Information and Learning in Limit Order Markets

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21st International Conference on Computing in Economics and Finance
22 June, 2015, Taipei, Taiwan

Background—Limit order book

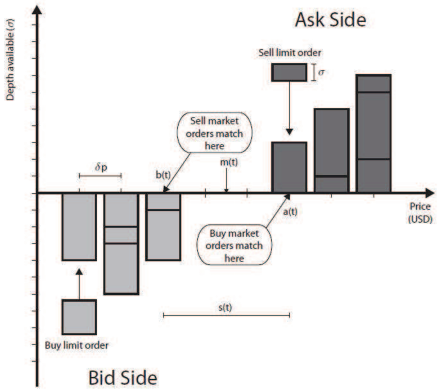


Figure 1: Limit order book (from Gould et al., 2013)

Limit order markets

- **Limit order books (LOBs)** used to match buyers and sellers provide more detailed information about order flow than has ever been available before
- **Limit order markets** are the dominant financial markets; more than half of the world's financial markets (Rosu, 2009)
  - *Pure LOB*: Euronext, the Australian Securities Exchange, the Helsinki, HK, Swiss, Tokyo, Toronto, Vanvouver, and Shenzhen Stock Exchange
  - *Hybrid LOB*: the NYSE, NASDAQ, LSE.
- **Key features and issues**:
  - Order choice and optimal order execution strategies
  - Market impact, empirical regularities and implications
  - Complex system—Many interacting agents trade globally across different markets

Limit order market models: Perfect rationality

- Agents maximize their utility in market by reacting to the changing state of the market driven by information (Parlour and Seppi, 2008)
- Cut-off strategies:
  - Single-period game via market maker: Chakravarty and Holden (1995) based on the difference between private valuation and the quotes (MM)
  - Sequential game: Foucault (1999), sequentially arrived trader chooses to submit a market or limit order based on a cut-off strategy, then left the market forever;
  - Information asymmetry and long/short-lived informationm, Goettler at al. (2006), Rosu (2009)

Issues and challenges

- Challenges—Complexity: huge state space; order flow dynamics, feedback between LOBs and market participant behavior; hidden liquidity (iceberg orders); ...Parlour, C. A. and Seppi, D. J. (2008)
- Focus on informed traders' order-choice and simplify uninformed traders' behaviours, such as private values in Goettler, Parlour and Rajan (2009) and time preference in Rosu(2014)
- Learning— "*It is the uninformed traders who provide the liquidity to the informed, and so understanding their behaviors can provide substantial insight and intuition into the trading process...(an) open question is* **what traders can learn** *from other pieces of market data, such as prices*"—O'Hara, 2001

Limit order market models: Agent-based

- Agent-based computational economics (ACE) and Finance (ACF), Hommes (2006), LeBaron (2006), Chiarella, Dicei and He (2009), Chen (2012), Chen, Chang and Du (2012)
- A rich toolbox for investigating LOBs without extreme modelling assumptions, Chiarella and Iori (2002), Chen and Tai (2003), Yeh (2008, 2015), Yamamoto (2015)...
- Heterogeneity among different market participants in real markets can be incorporated directly

Limit order market models: Stylized facts

- **Stylized facts and econophysics**: Challet and Stinchcombe, 2003, Preis et al., 2006, Lillo, 2007, Zhou (2012, 2015), Gould et al., 2013
  - power-law in order size, relative prices, order cancellation, ...
  - hump shaped mean depth profiles
  - intra-day volatility pattern
  - event clustering
  - heavy-tailed return distribution, no ACs in returns, long memory in volatility
  - ....
- **Sentiment trading and the stylized facts in LOM**: Chiarella, He, Shi and Wei (2014)

Agent-based models with GA learning

- Genetic algorithm (GA) with a classifier system, introduced by Holland (1975), to examine learning and evolution in economics and finance

- Recent agent-based models with GA in limit order markets:
    - LeBaron and Yamamoto (2008) employ GA to capture the **imitation behaviour** among heterogenous beliefs
    - Kluger and McBride (2011): traders use GA to learn when to trade to explain **the humped shape order book depth**;
    - Anufriev, Arifovich, Ledyard and Panchenko (2013): traders use GA to improve **market allocating** ;
    - Wei, Zhang, He and Zhang (2014): uninformed traders use GA to learn **from market prices**
    - ...

Agent-based models of LOM

- **Next milestone**: to see whether the agent-based model can be used to replicate the features in market microstructure
- **Differences**:
  - Asymmetric information—informed and uninformed traders; long/short-lived information;
  - The evolution of trading strategies largely driven by the fundamental value, the focus of the microstructure literature in limit order markets, instead of the market price

Specific Questions and Approach of this Talk

- **Questions**
  (i) How do trading strategies evolve and what information is more useful than the other?
  (ii) How do the informed and uninformed traders interact with each others and how do they behavior differently?
  (iii) How does the learning affect their order profit, order submission, market information efficiency, liquidity, volatility, and the bid-ask spread?
  (iv) What is the effect/impact of high frequency trading (HFT)?
- **Approach**—GA learning with a classifier system, Holland (1975)
  - A unified framework of market microstructure and agent-based computational finance literature
  - Behavior heterogeneity and order choice of traders are endogenously emerged from their learning, trading, and the state of the LOBs

Outline

The limit order market with continuous double auction

- Risky asset. The fundamental value v_t follows a random walk process. Innovations with κ ticks occur (with equal probability) according to a Poisson process with frequency ϕ (innovation frequency).
- There are N_I informed and N_U uninformed risk neutral traders who arrive at the market according to a Poisson process at rate λ, $N_I + N_U = N$.
- Asymmetric and short-lived information:
  - **Informed traders** enter the market exactly and know v_t.
  - **Uninformed traders** know $v_{t-\tau}$ with a time lag $\tau > 0$, information lag or lived time
- Time and price: There may be several traders entering the market and transaction may happens at time t' in the period t such as one minute.

Market Information

- When entering the market, traders submit orders to buy or sell at most one unit of the asset
- Re-entry: Traders can reenter the market and cancel the previous order and submit a new order upon reentry. The limit order has a maximum survival horizon D periods to reduce pick-off risk
- **Market information**—the most useful information for trading based on empirical studies:
  - the best bid $b_{t'}$ and ask $a_{t'}$ prices, the mid-price $p^m_{t'} = (a_{t'} + b_{t'})/2$, the spread $s_{t'} = a_{t'} - b_{t'}$,
  - the depth at the best bid $d^b_{t'}$ and the best ask $d^a_{t'}$, the depth of the buy side $d^{buy}_{t'}$ and the sell side $d^{sell}_{t'}$,
  - the last transacted order was a buy or sell initiated transaction,
  - the average market price over the last τ periods, $\bar{p}_{t,\tau} = [p_{t-1} + p_{t-2} + \cdots + p_{t-\tau}]/\tau$.

Trading strategies

- Based on **genetic algorithm (GA)** with a **classifier system** to buy/sell one share
- **Order aggressiveness**: market orders (MO); aggressive limit orders (ALO); limit order at quote (LOA); or limit order away the quote (unaggressive limit orders) (ULO)
- Informed traders use private information to decide buy/sell while uninformed traders use GA; both of them use GA to choose order aggressiveness.
- The trading strategy in the GA is evaluated based on its historical performance and evolved through processes of selection, crossover and mutation.

Classified rules

Group	Num	CR	Description
FV	CR1	$p^m_{t'} > v^j_t$	The mid-price is higher than the expected fundamental value
TR	CR2	$\bar{p}_{t,\tau} > v^j_t$	The average market price of last τ periods is higher than the expected fundament value
	CR3	$p^m_{t'} > \bar{p}_{t,\tau}$	The mid-price is higher than the average market price of last τ periods
	CR4	$\bar{p}_{t,\tau/2} > \bar{p}_{t,\tau}$	The average market price of last $\tau/2$ periods is higher than the average market price of last τ
QS	CR5	$s_{t'} > s_{t'-1}$	The current spread is bigger than the last spread
	CR6	$a_{t'} > a_{t'-1}$	The current ask is higher than the last ask
	CR7	$b_{t'} > b_{t'-1}$	The current bid is higher than the last bid
DI	CR8	$d^a_{t'} > d^b_{t'}$	The current depth at the best ask is larger than the current depth at the best bid
	CR9	$d^{sell}_{t'} > d^{buy}_{t'}$	The current depth of the sell side is larger than the current depth of the buy side
TS	CR10	$p^{\pm}_{t'-1}$	Last transaction sign (market buy or sell order)

Actions

Table 1: The actions (order types).

Action (buy)	Binary code	Description
MB	000	Market buy
ALB	001	Aggressive limit buy
LBB	010	Limit buy at the bid
ULB	011	Unaggressive limit buy
Action(sell)	**Binary code**	**Description**
MS	111	Market sell
ALS	110	Aggressive limit sell
LSA	101	Limit sell at the ask
ULS	100	Unaggressive limit sell

Trading strategies

A typical trading strategy (chromosome) has two parts

- the condition part—e.g., "1011100110" indicates one possible market condition based on **classified rules (CRs)**, say for example, the current mid-price $p^m_{t'}$ is higher than the expected fundamental value v^j_t . When some market information become irrelevant and we use "#" to replace 1 or 0.
- the action part—e.g. market order, or aggressive limit order, or a limit order at quote, or a limit order away the quote (unaggressive limit orders).

A trader then chooses the best trading strategy according to its strength mainly determined by its historical performance

The evolution process of the GA

- The performance of the strategy i is updated according to a weighted average of the recent performance r_z^i and historical performance π_z^i, that is

$$\pi_{t'}^i = \pi_z^i = \beta r_z^i + (1 - \beta)\pi_{z-1}^i, \tag{1}$$

  where $z - 1$ means the last trading time, and $\beta \in [0, 1]$ is the weight of the recent performance
- Stronger trading strategies are selected as the parents
- After the selection process, new strategies are generated through the crossover and mutation to replace low strength strategies

A Benchmark Model (BM)

- There are 100 informed and 900 uninformed traders;
- Initial fundamental value $v_0 = 20$ and the tick size is 0.01;
- Volatility of fundamental value: $\phi = 1$ and $\kappa = 4$;
- Information lag $\tau = 360$ and a trading day $D = 360$ min.
- Traders' arrival rate $\lambda = \frac{1}{60}$; (traders enter the market once per hour on average)
- Every trader has $2^8 = 256$ trading strategies and every evolution replaces 25 weak trading strategies;
- The evaluation weight of recent performance $\beta = 0.6$;
- Crossover rate: 0.1; Mutation rate: 0.3;
- The GA evolves every 360 periods (one trading day) on average.
- We run 30 simulations for BM cases; each simulation runs 180,000 periods (500 trading days), so the GA evolves 500 generations.

Two-side vs One-side Learning and Robustness

To examine different learning effect of informed and uninformed traders, we consider

- **Two-side learning**: The benchmark model (BM) in which both informed and uninformed traders use a GA to learn;
- **One-side learning**:
  - **IL**: only informed traders learn
  - **UL**: only uninformed (UL) traders learn

For the robustness we consider

- **private value**:
- **the evolution speed**:
- **the fraction of informed traders**:
- the fundamental volatility;
- the information lag,
- the discount rate of the historical performance in the GA

The evolution of trading strategies

To examine the impact of learning on the limit order market, including market efficiency and traders' order submission behavior, we need to understand

- what kind of information traders use when making decision
- whether the evolution of the GA and hence the trading strategy becomes stationary
- how market information is processed differently by informed and uninformed traders

We introduce an average usage frequency of market information to measure the usage of different information under the classified rules.

The information usage frequency

- The information usage frequency γ_i: the ratio of the sum of using times of informed traders to the sum of trading times of informed traders in one generation, which is one generation(360 periods) multiply by the arrival rate $\lambda = \frac{1}{60}$ and the number of informed traders $N_I = 100$ in the BM case.
- For example, if CR1 to CR4 have been used 210, 220, 180 and 240, respectively, by the informed traders in one generation,

$$\gamma_{FV} = \gamma_{CR1} = \frac{210}{360 * \frac{1}{60} * 100} = 0.350,$$

$$\gamma_{TR} = \frac{1}{3}(\gamma_{CR2}+\gamma_{CR3}+\gamma_{CR4}) = \frac{\frac{1}{3}(220+180+240)}{360 * \frac{1}{60} * 100} \approx 0.356.$$

Information usage for informed traders

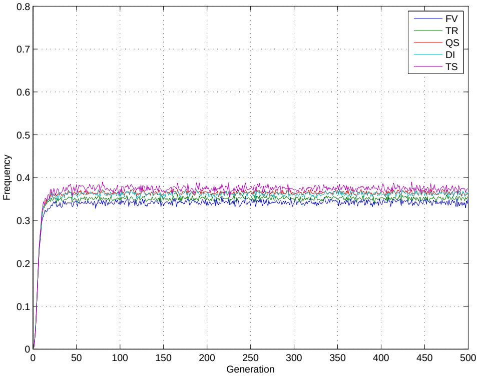


Figure 2: The average usage frequency per trade γ_i of each classified rule group i for informed traders in 500 generations(trading days).

Information usage for uninformed traders

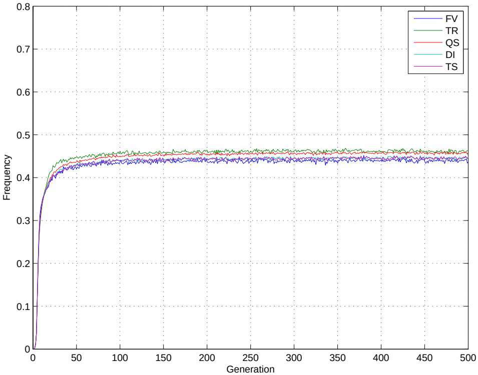


Figure 3: The average usage frequency per trade γ_i of each classified rule group i for uninformed traders in 500 generations (trading days).

Information Process

- The average usage of information increases very quickly in the early generations, becomes stationary;
- The speed is faster for informed than uninformed traders;
- The average usage for uninformed traders is higher (0.45) than for informed traders (0.35)
- Intuitively, compare to uninformed traders, informed traders do not need to use the GA learning to make the buy-sell decision and therefore their average usage of market information is lower.

The patterns of γ_i among CR groups for informed traders

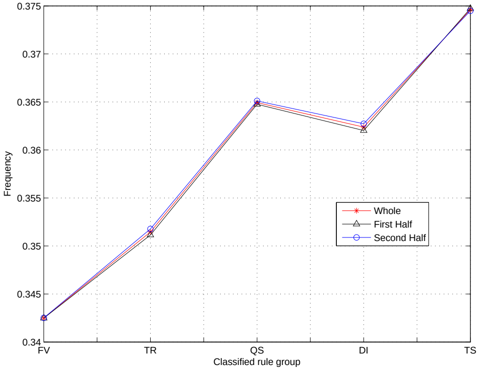


Figure 4: The patterns of the means of the whole series, the first half and the second half of γ_i among different classified rule groups for informed traders.

The patterns of γ_i among CR groups for uninformed traders

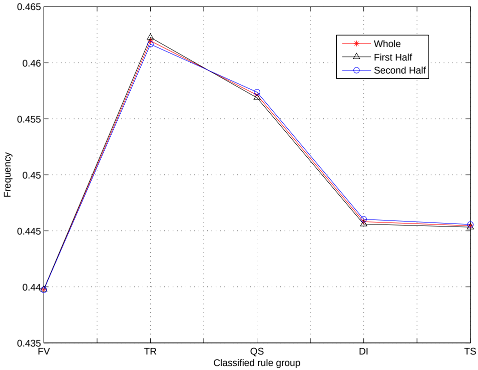


Figure 5: The patterns of the means of the whole series, the first half and the second half of γ_i among different classified rule groups for uninformed traders.

Information Process

- The average usage of the TS is the highest for the informed traders, while it is TR for the uninformed traders
- Note that moving averages is useful not only for the uninformed traders, but also for the informed traders
- With the classifier system, the GA learning becomes stationary in long-run; which helps to provide insight into how traders use the order book information to endogenously determine their order choice, a key issue (Rosu 2012)
- Arthur et al. (1997) find that the GA with a classifier system leads to rationally expected prices in a double auction market
- Goettler et al. (2009) employ a Markov perfect Bayesian equilibrium, which is stationary and symmetric and traders' strategies do not depend on market conditions.

The learning effect

Table 2: Order profit, information efficiency $MRE = \frac{1}{T}\sum_{t=1}^{T}\frac{|p_t - v_t|}{v_t}$ and bid-ask spread s.

Case	r_I	r_U	MRE	s
BM	**35.19**	**-3.51**	**2.21%**	**6.24**
	[4.61]	[0.45]	[1.37%]	[2.91]
UL	31.74*	-3.23*	1.49%*	7.28*
	[4.69]	[0.46]	[0.69%]	[3.58]
IL	59.84*	-8.37*	4.06%*	16.00*
	[4.49]	[0.59]	[1.90%]	[7.71]

- learning is more valuable for uninformed traders;
- learning of uninformed traders improves market information efficiency, but not always when informed traders learn;
- learning reduces spread, in particular when uninformed learn

The learning effect on order submission

$$MO_j = MB_j + MS_j, \qquad ALO_j = ALB_j + ALS_j,$$
$$LOA_j = LBA_j + LSA_j, \qquad ULO_j = ULB_j + ULS_j$$

Report the average order number per trading day, 600 trading periods.

The learning effect on order submission

Case	MO_I	ALO_I	LOA_I	ULO_I	ELO_I
BM	155	100	172	173	149
	[1]	[2]	[2]	[2]	[2]
UL	168*	97*	168*	168*	143*
	[1]	[3]	[1]	[2]	[1]
IL	147*	133*	161*	159*	227*
	[1]	[3]	[2]	[1]	[2]
Case	MO_U	ALO_U	LOA_U	ULO_U	ELO_U
BM	169	96	167	168	170
	[1]	[3]	[1]	[1]	[1]
UL	168*	97*	167	168	171
	[1]	[2]	[1]	[1]	[1]
IL	153*	133*	157*	157*	144*
	[1]	[3]	[2]	[1]	[1]

The BM Case

- Both informed and uninformed traders submit more limit orders than market orders, hence increases liquidity supply and reduces liquidity consumption.
- Relatively, informed traders submit more limit orders and less market orders than uninformed traders.

Informed Learning: comparing UL to BM

- Informed order profit increases by about 10%
- About 90% of informed profit is from the information and 10% is from the learning
- Informed traders' market orders decrease by about 8% but the limit orders increase by about 3%, no significant change for the uninformed traders
- The reduction in the bid-ask spread is due to an increase in liquidity supply and a decrease in liquidity consumption
- Informed traders may manipulate the market by setting the prices through submitting more limit orders and less market orders
- Reduces market information efficiency but increases market liquidity

Uninformed Learning: comparing IL to BM

- The learning has more significant impact on the order profit, reduces the profit of the informed traders (by about 41%) and the loss of the uninformed traders (by about 58%).
- Submit more market orders and limit orders at or away from the quote but less aggressive limit orders, hence increase liquidity consumption and reduce liquidity supply.

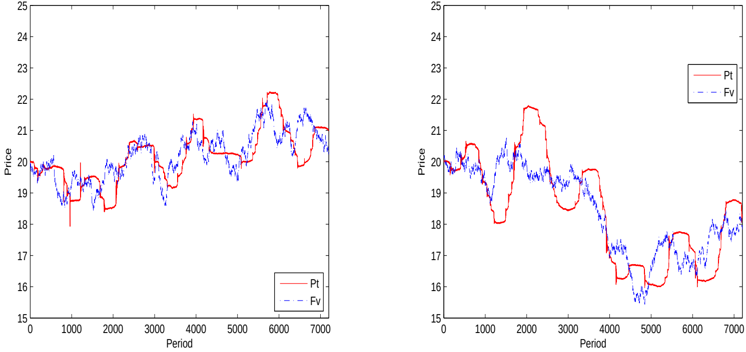
Robustness tests

- Private vlaue
- The evolution speed of the GA
- The fraction of informed traders:
  - LI: low fraction of informed traders, 50 informed traders and 950 uninformed traders.
  - HI: high fraction of informed traders, 200 informed traders and 800 uninformed traders.
- LV: low volatility, $\kappa = 2$ ticks.
- SL: short information lag, $\tau = 180$.
- LB: low weight on the recent performance, $\beta = 0.05$.

Private Information

- To motivate trading from uninformed traders, Gorttler et al. (2009) introduce private value for uninformative traders whose order submissions mainly depend on the private values.
- For example, an uninformed trader with a large positive private value prefers to submit a market buy order.
- The private value of the uninformed traders may have significant impact on the limit order market and order submission in short run, but the impact becomes less significant in long run.

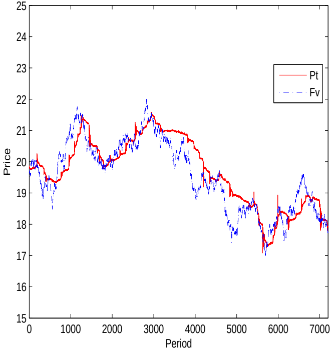
Private Information



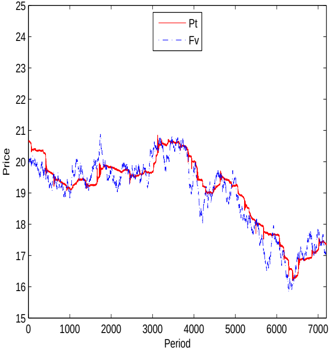
(a) BME (b) PV30E

Figure 6: The market price and fundamental value for the BM case (a) and PV30 case (b) over the first 7,200 periods

Private Information



(a) BME



(b) PV30E

Figure 7: The market price and fundamental value for the BM case (c) and the PV30 case (d) over the last 7,200 periods.

The evolution speed of the GA

A trade-off effect between the speed of the evolution and the effectiveness of the GA learning.

- When the evolution speed of the GA reduces (but not too much), trading strategies have more opportunity to be selected and tested and therefore better performed strategies are more likely to be selected for the evolution in the next generation.
- However, if the evolution speed slows down substantially, the GA may not be able to converge

The fraction of informed traders and the implication for PIN

- As the fraction of the informed traders increases from 5% to 90%, the calculated PIN is matching closely to the fraction of the informed traders.
- when the fraction of the informed traders increases, both the spread and volatility increase, while the trading volume decreases, and the relation are monotonic but nonlinear.
- The positive relation between PIN and spread/volatility is consistent with short-lived information.
- This provides a useful implication for estimating the PIN in limit order markets.

The fraction of informed traders and the implication for PIN

Table 3: The volatility of market-price return in basis points, the bid-ask spread in ticks and the probability of informed trading.

Fraction	PIN	Volatility	Spread	Volatility/Spread	volume
5%	4.70%*	28.70*	4.38*	6.55*	4.7
10%	9.06%	39.28	6.24	6.29	4.7
20%	17.53%*	49.19*	8.53*	5.77*	4.6*
50%	43.56%*	56.88*	10.83*	5.25*	4.4*
90%	86.98%*	69.31*	13.41*	5.17*	4.1*

Robustness

- A lower fundamental volatility or a higher share of informed traders reduces the order profit of informed traders, improves market efficiency, and makes traders submit more aggressive limit orders.

Summary

- Evolutionary dynamics: Measured by the information usage, trading strategies become stationary in the long run;
- Process of information: Uninformed traders use more information than informed traders; also, informed traders pay more attention to last transaction sign while uninformed traders pay more attention on the technical rules.
- Impact of Learning:
  - learning is more valuable for uninformed traders;
  - learning of uninformed traders improves market information efficiency, but not always when informed traders learn;
  - dominated by informed traders, learning reduces liquidity consumption and increases liquidity supply and hence reduces the bid-ask spread,
- Implication: the probability of informed trading (PIN) is positively related to the volatility and the bid-ask spread.

HFT: Debates and puzzles

- HTF: algorithm trading, large in size and scope, quick reaction to market information, high turnover and cancellation, tight position...
- Price discovery—**positive**: Brogaard (2012) and Brogaard et al. (2014); **negative**: Zhang (2010) and Hasbrouck (2013)
- Volatility—**increased**: Zhang (2010), Kirilenko et al. (2011) and Hasbrouck (2013); **reduced**: Brogaard (2012), Brogaard et al.(2014).
- Liquidity—**increased**: market making HFT, such as Hagströmer and Nordén (2013); **reduced**: Kirilenko and Lo (2013), ASIC (2013)
- Did HFT trigger the flash crash, May 6, 2010?

HFT: Types and Features

- Passive HFT: market-making, likely to use limit order to provide liquidity and manage inventories, Martinez and Rosu (2011);
- Active HFT: using sophisticated algorithm (such as statistical arbitrage), to exploit various sources of information (from fundamental or order flows).
- Empirically some **active HFT traders are informed**: Brogaard et al.(2014), Kirilenko et al.(2011), Hendershott et al.(2011);

Current Literature and Research Questions

- **Current Literature**: focusing on market making HFT, instead of LOM

- **Research Questions**:
  - (i) Information process: How do HF traders process information differently from LF traders?
  - (ii) HFT profit: speed, information, or learning?
  - (iii) Order flows and liquidity: How does HFT affect order submissions behavior of LF traders and order flows with respect to liquidity supply and consumption?
  - (iv) What is the impact on limit order market quality including information efficiency, volatility and liquidity?

The limit order market with HFT

- Traders: risk neutral $N_H + N_I + N_U = N$
  - N_H HF informed traders
  - N_I LF informed traders,
  - N_U LF uninformed traders, where $N_H < N_I < N_U$

- Time scale for LF and FT trading
  - HF trading in **short time period** t, traders enter the market in t following a Poisson process with λ_H
  - LF trading in **long time period** T, with $T = mt$, where m is a positive integer, following a Poisson process with λ_L

- The fundamental value v_t follows a Poisson process with frequency $0 < \phi < 1$ and κ ticks .

The limit order market with HFT

- Asymmetric information
  - HF informed traders enter the market exactly knowing v_t, faster than LF informed traders
  - LF informed traders enter the market knowing v_T
  - LF uninformed traders know $v_{T-\tau}$ with a time lag $\tau > 0$.

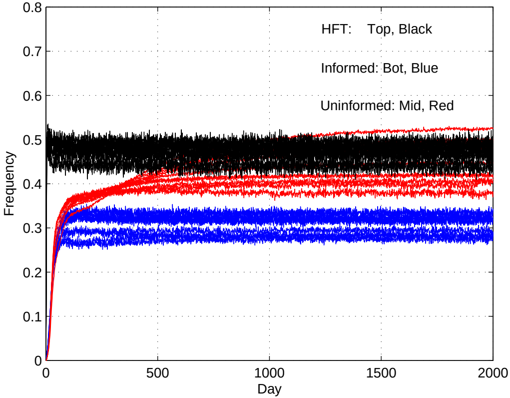
- Re-entry. Traders can reenter the market, cancel the previous order, and submit a new order upon reentry

The effect of HF trading, information and learning—Four cases

Case	Description	HFT	IN	UN	κ
BM	LFT with learning	0	100	900	2
HF	Informed HFT with learning	10	90	900	2
NL	Informed HFT without learning	10	90	900	2
HV	HF case high volatility	10	90	900	4

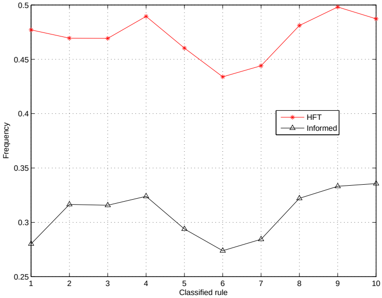
- the time period is 10 seconds for HF, and 1 min. for LF traders
- fundamental values change by $\kappa = 2$ ticks once every one min. on average for HF case; $\kappa = 4$ for HV case
- information lag $\tau = 60$ corresponds to one hour, which is also the maximum order survival time
- HF traders entry the market once per 10 sec., LF once per hour, on average
- One generation is 20 min for HFT and one day (6 hours) for LFT.

Evolution of information usage frequency of trading strategies: HF case



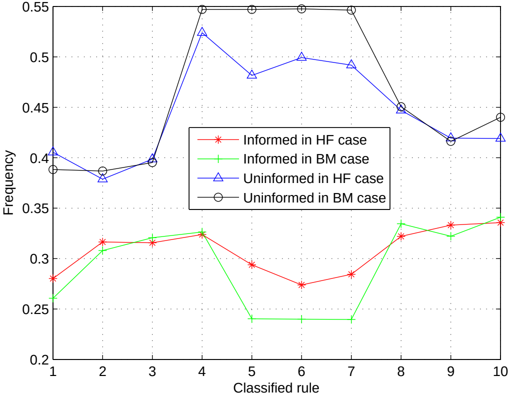
The patterns of γ_j^H among CRs for HFT traders

- HF traders pay much higher attention to all market information.
- Relatively, HFT traders use more information on the depth imbalance (CR9) and the fundamental value (CR1);



The impact of HFT on LF traders

- Informed traders pay more attention to the quotes and spread
- Uninformed traders pay less attention to the quotes and spread



Order profit

Case	r_H	r_I	r_U	R_H	R_I	R_U	$\bar{R}_H$	$\bar{R}_I$
BM	N/A	**13.76**	**-1.40**	N/A	8,545	-8,545	N/A	**85.45**
HF	**1.04**	**0.91**	**-0.51**	2,559	587	-3,146	**255.9**	**6.52**
NL	**0.72**	**0.26**	-0.23	649	77	-726	**64.9**	**0.86**
HV	**2.33**	2.22	-1.23	3,162	601	-3,763	**316.2**	6.68

- HFT reduces the profit of the LF informed traders and the loss of the LF uninformed traders
- For HF informed traders, the average order profit per trade is low but the total profit and average profit are much higher, the speed matters
- for the HF informed traders, the learning is the main driver of their profit; learning reduces the pick-off risk for the HF informed traders
- the HFT becomes more profitable for the informed traders as the fundamental volatility increases

Order profit

- Intuitively, it is the combination of information, learning and speed that makes the HFT more profitable
- More importantly, measured by the total profit, both **learning and information** play much more important role than the speed for the HF informed traders, though the speed also matters
- LF informed traders loss heavily from the HFT while the LF uninformed traders benefit from the HFT (by reducing their loss)

Order submission, market liquidity and order flow

Types of orders and order flows

$$MO_j = MB_j + MS_j, \qquad ALO_j = ALB_j + ALS_j,$$
$$LOA_j = LBA_j + LSA_j, \qquad ULO_j = ULB_j + ULS_j.$$
$$ELO = \text{Executed Limit order}$$

- MO: liquidity consumption, price risk, increase spread
- LO: liquidity supply, pick-off risk, no-execution risk
- ALO: narrows spread
- LOA: supply the immediate liquidity

Order submission, market liquidity and order flow

Total submission orders in four types of orders and four cases:

Case	MO	ALO	LOA	ULO	Total
BM	169,385	92,039	168,219	170,669	**600,312**
HF	238,593	192,766	264,675	258,690	**954,724**
NL	247,978	206,380	249,211	251,177	**954,746**
HV	236,478	221,157	252,056	245,014	**954,705**

Total submission orders in eight types for the four cases

Case	MB	ALB	LBA	ULB	MS	ALS	
BM	84,258	46,330	82,430	85,087	85,126	45,709	8
HF	118,672	97,693	128,397	127,441	119,921	95,073	13
NL	123,075	104,430	123,034	124,685	124,903	101,950	12
HV	118,439	109,926	124,467	122,382	118,039	111,230	12

Liquid, order flows and order submission

- HFT increases market liquidity (by about 60% in the total order submissions)
- HFT increases more ALO (by about 100%) than MO (by about 40%), **liquidity supply increases more than consumption**
- **Order imbalance**: more market sell and aggressive limit buy orders, to take the information advantage

Order submission, market liquidity and order flow

Table 4: Order submissions in %

Case	MO	ALO	LOA	ULO
BM	28.22	15.33	28.02	28.43
HF	24.99	20.19	**27.72**	**27.10**
NL	25.96	21.64	**26.09**	**26.30**
HV	24.77	23.16	26.40	25.57

- HFT traders submit less MO but more ALO, reduces liquidity consumption and increases liquidity supply
- Learning with HFT reduces MO, ALO, but increases LOA and ULO,
- Fundamental volatility reduces MO but increases ALO, reducing liquidity consumption and increasing liquidity supply

Liquid, order flows and order submission: Summary

- HFT increases overall market liquidity, but reduces liquidity consumption and increases liquidity supply, narrowing the spread
  - The reduction in liquidity consumption is mainly driven by learning
  - The increase in liquidity supply is mainly driven by private information
- HFT traders have low ELO rate (about 17%), cancelling limit orders more often to avoid pick-off risk
- HFT makes LF informed traders more aggressive (with more MO ALO and less LOA and ULO), less profitable
- HFT makes LF uninformed traders submit less MO and more aggressive LOs, reducing their loss

Conditional order submission and event clustering

Case	MB	ALB	LBA	ULB	MS	ALS	LSA	U
CP_BM	13.34	6.51	14.63	15.17	13.49	6.50	15.11	15
CP_HF	**14.63**	**13.31**	**20.72**	**20.07**	14.37	13.99	21.84	20
CP_NL	14.40	11.58	17.23	17.42	14.37	11.70	12.50	11
CP_HV	15.54	16.66	19.91	19.19	15.82	17.43	20.15	19
UCP_BM	12.42	10.23	13.44	13.36	12.57	9.94	14.29	13
UCP_HF	**14.00**	**7.97**	**13.77**	**14.09**	14.11	7.77	14.14	14
UCP_NL	12.88	10.96	12.87	13.05	13.08	10.68	13.22	13
UCP_HV	12.41	11.51	13.04	12.82	12.36	11.65	13.36	12

Note: CP: conditional prob.; UCP: unconditional prob. (%).

Conditional order submission and event clustering

- **Event clustering**—the incoming order type is more likely to follow the same order type, Biais, Hillion and Spatt (1995), Goettler, Parlour and Rajan (2005)
- HFT increases positive serial correlation of all the order types
- With HFT, the conditional prob of all limit order types are significantly higher than the corresponding unconditional prob. (by about 6%)
- HFT generates significant *event clustering* in order submission, becoming more significant with high fundamental volatility

Information efficiency and market volatility

$$MAE = \frac{1}{Y}\sum_{T=1}^{Y}|p_T - v_T|, \qquad MRE = \frac{1}{Y}\sum_{T=1}^{Y}\frac{|p_T - v_T|}{v_T}.$$

Case	MAE	MRE	Kurtosis	STD_{p_T}	$STD_{p_T^m}$
BM	14.26	0.84%	72.58	23.78	7.47
HF	**2.93**	**0.13%**	**45.57**	**27.31**	**10.62**
NL	4.94	0.19%	27.54	43.88	10.42
HV	7.39	0.30%	27.19	53.19	15.41

- HFT and learning improves information dissemination efficiency and hence price discovery
- HFT increases volatility (by about 15% - 40%), Martinez and Rosu (2011), different from market maker scenario

Liquidity

Case	Volume	D5	Spread
BM	4.7	13.9	3.7
HF	6.6	8.6	5.2
NL	6.9	8.4	10.1
HV	6.6	7.68	11.91

Note: D5 is the depth of 5 best quotes in the sell side, spread is in ticks

- HFT increases trading volume, Martinez and Rosu (2011)
- HFT increases the bid-ask spread and reduces order book depth, Brogaard(2010) Kim and Murphy (2013) and Gai et al. (2012)
- With HFT, learning narrows the spread, while information uncertainty increases the spread

Conclusion-I

- We provide a unified framework of market microstructure and agent-based computational literatures to examine the impact of learning, information and HFT on limit order markets
- We provide some insights into the recent debates and puzzles in the impact of HFT on market quality
  - HFT improves information efficiency, price discovery, increases market liquidity and trading volume;
  - it may also increase market volatility and the bid-ask spread, reduce order book depth

Conclusion-II

- With HFT, more information is used, including the history information related to moving averages
- It is the combination of speed, more importantly, information and learning that makes the HFT more profitable
- HFT reduces liquidity consumption (due to learning) and increases liquidity supply (due to information advantage)

Remarks-I

- Characteristics of limit order markets—Heterogeneity, dynamical interaction and complexity
    - Optimal order choice between limit orders and market orders—the trade-off between execution probability and price improvement
    - Risk neutral, quantity choice (of one share), no much on the endogenous decision about order aggressiveness and quantity
    - Extremely large state and action spaces, dynamical changing
    - Monitor, modify, order cancellation, splitting...
- Limitation of perfectly rational equilibrium framework
- Agent-based computational economics (ACE) is an ideal framework for studying LOM

Remarks-II

- Learning
  - Classifier system and machine learning, Kearns and Nevmyvaka (2013)
  - Individual learning, social learning, and social networks
- Chronological-time and volume-clock, Easley, de Prado and O'Hara (2013)
- Empirically to quantify the importance of the various relations, such as the relation between individuals' order submission strategies and the observed aggregate order flow process
- Develop optimized trading algorithms for practical use
- Market design, limit order markets or dealer markets, social welfare, and implication

Remarks-III

Chen, S-H., 2010 CEF planary lecture, ‘Varieties of agents in agent-based computational economics: A historical and an interdisciplinary perspective’, JEDC 36 (2012), 1-25.

“Various models of agents with personality, emotion and cultural backgrounds have been attempted. This research trend has been further connected to computational models of the brain, neuroscience and neuroeconomics. To what extent ACE can benefit from this enlarging interdisciplinary integration of agent research, an open mind is required.”