thickest market which they have simulated, the mean pricing error is only 2.17%. This figure suggests that the new classical simplification of a complex world may still provide a useful approximation if some conditions are met, such as, in this case, the market size.

Another series of contributions made by the AI-ECON Research Center is the study of *emergent properties* within the context of artificial stock markets. *Emergence* is about "*how large interacting ensembles exhibit a collective behavior that is very different from anything one may have expected from simply scaling up the behavior of the individual units*" ([35]; p.3). Consider the *efficient market hypothesis* (EMH) as an example. If none of the traders believe in the EMH, then this property will not be expected to be a feature of their collective behavior. Thus, if the collective behavior of these traders indeed satisfies the EMH as tested by standard econometric procedures, then we would consider the EMH as an emergent property. As another example, consider the *rational expectations hypothesis* (REH). It would be an emergent property if all our traders are boundedly rational, with their collective behavior satisfying the REH as tested by econometrics.

[25] applied a series of econometric tests to show that the EMH and the REH can be satisfied with some portions of the artificial time series. However, by analyzing traders' behavior, they showed that these aggregate results *cannot be interpreted as a simple scaling-up of individual behavior.* The main feature of **AIE-ASM** that produces the emergent results may be attributed to the use of genetic programming, which allows us to generate a very large search space. This large space can potentially support many forecasting models in capturing short-term predictability, which makes simple beliefs (such as that where the dividend is an iid series, or that when the price follows a random walk) difficult to be accepted by traders. In addition to preventing traders from easily accepting simple beliefs, another consequence of a huge search space is the generation of *sunspot-like signals* through mutually-reinforcing expectations. Traders provided with a huge search space may look for something which is originally irrelevant to price forecasts. However, there is a chance that such kinds of attempts may mutually get reinforced and validated. The generation of *sunspot-like signals* will then drive traders further away from accepting simple beliefs.

Using Granger causality tests, [25] found that dividends indeed can help forecast returns. Since by their experimental design, the dividend does not contain the information of future returns, what happens is a typical case of mutually-supportive expectations that make the dividend eventually contain the information of future returns.

As demonstrated in [24] and [25], one of the advantages of agent-based computational economics (the bottom-up approach) is that it allows us to observe *what traders are actually thinking and doing.* Are they *martingale* believers? Are they *sunspots* believers? Do they believe that *trading volume* can help predict returns? By counting the number of traders who actually use sunspots or trading volumes to forecast returns, one can examine whether *sunspots' effects* and the *causal relation between stock returns and trading volume* can be another two emergent properties ([16], [17]).

Agent-based artificial markets have a 5-year history. While their impact on computational finance is increasing, more has to be done before we can prove that they indeed deliver an entirely new way of studying finance. We believe that the challenge waiting for us ahead is to build *a multi-asset agent-based artificial stock market.* This advancement is important for applications to *portfolio theory.* [13] is moving in this direction. Using the extended agent-based artificial stock market, they are simulating the evolution of portfolio behavior, and investigating the characteristics of the long-run surviving population of investors.

5 Concluding Remarks

We review the development of agent-based artificial markets in the AI-ECON Research Center. Related studies which are not reviewed in this article include dynamic games, oligopolistic competition, double auction markets, and evolutionary models of R&D.

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