Network Topologies and Consumption Externalities

Shu-Heng Chen[1], Li-Cheng Sun[1], and Chih-Chien Wang[2]

[1] AI-ECON Research Center, Department of Economics,
National Chengchi University, Taipei, Taiwan 11623
chchen@nccu.edu.tw,
lester33@mail2000.com.tw
[2] AI-ECON Research Center, Department of Computer Science,
National Chengchi University, Taipei, Taiwan 11623
g9022@cs.nccu.edu.tw

Abstract. The economic implications of network topologies are studied via a monopolist's model of market networks originally proposed by Phan, et al. (2003). By embedding the market into a larger collection of network topologies, in particular, a class of *scale-free networks*, we extend the early analysis built upon a class of *ring networks*. To facilitate the study of the impacts of network topologies upon market demand, various measures concerning social welfare (the consumer's surplus), the avalanche effect, and the hysteresis effect, are formally established. Comparisons based on these measures show that *network topologies matter*, and their implied differences will remain even when the network size becomes large.

Keywords: Scale-Free Network, Ring Networks, Network Topologies, Consumption Externality, Avalanches, Hysteresis Effect.

1 Motivation and Literature Review

The impact of social networks on economic behavior appears to have become an important issue, which has just recently been recognized by a large number of economists.[1] The efforts devoted to exploring the effects of network topologies on the resultant economic equilibria are unprecedented. Obviously, a network serves as a fundamental support for the operation of an economic system, even though its existence is sometimes only implicitly granted. The conventional neoclassical theory, which did not explicitly acknowledge the existence of networks,

[1] The literature can be roughly classified into two kinds. The first kind of literature regards networks as *exogenous*, and studies their economic implications. The second kind of literature treats networks as endogenously determined, and studies the formation process of networks. Given a set of cost and profit conditions, economists study the efficiency and stability of different networks, and determine the optimal or a sustainable network. See Radner (1993), Bolton and Dewatripont (1994), Jackson and Wolinsky (1996, 2003), Bala and Goyal (2000), and Goyal (2003) . Nonetheless, to the best of our knowledge, little work has been done on the interaction between network formations and their consequent performance.

T. Washio et al. (Eds.): JSAI 2005 Workshops, LNAI 4012, pp. 314–329, 2006.

seems to suggest that networks, whatever their structure may be, have no effect on resource allocation, at least, in the long run. Nevertheless, in their pioneering work on network formation, Jackson and Wolinsky (1996) proved the Pareto optimality of some well-known network topologies such as the fully-connected network and the star network. They also proved the intuition that when transaction costs are exceedingly high, there will be no network formed, i.e. these is complete independence of agents. These initial theoretical results are convincing enough for us to reconsider more generally the relationship between network topology and resource allocation.

Maybe the most natural start to establishing the link between network topologies and resource allocation, is to look at some fundamentals of economic analysis. Phan, Pajot and Nadal (2003) is a pioneering example. They use network topologies to illustrate the working of *consumption externalities* and from there to show the impact of network topologies upon market equilibrium. First, by using the case where consumers are perfectly independent (the scale of the market network is zero) as the benchmark, they find that the minimum price to win the marginal consumer (the one with the lowest willingness to pay) increases with the *scale* of the market network. In a sense, *the demand curve shifts outward.* Secondly, they also find that the emergence of *avalanches* becomes apparent when the network becomes fully-connected.[2]

Third, even though the scale of the market network remains unchanged, the demand curve still shifts outward when the market topology changes from the *regular network* to the *small-world network.* Fourth, there is also evidence that the network topology can impact the dynamic patterns (the growth or the decay) of the market when a new price is attempted.[3] Fifth, what is particularly intriguing is the hysteresis effect of the demand. With the working of the network externality, once the market is expanding to a certain level with a decreasing price, it will get stuck there and become less sensitive to the price reversal coming later. This unique feature, despite its theoretical foundation, has been well acknowledged in marketing and advertising.

Finally, maybe the most impressive part of Phan, et al.'s paper is that it provides a fully-fledged version of demand and supply analysis, which makes it feasible for market equilibrium analysis with respect to different network topologies. From there, we can see the effect of the network topology on the equilibrium price, profits, and quantity supply (penetration rate).

Despite its rich insights from exploring network topologies, Phan, et al.'s pioneering work is still largely confined to a very specific type of network typology, namely, the *ring network.*[4] A follow-up study, as they also mention, would be

[2] The avalanche can be understood as a phase transition with a threshold,which in our case, is a critical price. The sales can be dramatically different if the price is just a little below on above the threshold.

[3] Of course, the network topology is not their only concern. The other aspect to which they devoted a lot of space is the homogeneity of the consumers.

[4] The ring network is, however, still very useful for the purpose of tutoring. The essence of the random network, regular network, and the small-world network can easily be presented with a ring.

to consider a larger class of network topologies, even including networks which are *evolving*. In this study, we are moving in this direction. Specifically, we are considering the well-known *scale-free network*, which is probably the first kind of evolving network in the literature.[5]

2 Basic Descriptions of a Network

Following the standard notation in graph theory, one can represent a network by $G(V, E)$, where G is the name for the network in question, and V and E denote sets of *vertices* and *edges* respectively, which are the main stays of a network. $V = \{1, 2, , ...n\}$ represents all n constituents of V, and the number n also refers to the *size* of the network. In our specific application, n is the number of consumers in the market, and can, therefore, serve as a measure of the *market size*. $E = \{b_{i,j} : i, j \in V\}$ encodes the relationship between any two vertices in the net. In a special case, $b_{ij} = 1$ if there exists an edge (connection, relation) between i and j; otherwise it is zero. In this special case, $b_{ij} = b_{ji}$, implying that direction is irrelevant,which is also known as the *non-directed network*.

Given $G(V, E)$, let $d(i, j)$ be the length of the shortest path between the vertices i and j, These the mean shortest length of $G(V, E)$ is simply the mean of all $d(i, j)$,

$$L = \frac{1}{\frac{1}{2}n(n-1)} \sum_{i \geq j} d(i, j). \tag{1}$$

The definition above may be problematic if there is an isolated vertex which actually has no edge on any other vertices. So, $G(V, E)$ with isolated vertices are not considered here.

In addition to the distance measure, there are also a few density measures, and the one which is used most popularly is known as the *cluster coefficient*. Given a vertex i in a $\mathbf{G}$, we would first like to measure how well its neighbors get connected to each other. Specifically, if j is connected to i, and k is also connected to i, is j also connected to k? Formally, we define the set of neighbors of i as

$$\vartheta_i = \{j : b_{ij} = 1, j \in G\}. \tag{2}$$

Then the cluster coefficient in terms of i is

$$C_i = \frac{\#\{(h, j) : b_{hj} = 1, h, j \in \vartheta_i, h < j\}}{\#\{j : j \in \vartheta_i\}}, \tag{3}$$

and the cluster coefficient of $\mathbf{G}$ is the average of all C_i,

$$C = \frac{1}{n} \sum_{i=1}^{n} C_i. \tag{4}$$

[5] The study suggested here can be further extended into two directions: first, to extend to various probabilistic combinations of random networks and scale-free networks, and second, to take into account the idea of *community*.

The size of the neighborhood ϑ_i is called the *degree* of i. Denoted by $f(k)$, the *degree distribution* of $G(V, E)$ gives the percentage of vertices of degree k. The behavior of the degree distribution has become one of the most intensively studied features of networks.

The three statistics above, *the average shortest length* (L), *the cluster coefficient* (C), and the *degree distribution* ($f(k)$) provide the basic description of a network $\mathbf{G}(\mathbf{V}, \mathbf{E})$.[6]

3 Matrix Representation of the Network Externality

The most natural way to think of a network is to consider all possible $\binom{n}{2}$ connections represented by a *matrix*, of which the geographical information can be implicit. The matrix representation also highlights the essential difference between social networks and other physical networks such as transportation networks. For the former, the geographical information is only implicit and may not even be relevant, whereas, for the latter, it is explicit and is very substantial.

Basically, space or location, those explicit geographical variables, may not be the most important variables in determining the formation of social networks. Unlike the regular ring network, agent i and agent j are connected to each other, not necessarily because their offices are just next to each other, but more because they share some common interests or attributes which connect them together. It would, therefore, leave us more freedom to form a social network if we could disentangle the *physical distance* from the *social distance*.[7]

For a market network model, we start with Phan, et al.'s (2003) set-up and will add our modification later. Phan, et al. model agents' decisions to buy as the following optimization problem:

$$\max_{\omega_i \in \{0,1\}} V_i = \max_{\omega_i \in \{0,1\}} \omega_i (H_i + \sum_{k \in \vartheta_i} J_{ik} \omega_k - p). \quad (5)$$

ω_i is an indicator for a binary decision.

$$\omega_i = \begin{cases} 1, & if \;\; agent \;\; i \;\; decides \;\; to \;\; buy, \\ 0, & otherwise. \end{cases} \quad (6)$$

[6] Other quantitative descriptions include.

[7] We do not intend to present a fully-fledged version of this type of social network. We just give a sketch. Consider an agent i, who is completely described by a vector of social attributes, say $x_i = (x_{1i}, x_{2i}, ..., x_{ni})$, i.e. a point on n-dimensional space. The distance between two agents i and j, $d(i, j)$ can, therefore, just be the standard Euclidean metric:

$$d(i, j) = ||x_i - x_j||.$$

An effective network of agent i, ϑ_i is determined by a $\{d(i, j) : j = 1, 2, ..., n, j \neq i, \}$. As feedback, the network therefore provides a value to agent i, i.e. the value or utility of the network, denoted by $u_i(\vartheta_i)$ At a point in time, to increase his well-being, agent i may like to change his network by modifying his attribute variables by, ϵ_i as $x_i + \epsilon_i$, is associated with the cost of making these changes, $c_i(\epsilon_i)$. The cost is subtracted from his u_i, which leaves agent i with utility $u_i(\vartheta_i) - c_i$.

What is inside the bracket is the classical linear willingness-to-pay function. H_i represents agent i's idiosyncratic utility of consuming the commodity. Written in terms of money value, it functions as the *reservation price* of agent i, i.e. his idiosyncratic willingness-to-pay for the commodity. The difference between H_i and p can be taken as the familiar *consumer's surplus* when agent i decides to buy ($\omega_i = 1$).

Now, agent i is situated in a network, and is directly connected to a collection, ϑ_i, of some other agents, called agent h, where $h \in \vartheta_i$. The network effect is captured by the parameter J_{ik}, which characterizes the influence on agent i of the decision of agent h. Clearly the larger the absolute value of J_{ih}, the larger the influence of agent h on agent j.[8] If $J_{ih} = 0$, for all $h \in \vartheta_i$, then there is no network effect on agent i; essentially, agent i makes his decision *independently.*

Consider the literature on *attention control.* If agent i's attention to the external world has an upper limit, say J, then

$$\sum_{h \in \vartheta_i} J_{ih} = J. \tag{7}$$

Furthermore, if agent i is indifferent to all his neighbors h, then J is uniformly distributed over ϑ_i, i.e.

$$J_{ih} = \frac{J}{N_{\vartheta_i}}, \quad \forall h \in \vartheta_i. \tag{8}$$

N_{ϑ_i} is the number of agents (vertices) connected to agent i. Under this setting, as the size of his neighborhood becomes larger, the influence of each individual neighbor on agent i becomes smaller.

The *external influence* of the network is demonstrated by the feature that agents are *heterogeneous* in preference, H_i.[9] However, the way to model heterogeneous preferences is not unique.[10] We shall come back to this point later.

[8] J_{ih} can in general be both positive and negative, while in our specific application below, we only consider the positive externality.

[9] Obviously, if all agents share the same H, there will be no need to study the network influence in this specific context, since "influence" means causality: someone will have to buy first, and someone will follow. If the agents are homogeneous in terms of preference, then from Equation (5), their decision will be perfectly homogeneous, including their timing, so that the network influence does not exist. Nevertheless, this does not mean that network topology has no effect on the market's behavior. In fact, as Phan, et al. (2003) have already shown, when agents are homogeneous in preference, then we shall have the most significant avalanche effect as well as the hysteresis effect. In fact, *it may be useful to distinguish the avalanche due to the homogeneous preference and also due to the market contagious influence of the market*

[10] For example, the *random field Ising model* is used in Phan, et al. (2003). More precisely, in their model,

$$H_i = H + \theta_i, \tag{9}$$

where θ_i follows a logistic distribution with mean 0 and $\sigma^2 = \frac{\pi^2}{3\beta^2}$.

It should be clear now that the optimization problem (5) leads to a very simple solution, namely,

$$\omega_i = \begin{cases} 1, & if \;\; H_i + \sum_{k \in \vartheta_i} J_{ik}\omega_k - p \geq 0, \\ 0, & otherwise. \end{cases} \tag{10}$$

To fully represent the network dynamics, it will be useful to extend Equation (5) as a individual decision to collective decisions as in (11).

$$\mathbf{W_{t+1}} = g(\mathbf{H} + \mathbf{J} \cdot \mathbf{B} \cdot \mathbf{W_t} - \mathbf{P}), \tag{11}$$

where

$$\mathbf{H} = \begin{bmatrix} H_1 \\ H_2 \\ \cdot \\ \cdot \\ \cdot \\ H_n \end{bmatrix}, \mathbf{J} = \begin{bmatrix} J_1 & 0 & \dots & 0 \\ 0 & J_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & J_n \end{bmatrix}, \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix}, \mathbf{W_t} = \begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \\ \cdot \\ \cdot \\ \cdot \\ \omega_{n,t} \end{bmatrix}, \mathbf{P} = \begin{bmatrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ \cdot \\ p_n \end{bmatrix}.$$

The vector matrix **H** just stacks up an individual's idiosyncratic preferences H_i. The diagonal matrix **J** has the contribution from each individual i's neighbor to i as described in Equation (8). The matrix **B** is a the general representation of the network. Each entry b_{ij} represents a connection (edge) between agent i and agent j. While a great flexibility of the connection may exist, here we make two assumptions. First, either the connection exists or it does not, i.e. there is no *partial* connection, nor is there any difference in the degree of connection. Therefore,

$$b_{ij} = \begin{cases} 1, & if \;\; agent \;\; i \;\; is \;\; connected \;\; to \;\; agent \;\; j, \\ 0, & otherwise. \end{cases} \tag{12}$$

Second, we assume that the connection (edge), if it exists, is *bi-directional.* Hence

$$b_{ij} = b_{ji}. \tag{13}$$

In other words, the matrix **B** is *symmetric.* Notice that, according to Equation (8), the matrix **J** is immediately determined by the matrix **B**.

For sack of illustrations, the matrix representations of a ring, star and fully-connected network with $n = 4$, denoted by $\mathbf{B}_{\bigcirc}$, $\mathbf{B}_{\star}$ and $\mathbf{B}_{\otimes}$, respectively, are given below from left to right:

$$\mathbf{B}_{\bigcirc} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \mathbf{B}_{\star} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \mathbf{B}_{\otimes} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

With these connections, the corresponding influence matrix representations $\mathbf{J}_{\bigcirc}$, $\mathbf{J}_{\star}$, and $\mathbf{J}_{\otimes}$ are

$$\mathbf{J}_{\bigcirc} = \begin{bmatrix} J/2 & 0 & 0 & 0 \\ 0 & J/2 & 0 & 0 \\ 0 & 0 & J/2 & 0 \\ 0 & 0 & 0 & J/2 \end{bmatrix}, \mathbf{J}_{\star} = \begin{bmatrix} J/3 & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & J & 0 \\ 0 & 0 & 0 & J \end{bmatrix}, \mathbf{J}_{\otimes} = \begin{bmatrix} J/3 & 0 & 0 & 0 \\ 0 & J/3 & 0 & 0 \\ 0 & 0 & J/3 & 0 \\ 0 & 0 & 0 & J/3 \end{bmatrix}.$$

The vector $\mathbf{W_t}$ stacks up the binary decisions made by all agents in period t, with each entry described by Equation (6). To trace the dynamics of binary choices, binary decisions are now indexed by t. Initially, $\omega_{i,0} = 0$ for all $i = 1, ..., n$. The function g then drives the dynamics of $\mathbf{W_t}$ based on each agent's optimal decision (10). The vector $\mathbf{P}$ is the price charged to each individual. Notice that we have indexed this price by individual i, which indicates the possibility that the price does not have to be homogeneous among agents, considering that we want to study the effect of price discrimination under this framework. The simulation conducted in this paper, however, does assume a homogeneous price.

This matrix representation leaves us with great flexibility to deal with various network topologies. As we shall see in this paper, it helps us to deal with the scale-free network to be detailed in the next section. In addition, by varying $\mathbf{B}$, there can be other advantages. Firstly, $\mathbf{B}$ can be *asymmetric*. This is a desirable variation since social influence in general is not symmetric. Secondly, it does not have to be a binary matrix. In fact, b_{ij} can be any continuous variable between 0 and 1. This can help us to capture different degrees of connection. In sum, a continuous and asymmetric $\mathbf{B}$ provides us with an opportunity to study more complex network topologies which are beyond simple geometric representations.

4 The Scale-Free Network

The purpose of this paper is to attempt to extend Phan, et al.'s analysis from their *ring-based networks* (including small world networks) to *scale-free networks*. In this section, we shall give a brief introduction to the scale-free network.

The *scale-free network* was first proposed by Barabasi and Albert (1999), and hence is also known as the *BA model* (the Barabasi-Albert model). The BA model is based on two mechanisms: (1) networks grow incrementally, by the adding of new vertices, and (2) new vertices attach *preferentially* to vertices that are already well connected.

Let us assume that initially the network is composed of m_0 vertices, and that each is connected to m other vertices ($m < m_0$). Then, at each point in time, a number of new vertices, m_T are added to the network, each of which is again connected to m vertices of the net by the *preferential linking*. This idea of *preferential attachment* is similar to the classical "*rich get richer*" model originally proposed by Simon (1955).[11] It is implemented as follows. At time T, each of the new m_T vertices is randomly connected to a node $i \in V_T$ according to the following distribution

$$\pi_i = \frac{k_i}{\sum_{j \in V_T} k_j}, i \in V_T, \tag{14}$$

[11] In fact, the BA model which leads to the power-law degree distributions is an independent rediscovery of earlier work by Simon (1955) on systems with skewed distributions. It can be interpreted as an application of Simon's growth model in the context of networks, readily explaining the emergent scaling in the degree distribution.

where $V_T = \{1, 2, ..., \sum_{t=0}^{T-1} m_t\}$. That is the probability of becoming attached to a node of degree k is proportional to k, $\pi(k)$, and nodes with high degrees attracts new connections with a high probability. To avoid redundancy, the random attachment with (14) is done by sampling *without* replacement.

The scale-free networks have a power-law distribution of degree

$$f(k) \sim k^{-\lambda}, \tag{15}$$

and they have smaller L (average shortest length) and larger C (cluster coefficients), as compared to the random network.

In addition to the scale-free network, this paper also considers something in between the scale-free network and the random network, i.e. a mixture of the two. The motivation behind this device is mainly to capture some possible degree of randomness in social network formation. While the preferential attachment defines the *rationality* behind the social network formation, one should not neglect the effects of random events on the network formation. Therefore, we allow each of the incoming m_t agents (vertices)to have a probability, denoted by $1-q$, of being connected to the incumbent agents simply *randomly*, and hence a probability of q being connected to them with preferential linking. By controlling q, one can have a mixture network which is very close to the random network (e.g., $q \approx 0$), and another mixture network which is very close to the scale-free network (e.g., $q \approx 1$). It is, therefore, feasible to examine how the emergent properties may change along this spectrum. For simplicity, we shall call this the q-network. Clearly, the q-network is a random network if $q = 0$, and is a scale-free network if $q = 1$.

5 Measuring the Network Effect on Demand

It is important to know what useful observations should be seen in order to justify the non-trivial differences induced by network topologies? We notice that, if the network topology does have non-trivial effects, then the demand curve, as a summary of these changes, should naturally be the first thing to look at. As compared to the extreme case, the isolated network, the demand should shift outward when a network with some degrees of connection is introduced. However, what is not clear is which network topology should have the strongest outward shift. This question becomes more perplexing when demand curves associated with different network topologies may cross each other.

Let us define the *penetration rate* as the percentage of buyers in the market, i.e.

$$r \equiv \frac{\#\{i : w_i = 1\}}{n}. \tag{16}$$

Since each consumer buys at most one unit, $0 \leq r \leq 1$. To make the roles of the price and the network topology explicit, we write r as $r(p, \bigoplus)$, where $\bigoplus$ is the respective network topology. A network topology $\bigoplus$ is said to *uniformly dominate* other network topologies and is denoted by $\bigoplus^*$, if

$$r(p, \overset{*}{\bigoplus}) \geq r(p, \bigoplus), \forall p, \forall \bigoplus. \tag{17}$$

Since the uniformly-dominating network topology may not exist, an alternative measure is to define a *maximum price*, p_{max}, as follows:

$$p_{max} \equiv \max_{p}\{p : r(p) = 1\}. \tag{18}$$

Again, to acknowledge the influence of the network topology, p_{max} is also written as $p_{max,\bigoplus}$.

Instead of p_{max}, we may consider a weighted average of $r(p)$ with respect to a distribution of p, and compare the weighted averages among different network topologies:

$$\mu_p = E(p) = \int_{\underline{p}}^{\overline{p}} r(p) f_p(p) dp, \tag{19}$$

where $\underline{p}$ and $\overline{p}$ define an effective range of p, such that

$$\begin{cases} r(p, \bigodot) = 0, & if \;\; p \geq \overline{p}, \\ r(p, \bigodot) = 1, & if \;\; p \leq \underline{p}, \\ 0 < r(p, \bigodot) < 1, otherwise, \end{cases} \tag{20}$$

where $\bigodot$ denotes the isolated network. f_p is a density function of p;and when it is uniform over $[\underline{p}, \overline{p}]$, μ_p in a sense can be regarded as a *social welfare measure* if the marginal cost is zero. In this specific context, what concerns us is how the network topology impacts social welfare.

Another interesting type of behavior of the demand curve is its *jump* or *discontinuity*, known as an *avalanche*, which can be formulated as follows. The demand curve is said to demonstrate an avalanche at price p_a if

$$d_{p_a} = r(p_a - \epsilon) - r(p_a) \;\; is \;\; large. \tag{21}$$

d_{p_a} is not actually well-defined because the word "large" is not precise. Certainly, one can substantiate its content with a threshold parameter, say θ_a, and modify Equation (21) as

$$d_{p_a} = r(p_a - \epsilon) - r(p_a) > \theta_a. \tag{22}$$

The avalanche effect can then be defined as a probability A as follows:

$$A = Prob(d_{p_a} > \theta_a). \tag{23}$$

However, as we mentioned earlier (see footnote 9), the avalanche effect may have nothing to do with the network topologies, and can purely come from the homogeneous group of agents. To avoid "*spurious*" avalanches and to disentangle the effect of homogeneity from the effect of the network topology, it will be useful to maintain a great degree of heterogeneity to examine the chance of observing avalanches with respect to different topologies.

Coming to the next issue is a more subtle one in that the demand curve may not be unique and is *scenario-dependent*,an issue that is described as the *hysteresis effect* by Phan, et al. (2003). That is, the demand given the price p can depend on what happens before. Has the price before been higher or lower? This phenomenon known as the hysteresis effect arises because the demand curve, or equivalently, the penetration rate, derived by decreasing the price is different from the one derived by increasing the price.[12] Formally, hysteresis happens at price p when

$$r_u(p) > r_d(p), \tag{24}$$

where r_u and r_d refer to the penetration rates derived by moving downstream and upstream respectively. Nevertheless, with the avalanche, the hysteresis effect may occur simply because of the great homogeneity of the agents. Therefore, to see whether network topologies can have a real effect on the appearance of hysteresis, it is important to keep agents as heterogeneous as possible. The hysteresis effect of a network topology can then be measured by

$$R \equiv \int_{\underline{p}}^{\overline{p}} (r_u(p) - r_d(p)) f_p(p) dp. \tag{25}$$

$R(\bigoplus)$ denotes the hysteresis effect of the network topology $\bigoplus$.

None of the questions discussed so far may be independent of the market size n (the size of the network) or k, the degree of local interaction, It is then crucial to know the limiting behavior as well. For example, would

$$\lim_{n\to\infty} A_n = 0? \tag{26}$$

and

$$\lim_{n\to\infty} R_n = 0? \tag{27}$$

If Equations (26) and (27) are valid, then in a sense the network topology will not matter when the market becomes thick. In this spirit, we can even ask whether the demand curves associated with different network topologies will be *asymptotically equivalent*.

Similarly, we can pose the same question regarding the dimension of k, ranging from an isolated network to a full network. In addition, the asymptotic issue can be framed in time: given the same penetration rate associated with the same price, which network topology has the fastest *convergence speed* to the penetration rate?[13]

6 Experimental Designs

Experimental designs mainly concern the determination of those parametric matrices appearing in Equation (11). First, *network topologies* (Matrix **B**). To isolate the working of the network topology, it is important to always have the *isolated network* as one of the our benchmarks. Other network topologies

[12] This is also known as the *captive effect* in marketing.

[13] Similarly,when considering the degree of heterogeneity, we may ask: how do the network topologies work with the heterogeneity of agents? Does heterogeneity amplify or depress some of the workings of network topologies?

Table 1. Experimental Design

Basic Design	
Network Topology	Isolated, Scale-free, World Ring (Regular), q-network
Network Size (N)	1000
Idiosyncratic Preference (H_i)	$U[1,2]$
Price (p)	$[1,2]$
Incremental Size of Price	0.02
Some Details	
Degree of Regular Networks	2
Degree of Scale-free Networks	m_0=10 , m=2
q of the q networks	0, 0.1, 0.2,...,0.9

included for comparison purposes are *random networks*, *regular networks* (*ring networks*), *scale-free networks*, and *mixture networks.* Once the network topology is fixed, matrix **J** in Equation (11) is determined accordingly.

Secondly, *idiosyncratic preference* or *willingness to pay*, H_i. As what has been printed it out in footnote (9), it is important to maintain a degree of heterogeneity among agents in terms of their idiosyncratic preference. Accordingly,as entities of **H**,the H_i are *uniformly* sampled from the interval $[1,2]$. With this design of **H**, the corresponding demand curve $r(p)$ is, therefore, restricted to the same interval, and is discretized with an increment of 0.02.

Once all these matrices are determined, we can have the dynamics of $\mathbf{W}_t$, which allows us to derive the demand curve r_p and other related measurements discussed in Section 5.

7 Experimental Results

7.1 Consumer‘s Surplus

Figure 1 is the demand curve $r(p)$ under different network topologies. The resultant demand curve is based upon 100 independent runs each with a new initialization of the matrix **H**. So, each single point along $r(p)$ is an average of 100 observations. Based on these estimated $r(p)$, we calculate the consumer‘s surplus. Not surprisingly, the world network and the isolated network provide the two extremes of the consumer‘s surplus: a maximum of 0.876 is given by the world network, whereas a minimum of 0.509 is given by the isolated network. The consumer‘s surplus of partially connected networks, the scale-free network and the regular network lie in between, and their differences are not that significant (0.776 vs. 0.766).

Experiment one and Figure 1 basically confirm a simple intuition that the consumer‘s surplus is a positive function of *degree* or *connection intensity.* The second experiment tries to explore other determinants of the consumer‘s surplus, particularly, the *cluster coefficient* and *average shortest length* of a network. To do

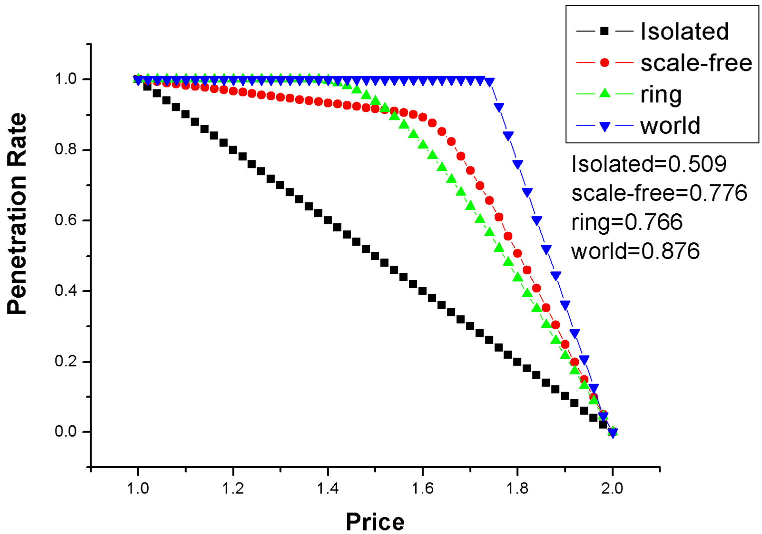

Fig. 1. Demand Curves in Various Networks

Table 2. Consumer's Surplus, Cluster Coefficient and Average Shortest Length

$CS = \alpha_0 + \alpha_1 C + \alpha_2 L$		
Regressors	coefficient	p-value
Constant	0.78019 (α_0)	0.0000
Cluster Coefficients (C)	0.52018 (α_1)	0.0041
Average Shortest Length (L)	-0.0092 (α_2)	0.1112

The R^2 of the above simple linear regression is 0.46676(R^2-adjusted=0.44625), and the mean square error is 0.00655.

so, we consider a class of q networks by controlling q from 0, 0.1,..., to 0.9, and 1. Five independent runs are conducted for each q network, and this gives us totally 55 networks.[14] The resulting consumer's surplus of these 55 networks is then regressed against two independent variables, namely, the cluster coefficient (C) and the average shortest length (L). The regression results are shown in Table 2. It can be found from Table 2, that these two variables C and L can jointly explain almost 50% of the variation in the consumer's surplus. In addition, both regression coefficients have signs consistent with our intuition: the consumer's surplus is positively related to the cluster coefficient, whereas it is adversely affected by the average shortest length.

[14] Only the **B** matrix is regenerated for each independent run. The **H** matrix remains unchanged for all the 55 matrices.

Table 3. Avalanche Effect

Topology/p_a	1.98	1.94	1.84	1.74	1.54
Random	0.01	0.04	0.05	0.06	0
q ($q = 0.5$)	0.07	0.05	0.11	0.03	0
Scale-free	0.06	0.06	0.04	0.09	0
World	0	0.2	0.35	0	0
Regular	0	0	0	0	0

The avalanche effect is defined in Equation 23. θ_a is set to 10%, i.e. 10% of the market capacity. ϵ= 0.02.

7.2 Avalanche Effect

Network topology can matter because it can introduce critical points to market demand, which in turn cause the demand curve to no longer be continuous as conventional economics assumes. This phenomenon known as the *avalanche effect* is what we study in the third experiment. A measure of the avalanche effect is defined in Equation (23), which depends on three parameters, namely, a threshold (θ_a),the perturbation size (ϵ) and the evaluation point (p_a). Since it would be meaningless to consider small jumps when talking about "avalanches", we, therefore, set θ_a to 0.1, i.e. 10% of the market capacity, and ϵ to 0.02. In other words, if by discounting 2 cents only, one can suddenly increase sales by 10% of the market capacity, then an avalanche is detected. Finally, since H_i, uniformly distributed over the range [1,2], we choose five evaluation points of p_a from 1.98 to 1.54 (see Table 3). This is roughly the upper half of the distribution of H_i, which should be the ideal place to monitor the avalanche if there is one.

As to the network topology, except for the isolated network, all other partially-connected or fully-connected network topologies listed in Table 1 are tried in this experiment. 100 runs are conducted for each network topology. The results are shown in Table 3. From Table 3, we find that except for the regular network, avalanches unanimously exist in all four other types of network, while their structures are different in terms of the distribution of the tipping points (p_a) and the tipping frequencies (A). For example, the world network has more concentrated tipping points ($p_a = 1.94, 1.84$) and a high tipping frequency at these tipping points ($A = 0.2, 0.35$), whereas the other three network topologies have evenly distributed tipping points, although with much lower tipping frequencies.

7.3 Hysteresis Effect

The third question to address is the examination of the *hysteresis effect* as defined by Equation (25). What we do here is first derive the demand curve by running the price downstream , and then by running the price upstream. In this experiment, we consider all network topologies listed in Table (2). For the q networks, we only consider the cases $q = 0, 0.5$, and 1. 100 runs are conducted for each network topology. The result shown in Table 4 is, therefore, the average of these

Table 4. Hysteresis Effect

Topology	Downstream ($r_d(p)$)	Upstream ($r_u(p)$)	R
Isolated	0.510	0.510	0
World	0.876	0.889	0.013
Scale-free	0.776	0.851	0.075
Random	0.738	0.816	0.079
q ($q = 0.5$)	0.758	0.847	0.089
Regular	0.766	0.943	0.177

100 runs. Table 4, columns two and three, show the consumer‘s surplus associated with the $r_d(p)$ and $r_u(p)$. The fourth column of Table 4 shows the difference between the two surpluses, i.e. the measure of the hysteresis effect R. From this column, we can see that both the isolated network and the fully-connected network have very little hysteresis effect. As expected, it is identically 0 for the isolated network, and is only 0.013 for the fully-connected network. However, all partially connected networks show some degree of hysteresis. Among them, the scale-free, random and q ($q = 0.5$) networks are close, whereas the regular network has a strong hysteresis effect.[15]

7.4 Network Size

It is interesting to know whether the property of hysteresis and avalanches obtained above is sensitive to the *size* of the network. In particular, we are interested in knowing, when the network‘s size becomes very large (ideally infinite), whether these two properties can still be sustained. We, therefore, simulate networks with sizes of 1000, 3000, and 5000. The results are shown in Figures 2 and 3.[16] What is shown on the left part of these two figures are the demand curve $r(p)$ associated with an isolated network and a scale-free network. By looking at these two graphs visually, we can see that the avalanche effect, characterized by the noticeable jumps in the demand curve, does not disappear as the size gets larger. Furthermore, the right part of the two figures shows the demand curve derived by running downstream and upstream. The bifurcation is clearly there with hysteresis measures of 0.0517 ($N = 3000$) and 0.0597 ($N = 5000$). Compared to the R of the scale-free network from Table 4, these two Rs become smaller, but are still quite distinct from the fully-connected network. Therefore, the asymptotic equivalence of network topologies does not hold here as far as these two properties are concerned. Generally speaking, the finding that network topology matters does not depend on network size.

[15] This is definitely a very interesting property of the regular network. We, however, cannot say much about the cause of it except for pointing out that it deserves further research.

[16] When the size becomes large, computation becomes very time-consuming. As a result, results based on multiple runs are not available yet. What is presented here is based on the results of a single run.

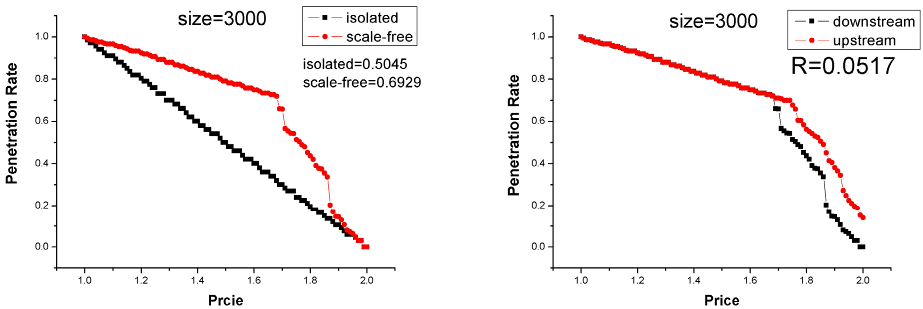

Fig. 2. Size=3000

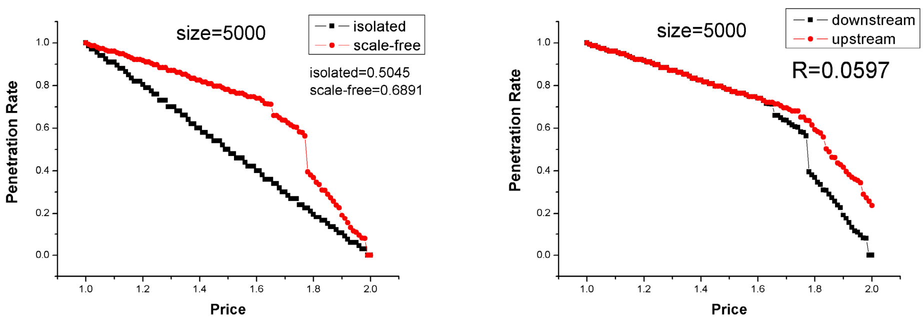

Fig. 3. Size=5000

8 Concluding Remarks

In this paper, five experiments are conducted to examine the economic implications of network topologies. To the best of our knowledge, this is probably the first systematic and extensive study of this kind. It is extensive because six different network topologies are considered, ranging from fully-connected and partially-connected networks to isolated networks. Specifically, these six networks are world networks, ring networks, random networks, scale-free networks, q networks, and isolated networks. By using a simple demand analysis based on a monopolist's market, the study is also extensive in the sense that it covers the four aspects of market behavior: the consumer‘s surplus, the avalanche effect, the hysteresis effect and the size effect. Besides, it is systematic because different measures have been developed to facilitate comparison of the market behavior of different network topologies.

The general results are as follows. First, the network topology will impact social welfare as conventionally described in terms of the consumer‘s surplus. We have further found that it is positively affected by the cluster coefficient, whereas it is negatively affected by the average shortest length. Second, the avalanche effect and the hysteresis effect are observed for some network topologies, but not others. For example, the avalanche effect does not occur in the case of the ring network (regular network), whereas it does exist for other partially- or fully-connected networks. Despite its occurrence, its extent in terms of our measure can differ among different network topologies. Finally, within our limited number of trials, it is also found that those avalanche effect and hysteresis effect will not disappear when the network size becomes larger. In other words, the asymptotic equivalence of network topologies may not be sustained and the network topology may matter to a quite general extent.

References

1. Bala, V. and S. Goyal (2000)," A Noncooperative Model of Network Formation", *Econometrica*, 68(5), pp 1181-1229.
2. Barabasi, A-L and R. Albert (1999), "Emergence of Scaling in Random Networks", *Science*, 286, pp 509-512.
3. Bolton, P., and M. Dewatripont (1994),"The Firm as a Communication Network", *Quarterly Journal of Economics*, 109(4), pp 809-839.
4. Jackson, M.O. and A. Wolinsky (1996), "A Strategic Model of Economic and Social Networks", *Journal of Economic Theory* , 71(1), pp 44-74.
5. Jackson, M.O. (2003),"A Survey of Models of Network Formation: Stability and Efficiency", In: Demange, G. and M. Wooders (Eds.), *Group Formation in Economics: Networks, Clubs, and Coalitions*. Cambridge University Press, Cambridge.
6. Goyal, S. (2003),"Learning in Networks: A Survey" Forthcoming In: G. Demange and M. Wooders(Eds.), *Group Formation in Economics: Networks, Clubs, and Coalitions*, Cambridge University Press, Cambridge.
7. Phan, D., S. Pajot , Nadal J.P. (2003) ,"The Monopolist's Market with Discrete Choices and Network Externality Revisited: Small-Worlds, Phase Transition and Avalanches in an ACE Framework", *Ninth annual meeting of the Society of Computational Economics University of Washington*, Seattle, USA, July 11-13.
8. Radner,R. (1993),"The Organization of Decentralized Information Processing", *Econometrica*, 61, pp 1109-1147.
9. Simon, H. A. (1955)," On a Class of Skew Distribution Functions", *Biometrika*, 42, pp 425-440.