

# TRADING RESTRICTIONS, PRICE DYNAMICS AND ALLOCATIVE EFFICIENCY IN DOUBLE AUCTION MARKETS: ANALYSIS BASED ON AGENT-BASED MODELING AND SIMULATIONS

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In this paper we conduct two experiments within an agent-based double auction market. These two experiments allow us to see the effect of learning and smartness on price dynamics and allocative efficiency. Our results are largely consistent with the stylized facts observed in experimental economics with human subjects. From the amelioration of price deviation and allocative efficiency, the effect of learning is vividly seen. However, smartness does not enhance market performance. In fact, the experiment with smarter agents (agents without a quote limit) results in a less stable price dynamics and lower allocative efficiency.

Keywords: Agent-based DA market; genetic programming; quote limit; allpa value; allocative efficiency.

#### 1. Introduction

In their seminal paper "Allocative Efficiency of Market with Zero-Intelligence Trader," Gode and Sunder posed an interesting question: How much intelligence is required of an agent to achieve human-level trading performance? The answer, as it first appeared, is surprising: little. How little? To make that message clearer, they called their agents zero-intelligence agents. These agents, when assigned a bargaining position in a standard double auction market, were simply bidding or asking randomly as if they had no capability to extract any useful information from the market. While these agents individually cannot bargain in an intelligent manner, their interactions via the market did collectively result in a near-100% allocative efficiency. They attributed this magic to Adam Smith's invisible hand:

<sup>&</sup>lt;sup>a</sup> Allocative efficiency is the efficiency with which markets allocate the resources. A market is allocatively efficient if the maximum possible net benefit (surplus) from market transactions has been realized.

"Adam Smith's invisible hand may be more powerful than some have thought; it can generate aggregate rationality not only from individual rationality, but also from individual irrationality." (Ref. 7, p. 119, Italics added).

It may be delicate to show that aggregate rationality does not rest upon individual rationality in some economic contexts. However, that does not include Gode and Sunder's case, and as a matter of fact, in their case it is trivial to show that allocative efficiency (aggregate rationality) does not rest upon individual rationality. To see this, let us simply assume that all agents are truth-tellers. In this regard, buyers would bid with their redemption values and sellers would ask with their unit costs. Obviously, allocative efficiency would then automatically be 100%.

Gode and Sunder did seem to assume that individual rationality would necessarily imply aggregate rationality (see the quotation above). They might also suppose that the allocative efficiency would be near 100% if all traders are smart enough. Consequently, their focus was to look for the minimum intelligence level which can generate aggregate rationality. What, however, may surprise them is that this minimum level is so low that you only need dumb agents.

Our picture is different: we do not assume that individual rationality necessarily implies aggregate rationality. Therefore, instead of the minimum level, we are looking at the other direction. We have already argued that the minimum level is not a problem at all: a group of innocent (honest) traders would result in a 100% allocative efficiency. What may make things uncertain is the case when traders are no longer innocent, but rather sophisticated. Hence, for us, the interesting question to ask is: Would smart or smarter traders reduce allocative efficiency? As we shall see in this paper, the answer is positive.

In this paper we conduct agent-based simulations of DA markets with different intelligence levels of traders. Traders in one case are endowed with more space to act than traders in the other case. This extra space makes more sophisticated trading strategies possible to emerge, and hence can make traders in one case potentially smarter than traders in the other case. b Traders who are given this favor are called the *smart trader*, and traders who are not are called the *mediocre trader*. We then examine the allocative efficiency achieved within these two different setups. It is found that markets composed of mediocre traders realized 96% of the potential social surplus, whereas markets composed of smart traders realized only 88% of it.

<sup>&</sup>lt;sup>b</sup>Chen and Chie [3] examined some *smart strategies* that evolved from their agent-based simulation

<sup>&</sup>lt;sup>c</sup> "Consumer surplus is the difference between what a consumer is willing to pay for a good and what the consumer actually pays when buying it," and "the producer surplus of a firm is the sum over all units produced of the difference between the market price of the good and the marginal cost of production." (Ref. 9, p. 113 & p. 255). The social surplus is the sum of consumer surplus and producer surplus. It, therefore, represents total net gain from market transactions.

Our finding for smart traders can be compared to the one Gode and Sunder saw for their zero-intelligence traders. While Gode and Sunder's finding shows that the invisible hand is more powerful than we thought, our finding shows the opposite. Moreover, to attain a higher allocative efficiency, the privilege given to traders should be deprived, and this deprivation can be interpreted as a kind of an intervention used to protect the market. Therefore, one way to summarize our study is as the following: Smart agents do not necessarily bring goodness to the market. One purpose of regulation is to annihilate the evil-side of smartness.

The rest of the paper is organized as follows. Section 2 shall briefly review the agent-based double auction markets used in this paper. Section 3 proposes experimental designs and measures of market performance. Section 4 presents and analyzes the simulation results, as Sec. 5 provides the concluding remarks.

# 2. Agent-Based Double Auction Markets: AIE-DA

Agent-based double auction (DA) markets were first seen by Dawid [6], who applied the single-population genetic algorithm (SGA) to evolve traders' bids and asks, but not bargaining strategies per se. Chen [1] proposed an agent-based DA market which is suitable for studying the evolution of bargaining strategies and which can be implemented with the software AIE-DA, developed by the AI-ECON Research Center. Chen [2] and Chen, Chie, and Tai [4] prepared a documentation accompanying this software, which is written in the language Delphi, and is largely motivated by object-oriented programming (OOP). The experiments conducted in this paper will be based on this software.

All buyers and sellers in AIE-DA are artificial adaptive agents as described in Ref. 8. Each artificial adaptive agent is built upon genetic programming. The architecture of genetic programming used in AIE-DA is what is known as multipopulation genetic programming (MGP). Briefly, we view or model an agent as a population of bargaining strategies. d Genetic programming is then applied to evolving each population of bargaining strategies. In this case, a society of bargaining agents consists of many populations of programs. This architecture is shown in Fig. 1.

The evolution of bargaining strategies for each agent proceeds as follows. First, consider a counter called *generation*. At generation t, each agent is assigned K bargaining strategies, collectively denoted by  $Gen_t$ , whose determination will be explained later. For each trading period h, the agent takes a random strategy I as follows:  $I \sim Uniform$  [1, K]. At the end of the trading period, the profits of the chosen strategy i  $(i \in [1, K])$  will be recorded as  $\pi_{i,h}$ . If strategy i is not chosen, then its profits will be counted as zero. After every H periods of trading, these

<sup>&</sup>lt;sup>d</sup>The number of bargaining strategies assigned to each bargaining agent is called the *population* size. AIE-DA Version 2 allows each agent to have at most 1000 bargaining strategies.

#### **Buyer 1** Seller 1 GP GP **Buyer 2** Seller 2 GP GP Double **Buyer 3** Seller 3 GP GP Auction Market Seller N<sub>2</sub> Buyer N<sub>1</sub> GP GP

# AIE-DA: DA Market Architecture

Fig. 1. The AIE-DA architecture: multi-population genetic programming.

k strategies will be revised and renewed by standard genetic programming. The fitness is the mean profits:

$$\pi_i = \frac{\sum_{h=1}^{H} \pi_{i,h}}{\sum_{h=1}^{H} 1_{i,h}},$$
(2.1)

where  $1_{i,h}$  is 1 if i is selected at period h, otherwise it is 0.<sup>e</sup> The revision and renew process will generate a new generation of K strategies, and at this point, the counter shall move to generation t+1. The new generation of K strategies will be denoted by  $Gen_{t+1}$ . This revision and renew procedure is summarized as follows:

$$Gen_{t+1} \Leftarrow \underbrace{[Reproduction] \bigvee [Crossover] \bigvee [Mutation]}_{Genetic\ Operators} \bullet (Gen_t). \quad (2.2)$$

As for the initial generation  $Gen_0$ , it will be generated by the ramped half-and-half method. This process will continue when t hits a prespecified number T. For the simulations in this paper, K is set to 50, H is 100, and T is 100.

#### 3. Experimental Designs

## 3.1. Quote limit

To test the effect of smartness on allocative efficiency, a quote limit (upper bound for bid and lower bound for ask) is imposed in one experiment, but not the other. The purpose of a quote limit is to prevent buyers and sellers from bidding and asking an unprofitable price, and so a buyer cannot bid a price higher than his redemption value, and a seller cannot not ask a price lower than his unit cost. One may wonder why traders would be so foolish to bid or ask a price outside the quotation limit.

 $<sup>^{</sup>m e}$ For the case when i is never selected, its mean profits would automatically be zero.

<sup>&</sup>lt;sup>f</sup>In other words, in total there are  $H \times T = 10,000$  periods of trading for a single run of simulation.

Would they definitely make a loss? The answer is negative. Quoting a price outside the limit makes a trader's offer more lucrative, and enhances the chance of making a deal. Once the deal is won, depending on the trading mechanism, the trader may actually fulfill the transaction with a price which is different from his original quote.

To show an example, consider the AURORA computerized trading system developed by the Chicago Board of Trade. AURORA rules stipulate that only the holder of current bid (CB) or current ask (CA) is allowed to trade if CB > CA.<sup>g</sup> By the AURORA rule, the actual transaction price (P) to fulfill a transaction will be somewhere between the current ask and the current bid. Let us assume the middle of them, h i.e.

$$P = \frac{\text{CA} + \text{CB}}{2} \,. \tag{3.1}$$

It is therefore clear that  $CA \leq P \leq CB$ , and so neither side would have to fulfill the transaction with their original quotes. This explains why the trader may take a risk of quoting a price outside the limit. This aggressive quotation should therefore be considered as a strategic behavior instead of a foolish one. Furthermore, this kind of aggressive bargaining strategy can actually emerges from the evolution of DA markets [3].

# 3.2. Designs of markets and traders

Once the meaning of the quote limit is clear, we run two series of experiments. The first series of experiments are conducted without the quote limit, whereas the other series are conducted with it. This is essentially to say that we do not allow traders in the second series of experiments to evolve and develop aggressive bargaining strategies, while they are free to do so in the first series. By this limit, the "smartness" of the traders in the second series is restricted, whereas traders in the first are not. We therefore call traders in the first series smart traders, and traders in the second series mediocre traders.

Twenty experiments were conducted for each series. In each experiment a tokenvalue table, which is randomly generated, is applied to both series. This table allows for four buyers and four sellers each with four units of tokens to trade. The 20 tokenvalue tables (markets) generated are depicted in Fig. 2. In each market, buyers and sellers' trading strategies evolved with genetic programming, whose control parameters are specified in Table 1.

gCurrent bid refers to the highest bid at the current trading step, and current ask refers to the lowest ask. When CA is greater then CB, there shall be no match between buyers and sellers at the current step.

<sup>&</sup>lt;sup>h</sup>While the Aurora rules allow a random determination between CA and CB, we shall only consider the case by taking the average. See also Ref. 6.

<sup>&</sup>lt;sup>i</sup>Notice that none of them in both series are born intelligent. They all have to learn and adapt to be intelligent. These terms only refer to the potentials which they may later develop.

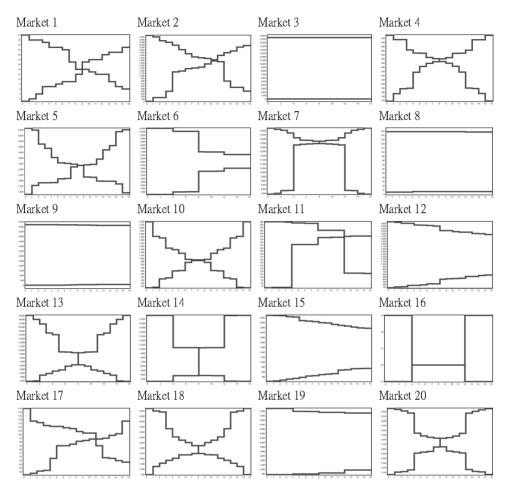


Fig. 2. 20 different markets used in the experiments.

Table 1. Tableau of control parameters for genetic programming.

| Number of generations $(T)$ | 100          |
|-----------------------------|--------------|
| Population size $(K)$       | 50           |
| Evaluation cycle $(H)$      | 100          |
| Fitness function            | Mean profits |
| Elitist strategy            | On           |
| Number of elites            | 1            |
| Selection scheme            | Tournament   |
| Tournament size             | 5            |
| Mutation rate               | 0.05         |
| Tree mutation               | 0.1          |
| Point mutation              | 0.9          |
| Max depth                   | 17           |
|                             |              |

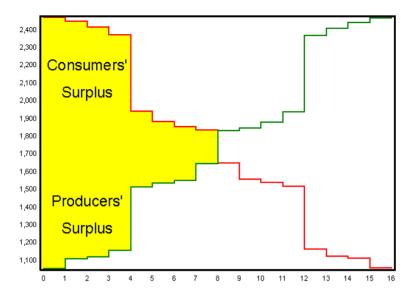


Fig. 3. Consumers' surplus and producers' surplus.

# 3.3. Measures of market performance

In experimental economics, two measures are frequently used for market performance. One is the so-called *alpha value*, and the other is the *efficiency ratio*. The alpha value takes the *competitive equilibrium price*  $(p^*)$  as a benchmark, and attempts to measure the deviation degree between *market prices* and the *competitive equilibrium price*. More precisely, it is defined as

$$\alpha = \frac{\sqrt{\sum_{i=1}^{n} (P_i - P^*)^2}}{P^*},$$
(3.2)

where n is the number of transaction made in each trading period, and  $P_i$  is the market price of the ith transaction. One technical issue involved in this definition is the location of  $P^*$ . The meaning of competitive equilibrium price is clear when the demand and supply curves intersect at a single point, e.g. Markets 10 and 16 (see Fig. 2). However, it is less clear when they intersect at an interval (Markets 7 and 20) or have no intersection at all (Markets 3 and 9). For the former case, we would define  $P^*$  as the midpoint of the intersecting interval, while for the latter case it is the midpoint between the lowest redemption value and the highest unit cost.

As to the second measure, we first take the sum of consumers' surplus (CS) and producers' surplus (PS) as the *potential social surplus* (see Fig. 3). We then divide the realized surplus by the potential surplus, and post-multiply the quotient by 100%. The result is then a measure for allocative efficiency. It is mathematically described as follows. Let  $\pi_j$  be trader j's profits at a specific trading period,

$$\pi_j = \sum_{q \in bought} (V_{j,q} - P_q), \quad j \in Buyer,$$
(3.3)

and

$$\pi_j = \sum_{q \in sold} (P_q - V_{j,q}), \quad j \in Seller,$$
(3.4)

where  $V_{j,q}$  is the redemption value of the qth unit for the jth buyer, or the unit cost of the qth unit for the jth seller. Term  $P_q$  is the transaction price of that unit. The realized surplus (RS) is then simply the sum of profits earned by all traders, namely,

$$RS = \sum_{j} \pi_{j}, \quad j \in Traders.$$
 (3.5)

The efficiency ratio  $\beta$  is

$$\beta = \frac{RS}{PS}, \tag{3.6}$$

where PS is the potential surplus. By this definition,  $0 \le \beta \le 1$ .

#### 4. Experimental Results

# 4.1. Price dynamics

Our analysis of the simulation results will be based on the two measures introduced above. However, before proceeding to those measures, it will be useful to first look at the price dynamics of those markets. For this purpose, we demonstrate time series plots of the price for the following six markets: Markets 3, 7, 9, 10, 16, and 20 (Figs. 4–6). For each figure, there are three plots. The leftmost plot is the market with its equilibrium price or equilibrium price interval. The middle plot is the time series of the price of experiment 1, whereas the rightmost plot is that of experiment 2. We shall distinguish these results by three separate cases. The first case refers to Markets 7, 10, and 20 (Fig. 4). In these markets, we either have a unique equilibrium price (Market 10) or a tight equilibrium interval (Markets 7 and 20). Market prices in this case quickly move toward the equilibrium price (or price interval), and then slightly fluctuate around there. This result is basically consistent with what we learned from experimental economics with human subjects [10].

The distinguishing feature of our DA markets actually starts from the second case, Markets 3 and 9 (Fig. 5). The common characteristic of these two markets is that demand and supply curves are completely flat with no intersection. This case, while very intriguing, has almost been neglected in experimental economics literature. How the price shall be determined under this circumstance is still an open question. Our simulation results from both markets seem to indicate that the price can shop around the whole interval, and is difficult to settle down to a narrow niche, but the price does not just randomly fluctuate. In fact, in Experiment 2, we see evidence of a slowly-upward moving trend. It begins with a price favorable to

<sup>&</sup>lt;sup>j</sup>To the best of our knowledge, Ref. 6 is the only study that has drawn attention to this case.

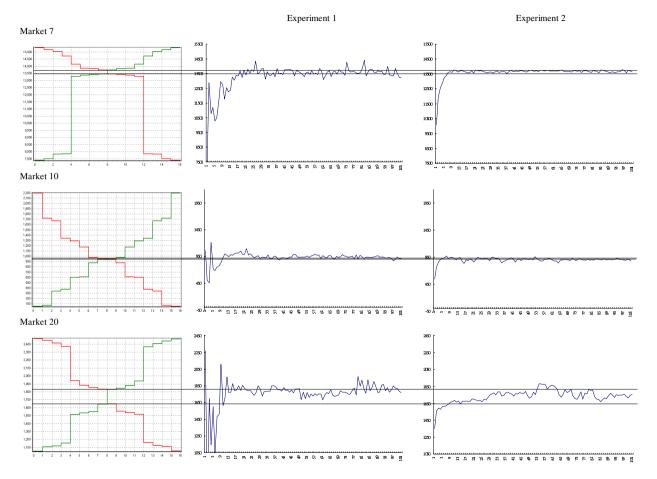


Fig. 4. CASE 1: Time series plots of price for Market 7, 10, and 20.

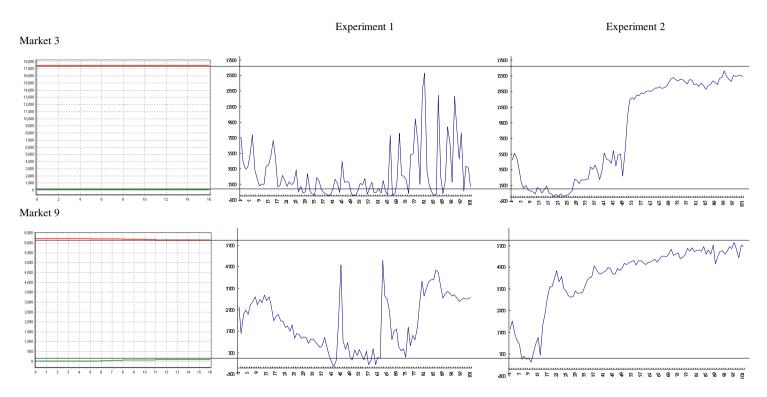


Fig. 5. CASE 2: Time series plots of price for Market 3 and 9.

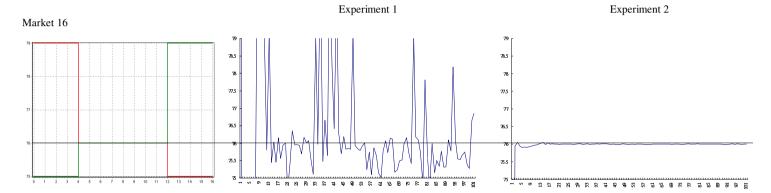


Fig. 6. CASE 3: Time series plots of price for Market 16.

buyers, but then eventually turns to the seller side. For Experiment 1, Market 9 in particular, the price path is even more complicated. It starts with a downward trend, but then ends up with an upward trend. In the middle, the trend breaks occur several times.<sup>k</sup>

We have so far not seen any qualitatively differences between Experiments 1 and 2. Their divergence appears to be clear in Case 3, Market 16 (Fig. 6). In this market, demand and supply do intersect at a unique price. Based on our experience with Case 1, one might predict that the price will converge to this competitive equilibrium price; nevertheless, these two curves are completely flat before the intersection, and, as we have encountered in Case 2, the price may find it difficult to converge. The perplexity of this situation is also seen in our simulation results: Experiment 2 confirms the first conjecture, whereas Experiment 1 supports the second. Here is a good time to ask: Would a market with smart traders (a market without a quote limit) destabilize rather than stabilize the market? We shall now deal with this question with more delicate statistics.

# 4.2. Alpha value

As defined in Eq. (3.2), the Alpha value measures the price deviation of a market period. Let  $\alpha_{t,h}$  be the  $\alpha$  value observed at the hth market period in the tth generation. In an ideal case, where  $\lim_{t\to\infty} P_{t,h} \to P^*$ ,

$$\lim_{t \to \infty} \alpha_{t,h} = 0, \quad h = 1, \dots, H.$$

$$(4.1)$$

The distribution (histogram) of  $\{\alpha_{t,h}\}_{h=1}^{H}$  might also be expected to degenerate to the point zero. To see how far we are away from the ideal case, we draw the histogram of  $\{\alpha_{t,h}\}_{h=1}^{H}$  for every ten generations, i.e.  $t=0, 10, \ldots, 100$ . To economize the presentation, the histogram is drawn by pooling the  $\{\alpha_{t,h}\}_{h=1}^{H}$  of all twenty markets. Figures 7 and 8 give the results of Experiments 1 and 2, respectively.

These two figures roughly indicate that these distributions exhibit a mass at zero and a skew to the right. Furthermore, Fig. 9 plots the time series of the medium of  $\{\alpha_{t,h}\}_{h=1}^{H}$  for both experiments. Here, we can see the effect of learning on price deviation; alpha values in both experiments tend to decrease in time. Drawing the two lines together also makes the effect of smartness transparent; the experiment with smarter agents (Experiment 1) tends to have a higher medium value at any point in time. This may help us answer the question posed above; markets with smarter agents tend to be more fluctuating. Hence, smarter agents play a destabilizing role for the market.

<sup>&</sup>lt;sup>k</sup>Our results can be compared to Ref. 6. Dawid [6] found that price *always converges*, even though it did not necessary converge to the middle point of the demand and supply curves. However, there are several noticeable differences between our simulations and Dawid's. Firstly, Dawid did not use the Aurora Rule as the trading mechanism. Secondly, his market size is much bigger than us. Finally, his model of adaptive agents is also different from ours.

 $<sup>^{1}</sup>$ Since H is set to 100 in this paper, and there are 20 markets, there are 2,000 observations in each histogram.

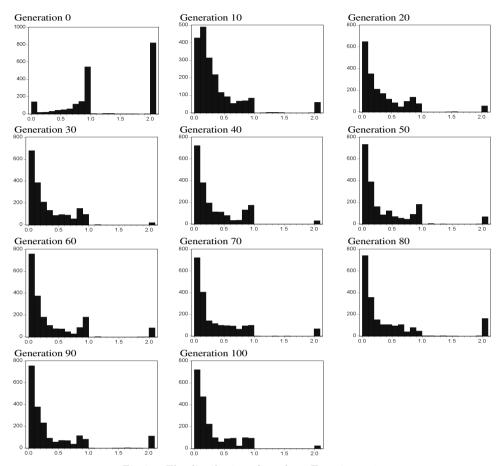


Fig. 7. The distribution of  $\alpha$  values: Experiment 1.

#### 4.3. Beta ratio

The  $\beta$  ratio, as defined in Eq. (3.6), measures the allocative efficiency of a DA market. From the proceeding analysis above, we are interested in two things: first, the role of learning in allocative efficiency, and second, the role of smarter agents. Our first concern is shown in Fig. 10. Figure 10 has three time series plots of the  $\beta$  ratio. The first time series plot shows that the  $\beta$  ratio increases with time. In this case, learning enhances the allocative efficiency, but, the second and the third do not. The third case even shows that allocative efficiency deteriorates after learning.

However, we have 20 markets for each experiment; therefore, we have numerous such time series plots. To economize our presentation, we use the sup function and the inf function to capture the essential characteristics of these three different time series plots.<sup>m</sup> Given an ordered series  $\{x_t\}_{t=1}^T$ , the sup function is defined as

$$\sup_{i} (\{x_t\}_{t=1}^T) = \max\{x_t\}_{t=1}^i, \quad i = 1, \dots, T,$$
(4.2)

<sup>&</sup>lt;sup>m</sup>This idea was first used in Ref. 5.

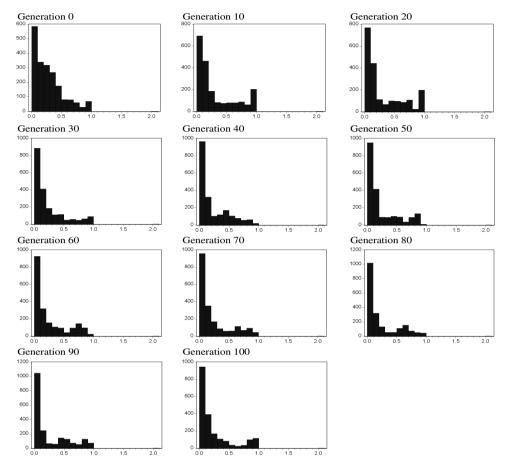


Fig. 8. The distribution of  $\alpha$  values: Experiment 2.

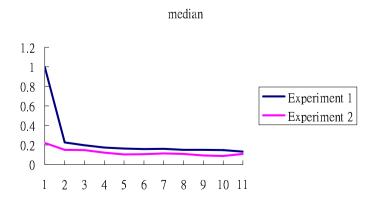


Fig. 9. Time series of the median of the distribution of  $\alpha$  values.

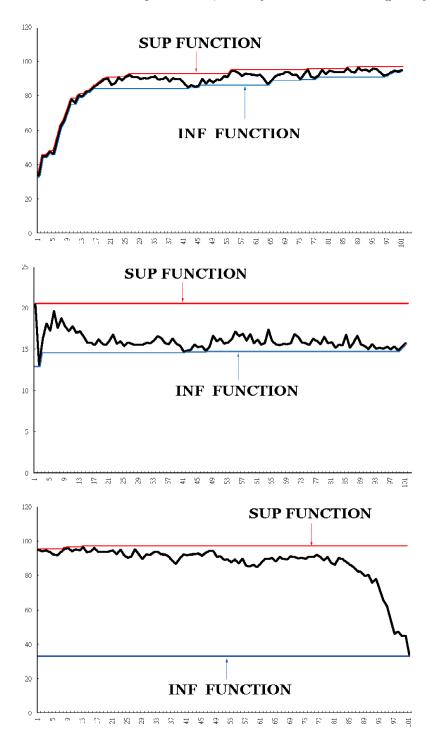


Fig. 10. Three possible time paths of beta ratio.

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and the inf function is<sup>n</sup>

$$\inf_{i}(\{x_t\}_{t=1}^T) = \min\{x_t\}_{t=i}^T, \quad i = 1, \dots, T.$$
 (4.3)

The corresponding sup and inf functions of the three series are also drawn in Fig. 10. As we can see from this diagram, if learning can enhance allocative efficiency, then these two functions should ideally display many upward jumps. On the contrary, if allocative efficiency deteriorates during the course of learning, then these two functions are totally flat with no jumps at all. Therefore, by counting the number of jumps and the total jump size, one can effectively summarize the role of learning in allocative efficiency over different markets.

Figure 11 plots the number of jumps (on the y-axis) and jump size for the inf and sup function of the  $\beta$  values over 20 markets. Since each market may start with different initial ratios of  $\beta$ , we distinguish them by two different symbols, namely, diamond and box. Diamond stands for a lower initial ratio of  $\beta$  (below 0.4), whereas box stands for a higher initial ratio of  $\beta$  (between 0.4 and 0.6). Since jumps are

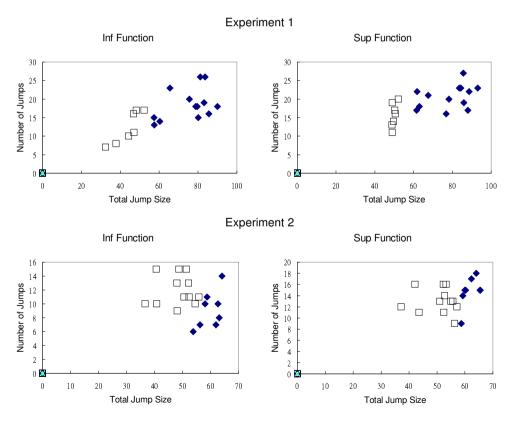


Fig. 11. The number of jumps and jump size of inf and sup functions.

<sup>&</sup>lt;sup>n</sup>Actually, this is the inf function in reverse order.

|         | Experiment 1 |       | Experiment 2 |       |
|---------|--------------|-------|--------------|-------|
| Market  | Initial      | End   | Initial      | End   |
| 1       | 29.09        | 94.67 | 56.93        | 97.56 |
| 2       | 32.09        | 85.60 | 44.01        | 96.14 |
| 3       | 45.56        | 83.44 | 41.19        | 97.00 |
| 4       | 8.59         | 90.91 | 45.32        | 93.37 |
| 5       | 11.11        | 92.28 | 33.75        | 96.78 |
| 6       | 46.66        | 89.28 | 46.74        | 94.73 |
| 7       | 6.41         | 86.45 | 59.69        | 96.29 |
| 8       | 48.32        | 96.63 | 46.72        | 97.25 |
| 9       | 46.09        | 98.31 | 38.50        | 97.23 |
| 10      | 9.92         | 79.33 | 42.12        | 94.41 |
| 11      | 33.42        | 90.91 | 55.26        | 96.00 |
| 12      | 45.50        | 92.62 | 39.66        | 97.66 |
| 13      | 4.47         | 80.07 | 34.59        | 98.59 |
| 14      | 8.57         | 84.27 | 33.78        | 96.34 |
| 15      | 48.42        | 95.18 | 36.86        | 98.78 |
| 16      | 6.44         | 89.19 | 46.69        | 98.00 |
| 17      | 34.14        | 89.10 | 43.62        | 98.11 |
| 18      | 3.80         | 89.37 | 38.82        | 92.55 |
| 19      | 47.26        | 79.40 | 38.02        | 94.26 |
| 20      | 5.46         | 89.03 | 45.80        | 94.51 |
| Average |              | 88.80 |              | 96.28 |

Table 2. The beta ratio of the initial and last generation.

prevalent for all markets, allocative efficiency becomes ameliorated as times goes on. Furthermore, for those markets with lower initial ratios of  $\beta$ , the amelioration degree is generally more significant than those with higher initial values.

To see the effect of smartness, the beta ratios of the initial generation and the last generation are shown in Table 2. The  $\beta$ s of Experiment 1 start with a range from 5% to 48%, and end up with a range from 79% to 98%. The  $\beta$ s of Experiment 2 are much higher: they start with a range from 33% to 59%, and end up with a range from 92% to 98%. If we compare these ratios in a pairwise manner, then Experiment 1 is uniformly beaten by Experiment 2 in its resultant allocative efficiency, except for Market 9. Therefore, smarter traders do not enhance allocative efficiency.

#### 5. Concluding Remarks

Our evidence is quite clear: smarter agents fail to enhance market performance.<sup>o</sup> They induce a relatively unstable price (higher alpha value) and lower allocative efficiency (lower  $\beta$  ratio). There is only one thing left to address in this concluding section. Why?

<sup>&</sup>lt;sup>o</sup>Conversely, making everybody dumb by imposing a quote limit does not have a negative impact on market efficiency.

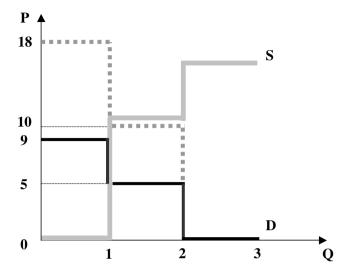


Fig. 12. Removal of the quote limit and the competition between intra- and extra-marginal agents.

While it is not easy to provide a mathematical proof, we try to make our argument as plausible as possible. We shall build our argument based on the competition between *intra-marginal* and *extra-marginal* agents, who are the trading partners of *intra-marginal* and *extra-marginal* tokens. Intra-marginal and extra-marginal tokens are the tokens which are just inside or outside the equilibrium frontier. Without loss of generality, let us consider a very simple demand and supply schedule, as shown in Fig. 12. Clearly, in this diagram only Seller 1 may have a successful trade. Let us also assume that Buyer 1 takes a random bid from the open interval (5,9). Given this bid and the imposition of the quote limit, Buyer 2 has no chance to beat Buyer 1 since his redemption value is only 5. In this case, Buyer 1 is an intra-marginal buyer, and Buyer 2 is the extra-marginal buyer.

Now consider the removal of the quote limit. Buyer 2 then *strategically* makes a bid up to ten by not making a loss deal.<sup>q</sup> Supposing now that Buyer 1 still takes a random bid from (5,9), the deal-winning chance for Buyer 2 will then jump from 0 to 0.2 if he takes a random bid from (5,10). If Buyer 2 wins the deal, then the allocative efficiency derived becomes lower as opposed to the case where Buyer 1 wins the case. This gives us an explanation for why allocative efficiency will be deteriorated when the quote limit is removed.

One may next ask why Buyer 1 would not enlarge his bidding area, say up to 18. Yes, he will, if Buyer 2 continues to undertake the ambitious bidding. Nonetheless,

PGiven a demand and supply schedule, the equilibrium frontier delimits the size of the feasible trading opportunities. Outside the frontier, any further trading is not affordable for either the consumers or the producers.

<sup>&</sup>lt;sup>q</sup>This upper limit is obtained by assuming that Seller 1 is a truth teller.

when Buyer 2 is scared away, Buyer 1 may shift down his bidding area, and hence open the gate for the intruder again. Since we are using multi-population genetic programming to model our agents, it would be useful to make a distinction between phenotype and qenotype in interpreting this dynamics. What we see from the outside is a competition between intra- and extra-marginal buyers, but what really happens inside (mentally) is the enduring competition between greedy strategies and cautious strategies. Greedy strategies nurse the growth of cautious strategies, which in turn do the same thing for the greedy strategies. The market just cannot settle its dynamics steadily. The in-and-out process is a generic property observed in many other agent-based markets.

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