# Behavioral Heterogeneity in U.S. Inflation Dynamics

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**Abstract**

In this paper we develop and estimate a behavioral model of inflation dynamics with monopolistic competition, staggered price setting and heterogeneous firms. In our stylized framework there are two groups of price setters, fundamentalists and random walk believers. Fundamentalists are forward-looking in the sense that they believe in a present-value relationship between inflation and real marginal costs, while random walk believers are backward-looking, using the simplest rule of thumb, naive expectations, to forecast future inflation. Agents are allowed to switch between these different forecasting strategies conditional on their recent relative forecasting performance. The estimation results support behavioral heterogeneity and the evolutionary switching mechanism. We show that there is substantial time variation in the weights of forward-looking and backward-looking behavior. Although on average the majority of firms use the simple backward-looking rule, the market has phases in which it is dominated by either the fundamentalists or the naive agents.

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# 1 Introduction

*"Any time seems to be the right time for reflections on the Phillips curve"*

Robert Solow

In recent years, pricing behavior has been described in the context of models that incorporate both nominal rigidities and optimizing agents with rational expectations.[1] One of the most popular versions of New Keynesian pricing models is derived from Calvo (1983) and it implies a forward-looking inflation equation (a "New Keynesian Phillips curve", NKPC henceforth) of the form

$$\pi_t = \delta E_t \pi_{t+1} + \gamma mc_t \ , \tag{1.1}$$

which relates inflation, $\pi_t$, to next period's expected inflation and to real marginal costs, $mc_t$.[2] An important implication of this model is that there is no intrinsic inertia in inflation, in the sense that there is no structural dependence of inflation on its own lagged values. As a result, this specification has often been criticized on the grounds that it can not account for the important empirical role played by lagged dependent variables in inflation regressions (see e.g., Rudd and Whelan (2005a,b) for a recent discussion). This critique resulted in various proposals for so-called "hybrid" variants of the NKPC, which take the form

$$\pi_t = \theta E_t \pi_{t+1} + (1-\theta)\pi_{t-1} + \gamma mc_t \ . \tag{1.2}$$

Hybrid models have been theoretically motivated in several ways. Fuhrer and Moore (1995) assume an alternative contracting specification in which workers bargain over relative real wages; Christiano, Eichenbaum, and Evans (2005) use a variant of the Calvo model in which firms that are unable to reoptimize their price

[1] See Clarida, Galí, and Gertler (1999) for a survey, and Woodford (2003) for a detailed treatment.

[2] Roberts (1995) shows that Eq. (1.1) can be derived from a number of different models of price rigidity.

instead index it to past inflation; Galí and Gertler (1999) assume the existence of a group of backward looking price setters.[3]

After the introduction of the hybrid NKPC, a significant strand of research focused on two important empirical issues, namely the relative importance of the forward looking component in price setting and the appropriate measure of inflationary pressure, generating mixed results. Here we briefly summarize some of the evidence obtained in previous studies.

Galí and Gertler (1999) estimate the hybrid NKPC by GMM using the labor share of income as driving variable for the inflation process, and they conclude that rational forward-looking behavior plays an important role in determining U.S. inflation. Sbordone (2005) estimates the closed form solution of the hybrid model where inflation is a function of lagged inflation and the discounted sum of expected future real marginal costs, proxied by the labor share. Using a two-step distance estimator that exploits an auxiliary autoregressive representation of the data as in Campbell and Shiller (1987) to estimate the present value form of the inflation dynamics model, Sbordone (2005) finds that the forward-looking component is quantitatively more relevant than the backward-looking component, confirming thus the results of Galí and Gertler (1999). Kurmann (2007) reports the maximum likelihood estimates of the hybrid model confirming the results of Galí and Gertler (1999) when the labor share of income is used as proxy for real marginal costs.

Lindé (2005) estimates a New-Keynesian sticky price model using a full information maximum likelihood approach with output gap as proxy for real marginal costs and suggests a hybrid version of the NKPC where forward-looking behavior is significant but about equally or less important than backward-looking behavior. Fuhrer (1997) considers the model developed in Fuhrer and Moore (1995), which extends the staggered contracting framework of Taylor (1980) in a way that imparts persistence to the rate of inflation. Using a maximum likelihood estimation procedure and the output gap as forcing variable he concludes that forward look-

[3]In the specification of Galí and Gertler (1999) the weights of lagged and expected future inflation are not constrained to sum to unity, unless the time discount factor $\delta = 1$.

ing behavior plays essentially no role in observed inflation dynamics. Rudd and Whelan (2006) estimate the closed form solution of the hybrid model (1.2) proxing real marginal costs with output gap measures as well as the labor share. Using both VAR-based methods and GMM estimation, they find no significant evidence of rational forward-looking behavior in U.S. data.

One possible explanation for the mixed evidence on the empirical relevance of rational forward-looking behavior stemming from previous tests of sticky price models may be rooted, as put forward by Rudd and Whelan (2006), in the reliance of these models on a strict form of rational expectations (RE henceforth). Rudd and Whelan (2006) conclude that:

*"...further research in this area is probably best aimed toward developing models that deviate from the standard rational expectations framework in favor of alternative descriptions of how agents process information and develop forecasts".*

Moreover, Carriero (2008) performs a simple test of the hybrid NKPC using VAR projections as a proxy for agents' expectations. Under the assumption of model-consistent RE, this procedure commonly used in the literature (see, e.g., Galí and Gertler (1999), Sbordone (2005), Rudd and Whelan (2006) and Kurmann (2007)) imposes cross-equations restrictions. Carriero (2008) tests for these restrictions finding that there does not exist a combination of structural parameters consistent with U.S. data, and concludes that this might be due to the assumption of model-consistent RE. In line with Rudd and Whelan (2006), Carriero concludes that further research should be aimed at providing alternative models for agents' expectations.

In this paper we take these criticisms seriously and propose a model of inflation dynamics characterized by an alternative behavioral heterogeneous expectation formation paradigm.

Standard New Keynesian models of price setting are based on the assumptions that: (i) prices are sticky; (ii) agents make optimal decisions given their beliefs about future inflation; (iii) individual expectations are formulated in a rational

(i.e., model-consistent) way. Empirical studies suggest that a significant degree of price stickiness is present in the U.S. economy, providing thus a rationale for firms trying to make predictions about future inflation when setting current prices. In our model we keep the assumption of sticky prices and optimizing behavior (given individual beliefs), so that expected future inflation has an important influence on current inflation, but we depart from standard models by replacing the assumption of homogeneous firms holding RE with the assumption of heterogeneous firms with subjective forecasting strategies and evolutionary selection of these strategies on the basis of their relative forecasting performance.

Our alternative modeling assumptions stem from two empirical stylized facts. First, a vast empirical literature rejects rational expectations (see for example Lovell (1986), Baghestani (1992), Ball and Croushore (2003)), and heterogeneity in individual expectations has been abundantly documented. For example, Frankel and Froot (1987, 1990), Allen and Taylor (1990) and Ito (1990) find that financial experts use different forecasting strategies to predict exchange rates. More recently, Carroll (2003), Mankiw, Reis, and Wolfers (2003), Branch (2004) and Pfajfar and Santoro (2010) provide empirical evidence in support of heterogeneous expectations using survey data on inflation expectations, while Hommes, Sonnemans, Tuinstra, and van de Velden (2005), Adam (2007), Pfajfar and Zakelj (2014), Assenza, Heemeijer, Hommes, and Massaro (2011) and Hommes (2011) find evidence for heterogeneity in learning to forecast laboratory experiments with human subjects. Moreover, Beechey, Johannsen, and Levin (2011) conclude that long-run expectations of US inflation vary noticeably over time and are characterized by a great disagreement across professional forecasters.

Concerning the second empirical stylized fact, while all the empirical studies on forward versus backward-looking behavior in inflation dynamics cited above take the distribution of weights of heterogeneous firms as fixed and exogenous, recent empirical analysis suggest that this assumption is overly restrictive. In fact, Zhang, Osborn, and Kim (2008), Kim and Kim (2008), Castle, Doornick, Hendry,

and Nymoen (2014) and Hall, Han, and Boldea (2012) find evidence for multiple structural breaks in the relative weights of forward- and backward-looking firms. See also Stock and Watson (2007) who acknowledge the possibility of the changing time-series properties of inflation. Moreover, Carroll (2003) and Mankiw, Reis, and Wolfers (2003) show that the distribution of heterogeneity evolves over time in response to economic volatility, while Frankel and Froot (1991), Bloomfield and Hales (2002), Branch (2004), Assenza, Heemeijer, Hommes, and Massaro (2011) and Hommes (2011), among others, provide evidence that the proportions of heterogeneous forecasting strategies evolve over time as a reaction to past forecast errors using survey data as well as experimental data. Therefore, on the basis of this empirical evidence, we introduce heterogeneous firms with subjective beliefs and endogenize the evolution of the distribution of heterogeneous firms by assuming that agents can switch between different forecasting regimes, depending on the recent prediction performance of their forecasting rules as in Brock and Hommes (1997).

Our stylized model includes two types of price setters. The first type are *fundamentalists*, who believe in a present-value relationship between inflation and real marginal costs. The second type believe instead that inflation follows a random walk, *random walk believers*, and use the simplest backward-looking rule of thumb corresponding to naive expectations (i.e., their forecast coincides with the last available observation) to forecast future inflation. We choose this specific set of forecasting rules in order to obtain a NKPC similar to the closed-form solution of hybrid models estimated in the literature. In fact, the models estimated by Sbordone (2005) and Rudd and Whelan (2006), among others, feature two components: a forward-looking term given by the discounted sum of expected future marginal costs, and a backward-looking term given by lagged inflation. Our fundamentalists are forward-looking in the sense that they believe that the evolution of inflation depends on the discounted sum of expected future values of real marginal costs, accounting thus for the forward-looking term in the hybrid NKPC, while random

walk believers are backward-looking, accounting for the lagged inflation term in the hybrid NKPC. Although we refer to fundamentalists as forward-looking, we note that their expectations differ from those of perfectly rational agents in the sense that they are *not* model-consistent because fundamentalists do not take into account the presence of non-rational agents. In fact, achieving the model-consistency requirement of RE models is especially difficult in a world with heterogeneous agents. Individuals would need to gather and process a substantial amount of information about the economy, including details about the beliefs of other agents in the market, in order to derive the objective probability distribution of aggregate variables (see Hommes (2006) for an extensive discussion of heterogeneous agents models). Moreover, the assumption of model-consistent RE in the formulation of inter-temporal optimization decisions has been criticized by Hendry and Mizon (2010, 2014) in the presence of unanticipated structural breaks on the grounds that the law of iterated expectations needs not to hold when distributions shift as integrals are taken over different weighted intervals. Castle, Doornick, Hendry, and Nymoen (2014) find evidence for such shifts when fitting the hybrid NKPC to U.S. inflation data and demonstrate that a potentially spurious outcome can arise when the NKPC is estimated under the assumption of RE. We allow firms to switch between relatively simple prediction rules according to their forecasting performance, providing a behavioral micro-foundation for the the structural breaks observed in the relative weight of forward-looking term in the NKPC. Obviously we are not the first to introduce a dynamic predictor selection mechanism in macroeconomic models. Recent theoretical papers analyzing inflation dynamics under endogenous selection of expectation rules include, among others, Brock and de Fontnouvelle (2000), Tuinstra and Wagener (2007), Brazier, Harrison, King, and Yates (2008), Branch and McGough (2010), De Grauwe (2011), Branch and Evans (2011), Anufriev, Assenza, Hommes, and Massaro (2013).

The main novelty of our paper consists in the estimation of a NKPC with heterogeneous expectations and endogenous switching between different beliefs using U.S.

macroeconomic data. To our knowledge, there are only a few empirical applications that attempt to estimate heterogenous agents models with fully-fledged switching mechanism. Other applications include the S&P500 market index (Boswijk, Hommes, and Manzan (2007)), commodity markets (Reitz and Westerhoff (2005, 2007)), the Asian equity markets (De Jong, Verschoor, and Zwinkels (2009)), the DAX30 index options (Frijns, Lehnert, and Zwinkels (2010)), and the U.S. housing market (Kouwenberg and Zwinkels (2014)).

Moreover, our paper contributes to the debate about the empirical relevance of forward-looking behavior in inflation dynamics. In fact our model, although with a different behavioral interpretation, is similar to the hybrid models estimated by Sbordone (2005) and Rudd and Whelan (2006) among others. Our measure of fundamental expectation is constructed in the same way as Sbordone (2005) and Rudd and Whelan (2006) obtain their estimation of the discounted sum of expected future values of real marginal costs in the closed-form solution of the model, i.e., using the Campbell and Shiller VAR methodology, while the expectations of naive firms account for lagged value of inflation in the hybrid specification of the NKPC. The main difference in our model stems from the time-varying weights assigned to fundamentalists and naive price setters, evolving over time according to past relative forecasting performances. As for the debate on the appropriate measure of inflationary pressure, we perform our empirical exercise using both the output gap and the labor share of income as proxy for real marginal costs and check for the robustness of our findings.

The results of our analysis provide empirical evidence for behavioral heterogeneity in U.S. inflation dynamics. Moreover, the data support the hypothesis of an endogenous mechanism relating predictors choice to their forecasting performance. In fact, our results suggest that the degree of heterogeneity varies considerably over time, and that the economy can be dominated temporarily by either forward-looking or backward-looking behavior. These findings are robust to the choice of the proxy for real marginal costs.

Our findings have important implications for monetary policy. Standard policy recommendations based on determinacy under RE may not be a robust criterion for policy advices in the presence of heterogeneous expectations; see Molnár (2007). In fact, recent papers have shown that multiple equilibria, periodic orbits and complex dynamics can arise in the presence of dynamic predictor selection, even if the model under RE has a unique stationary solution (see Anufriev, Assenza, Hommes, and Massaro (2013), Branch and McGough (2010), and De Grauwe (2011) among others).

The paper is structured as follows. Section 2 derives a NKPC with heterogenous expectations and endogenous switching dynamics. Section 3 presents the estimation results and describes the fit of the model. Section 4 discusses the robustness of the empirical results to alternative forecasting models for the driving variable in the NKPC and to alternative measures of real marginal costs. Section 5 concludes.

# 2 The model

This section derives a NKPC with heterogeneous, potentially nonrational expectations and endogenous switching between forecasting strategies.

## 2.1 The NKPC with heterogeneous expectations

We consider a model with monopolistic competition, staggered price setting and heterogeneous firms. There is a continuum of differentiated goods indexed by $j \in [0, 1]$. The demand curve for product $j$ takes the form:

$$Y_t(j) = Y_t(P_t(j)/P_t)^{-\eta},$$

where $\eta$ is the Dixit-Stiglitz elasticity of substitution among differentiated goods, $Y_t$ is the aggregator function defined as $Y_t = [\int_0^1 Y_t(j)^{(\eta-1)/\eta} dj]^{\eta/(\eta-1)}$, and $P_t$ is the aggregate price level defined as $P_t = [\int_0^1 P_t(j)^{1-\eta} dj]^{1/(1-\eta)}$. Each firm has a

production technology that uses labor as the only factor of production. Nominal price rigidity is modeled by allowing, in every period, only a fraction $(1-\omega)$ of the firms to set a new price along the lines of Calvo (1983). We assume a continuum of firms of each production type $j$, and that the same proportion of firms of each production type has subjective expectations $E_t^i$ of type $i$. Given that each firm hires labor from the same integrated economy-wide labor market, the prices chosen by the firms that can re-optimize in each period will only differ because of their subjective forecasts. We will therefore index firms and their prices according to their expectation type $i$. Firms that reset prices maximize expected discounted profits, which are given by

$$\max_{P_{i,t}} E_t^i \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} \left( \frac{P_{i,t}}{P_{t+s}} - mc_{t+s} \right) \left( \frac{P_{i,t}}{P_{t+s}} \right)^{-\eta} Y_{t+s},$$

where $Q_{t,t+s}$ denotes the stochastic discount factor and $mc_t$ are real marginal costs of production. Log-linearizing the first order conditions of this problem around a zero inflation steady state and defining $p_{i,t} \equiv P_{i,t}/P_t$ yields

$$\widehat{p}_{i,t} = (1-\omega\delta) E_t^i \sum_{s=0}^{\infty} (\omega\delta)^s \widehat{mc}_{t+s} + \omega\delta E_t^i \sum_{s=0}^{\infty} (\omega\delta)^s \pi_{t+s+1}, \tag{2.1}$$

where $\delta$ is the time discount factor, $\pi_t \equiv \widehat{p}_t - \widehat{p}_{t-1}$ is the inflation rate, and hatted variables denote log-deviations from steady state.

Optimal pricing decisions involve subjective forecasts of future macroeconomic variables, hence firms with different expectations will set different prices. The relative average price set by optimizing firms is given by $\widehat{p}_t^* = \int_i \widehat{p}_{i,t}$. Log-linearizing the aggregate price level equation yields

$$\pi_t = \frac{1-\omega}{\omega} \widehat{p}_t^*. \tag{2.2}$$

Under the assumption of a representative firm with rational expectations, Eqs. (2.1) and (2.2) can be used to derive the standard NKPC in Eq. (1.1), reported here for

convenience:

$$\pi_t = \delta E_t \pi_{t+1} + \gamma mc_t \ ,$$

where $\gamma \equiv (1-\omega)(1-\delta\omega)\omega^{-1}$ and we omitted hats for notational simplicity. Deriving an equation for inflation similar to Eq. (1.1) is not entirely obvious when expectations are heterogeneous. Following Kurz (2012), it is possible to aggregate the individual pricing rules in order to obtain an aggregate supply equation of the form

$$\pi_t = \delta \bar{E}_t \pi_{t+1} + \gamma mc_t + \xi_t \ , \tag{2.3}$$

where $\bar{E}_t = \int_i E_t^i$ denotes the average expectation of individuals and the term $\xi_t$ is defined as $\xi_t \equiv (1-\omega)\delta \int_i (E_t^i p_{i,t+1} - E_t^i p_{t+1})$.[4] Eq. (2.3) shows that, in the presence of heterogeneous expectations, inflation depends on real marginal costs, on the average forecasts of future inflation, and on an additional term $\xi_t$ representing deviations of average agents' forecasts of individual prices from average forecast of aggregate price. In the presence of heterogeneous agents, with possibly non-rational beliefs, there is no a-priori reason to believe that in every period the average forecast of individual prices will coincide with the average forecast of aggregate price. Given that we have no data on the deviations of average forecasts of individual prices from average forecast of aggregate price, in our empirical analysis we will consider $\xi_t$ as part of the error term and performs diagnostic checks on the properties of the residuals of our regression model.[5] This is in line with the ideas of, e.g., Kurz (2012) and Diks and van der Weide (2005), who consider expectations heterogeneity as a

[4]See Kurz (2012) for details.

[5]Notice also that we can not directly impose a structure on $\xi_t$ since we will make assumptions about how agents forecast inflation but not about how agents forecast prices. From a behavioral point of view, forecasting prices is rather different than forecasting inflation (see e.g., Tuinstra and Wagener (2007)). In fact, while we will make specific assumptions on inflation expectations on the basis of observable statistical or theoretical properties of the inflation process, it is more difficult to model price expectations since in reality agents rarely collect information or read news about prices in levels.

natural source of randomness.

## 2.2 Evolutionary selection of expectations

We assume that agents form expectations by choosing from $I$ different forecasting rules, and we denote by $E_t^i \pi_{t+1}$ the forecast of inflation by rule $i$. The fraction of individuals using the forecasting rule $i$ at time $t$ is denoted by $n_{i,t}$. Fractions are updated in every period according to an *evolutionary fitness* measure. At the beginning of every period $t$ agents compare the realized relative performances of the different strategies and the fractions $n_{i,t}$ evolve according to a discrete choice model with multinomial logit probabilities (see Manski and McFadden (1981) for details), that is

$$n_{i,t} = \frac{\exp(\beta U_{i,t-1})}{\sum_{i=1}^{I} \exp(\beta U_{i,t-1})}. \tag{2.4}$$

$U_{i,t-1}$ is the realized fitness metric of predictor $i$ at time $t-1$, and the parameter $\beta \geq 0$ refers to the *intensity of choice* reflecting the sensitivity of the mass of agents to selecting the optimal prediction strategy. Brock and Hommes (1997) proposed this model for endogenous selection of expectation rules.[6] The key feature of Eq. (2.4) is that strategies with higher fitness in the recent past attract more followers. The case $\beta = 0$ corresponds to the situation in which differences in fitness can not be observed, so agents do not switch between strategies and all fractions are constant and equal to $1/I$. The case $\beta = \infty$ corresponds to the "neoclassical" limit in which the fitness can be observed perfectly and in every period all agents choose the best predictor.

A strong motivation for switching among forecasting rules can be found in empirical works on individual expectations. Frankel and Froot (1991) find that professional market participants in the foreign exchange markets expect recent

[6]Eq. (2.4) can also be derived from an optimisation problem under rational inattention (see Matějka and McKay (2015)). In this context the parameter $\beta$ is inversely related to the shadow cost of information.

price changes to continue in the short term, while they expect mean reversion to fundamental value in the long term. Moreover, Frankel and Froot (1991) report survey evidence showing that professional forecasting services in the foreign exchange markets rely both on technical analysis and fundamental models, but with changing weights over time, and the weights appear to depend strongly on recent forecasting performances. Branch (2004) finds evidence for dynamic switching between alternative forecasting strategies that depends on the relative mean squared errors of the predictors using survey data on inflation expectations.[7] In addition, Bloomfield and Hales (2002), Assenza, Heemeijer, Hommes, and Massaro (2011) and Hommes (2011) document experimental evidence that participants switch between forecasting regimes conditional on recent forecasting performances.

## 2.3 A simple two-type example

We assume that agents can choose between two forecasting rules to predict inflation, namely fundamental and random walk beliefs. The fundamental rule is based on a present-value description of the inflation process. When all agents have rational expectations, repeated application of equation Eq. (2.3) gives

$$\pi_t = \gamma \sum_{k=0}^{\infty} \delta^k E_t mc_{t+k} \ . \tag{2.5}$$

We refer to (2.5) as the *fundamental inflation*. Fundamentalists use expression (2.5) to forecast future inflation. In particular, leading (2.5) one-period ahead we get

$$\pi_{t+1} = \gamma \sum_{k=1}^{\infty} \delta^{k-1} E^f_{t+1} mc_{t+k} \ , \tag{2.6}$$

[7] Branch (2004) estimates a model with dynamic selection among three predictors, namely naive, adaptive and VAR expectations, using the Michigan survey data. Moreover, the author assumes the presence of costs associated with the adoption of each predictor.

where $E^f$ denotes fundamentalists forecast. Applying the expectation operator $E_t^f$ on both sides we get

$$E_t^f \pi_{t+1} = \gamma \sum_{k=1}^{\infty} \delta^{k-1} E_t^f mc_{t+k} \ . \tag{2.7}$$

In deriving Eq. (2.7) we made use of the law of iterated expectations at the *individual* level. This is a reasonable and intuitive assumption which is standard in the learning literature (see, e.g., Evans and Honkapohja (2001) and Branch and McGough (2009)).[8]

From a behavioral point of view, fundamentalists can be considered as agents who believe in RE and use the closed form solution of the model to forecast the inflation path. There is, however, an important difference between fundamental expectations and RE. Fundamental expectations are *not* model-consistent because they do not take into account the presence of non-rational agents. As already mentioned in the Introduction, the assumption of model-consistent expectations has been criticized in the empirical literature on the NKPC (see, e.g., Rudd and Whelan (2006) and Castle, Doornick, Hendry, and Nymoen (2014)) and found to be inconsistent with U.S. data (see Carriero (2008)). Also intuitively, in a world with heterogeneous firms, model-consistent expectations would require agents to collect an incredible amount of information about the economy, including details about the beliefs of other agents in the market, in order to derive the objective probability distribution of aggregate variables (Hommes (2006)). More realistically, firms in our framework only have knowledge of their objectives and of the constraints that they face, and therefore they do not have a complete model of determination of aggregate variables.

In order to characterize the fundamental forecast (2.7) we use the VAR method-

[8]We justify the fact that the law of iterated expectations holds at the individual level in the presence of evolutionary switching by appealing to the learning literature which models the selection of forecasting rules as a distinct statistical problem. Thus agents choose a forecasting model and then use that model to solve for their optimal plan in the anticipated utility sense, as in Kreps (1998) and Sargent (1999).

ology of Campbell and Shiller (1987). Assuming that the forcing variable $mc_t$ is the first variable in the multivariate VAR

$$Z_t = A_t Z_{t-1} + \epsilon_t, \tag{2.8}$$

we can rewrite the sum of discounted future expectations of marginal costs (2.7) as

$$E_t^f \pi_{t+1} = \gamma \sum_{k=1}^{\infty} \delta^{k-1} E_t^f mc_{t+k} = \gamma e_1' (I - \delta A_t)^{-1} A_t Z_t,$$

where $e_1'$ is a suitably defined unit vector.[9] Notice that the matrix of coefficients $A_t$ in the VAR model (2.8) is time-varying. In fact, fundamentalist agents do not have perfect foresight into the future and, in every period $t$, they estimate a VAR model to produce their forecast using the information set available at time $t$. The coefficient matrix is then updated in every period as new information becomes available.

The second rule, which we call random walk or naive, takes advantage of inflation persistence and uses a simple backward-looking forecasting strategy:

$$E_t^n \pi_{t+1} = \pi_{t-1} \ , \tag{2.9}$$

where $E^n$ denotes the naive expectation operator. Notice that, although being an extremely simple rule, the naive forecasting strategy is optimal when the stochastic process is a random walk; hence for a near unit root process, as in the case of inflation, random walk expectations are close to optimal.

The specific choice of the set of forecasting rules, namely fundamental and random walk, will enable us to compare the outcome of our analysis with the results of previous empirical works based on the hybrid Phillips curve specification. In

[9]Technically, because the discounted sum of real marginal costs starts at $k = 1$, we measure it using $(I - \delta A_t)^{-1} A_t Z_t$ instead of $(I - \delta A_t)^{-1} Z_t$.

fact, fundamental expectations account for the forward-looking component in the estimated closed-form solution of the hybrid NKPC. We estimate the discounted sum of expected marginal costs using the VAR methodology as in Sbordone (2005) and Rudd and Whelan (2006) among others, with the only difference that, consistently with the behavioural interpretation of our model, the matrix of coefficients in the VAR model is not estimated using observations over the full sample (i.e., information which is not available to agents at the moment of their forecast) but it is recursively updated as new observations become available. The backward-looking component introduced in different ways in hybrid RE models is accounted for by the expectations of naive firms.

The main difference between traditional hybrid specifications of the NKPC and our model is the fact that the weights assigned to forward-looking and backward-looking component are endogenously varying over time. We assume that agents can switch between the two predictors based on recent forecasting performance. Defining the absolute forecast error in the previous $K$ periods as

$$FE_t^i = \sum_{k=1}^{K} \left| E_{t-k}^i \pi_{t-k+1} - \pi_{t-k+1} \right| ,$$

with $i = f, n$, we can then define the evolutionary fitness measure as[10]

$$U_{i,t} = -\frac{FE_t^i}{\sum_{i=1}^{I} FE_t^i} . \tag{2.10}$$

The evolution of the weights of different heuristics is then given by Eq. (2.4). Denoting the fraction of fundamentalists as $n_{f,t}$ we can summarize the full model as

$$\pi_t = \delta(n_{f,t} E_t^f \pi_{t+1} + (1 - n_{f,t}) E_t^n \pi_{t+1}) + \gamma mc_t + u_t , \tag{2.11}$$

[10]The estimation results are robust to alternative specifications of the fitness measure. We chose relative absolute forecast error for numerical convenience, since it restricts the support of the fitness measure to the interval $[-1, 0]$.

where $u_t$ is a composite error term including the component $\xi_t$ and potential errors due to measurement or linearization, and

$$
\begin{aligned}
E_t^f \pi_{t+1} &= \gamma e_1' (I - \delta A_t)^{-1} A_t Z_t \\
E_t^n \pi_{t+1} &= \pi_{t-1} \\
n_{f,t} &= \frac{1}{1 + \exp\left(\beta \left(\frac{FE_{t-1}^f - FE_{t-1}^n}{FE_{t-1}^f + FE_{t-1}^n}\right)\right)} \\
FE_{t-1}^i &= \sum_{k=1}^{K} |E_{t-k-1}^i \pi_{t-k} - \pi_{t-k}|, \qquad \text{with } i = f, n \ .
\end{aligned}
$$

# 3 Estimation results

This section describes data and methodology used to estimate the nonlinear switching model derived in the previous section.

## 3.1 Data description

We use quarterly U.S. data on the inflation rate, the output gap, unit labor costs, the labor share of income, hours of work and consumption-output ratio, from 1960:Q1 to 2010:Q4. Inflation is measured as log-difference of CPI. Output gap is measured as quadratically detrended log-real GDP. We use unit labor costs, labor share of income, detrended hours of work and detrended consumption-output ratio time series for nonfarm business sector in the construction of the VAR model (2.8). A more detailed description of data sources and variables definition is given in Appendix A.

## 3.2 The fit of the model

In this section we discuss the empirical implementation of model (2.11). In the "baseline" specification (the one used in the results reported below) we use the output gap as driving variable, and in section 4 we discuss the sensitivity of the results to the use of the labor share of income as measure of inflationary pressure.

**Baseline VAR specification**

The first step concerns the choice of the baseline VAR specification to estimate the matrix $A_t$, needed to construct the forecasts of fundamentalists,

$$E_t^f \pi_{t+1} = \gamma e_1' (I - \delta A_t)^{-1} A_t Z_t.$$

In principle the specification of the VAR model (2.8) could be selected period by period on the basis of standard econometric tests (e.g., unit root, Granger causality, lag selection). However, in order to avoid potential problems due to the small sample size (lack of power of the tests), we select the best VAR specification using the full sample and we keep it fixed over time. The VAR coefficients' estimates are instead updated in every period as new information becomes available. In order to construct the baseline VAR specification for the fundamentalists' forecast we started with a very broad VAR model in the output gap ($y_t$), unit labor costs ($ulc_t$), the labor share of income ($lsi_t$), and the (past) inflation rate ($\pi_{t-1}$).[11] This specification extends the baseline specifications of previous empirical works, e.g., Woodford (2001) and Rudd and Whelan (2005a), by adding lagged values of inflation in the output gap equation. ADF and KPSS unit root tests show that unit labor costs and labor share of income are I(1) processes.[12] Therefore we estimated VAR models which include the rate of change of unit labor costs ($\Delta ulc_t$) and of labor share ($\Delta lsi_t$). The number of lags was chosen optimally on the basis of the comparison of standard information criteria, namely the sequential modified LR test statistic (LR), the Akaike information criterion (AIC), the Bayesian information criterion (BIC) and the Hannan-Quinn information criterion (HQ). We then

[11] Note that we use $\pi_{t-1}$ in the construction of the VAR to be consistent with the information set of fundamentalist firms in the model. In fact, as standard in learning models, current values of endogenous variables are not observable at time $t$ because they depend on the heterogeneous beliefs in the economy which are not known to the individual firm.

[12] The presence of a unit root in the labor share time series was not detected in previous empirical works such as Woodford (2001) and Rudd and Whelan (2005a). This is due to the fact that our dataset incorporates observations until 2010:Q4. Unit root tests performed on the same sample considered by Woodford (2001) and Rudd and Whelan (2005a) confirm the results found by these authors, i.e., the presence of a unit root in $lsi_t$ is rejected.

performed pairwise Granger causality tests and proceeded iteratively, eliminating insignificant regressors, highest $p$-value first. We found evidence that neither inflation nor the rate of change of unit labor costs Granger cause the output gap, therefore we excluded the variables $\Delta ulc_t$ and $\pi_{t-1}$ from the VAR and we chose a four-lag bivariate VAR in the output gap and labor share of income growth as our baseline specification. One might argue that if marginal costs are to successfully explain the observed dynamics of inflation, then it might be the case that lagged inflation is a useful predictor for marginal costs. We exclude lagged inflation from our baseline VAR specification on the statistical grounds that it does not Granger cause the output gap. However, in Section 4 where we perform robustness checks to alternative specifications of the forecasting VAR, we include lagged inflation in the VAR specification to verify the sensitivity of our results. Denoting by $Y_t$ the vector of dependent variables, $Y_t = [y_t, \Delta lsi_t]'$, the vector $Z_t$ is defined as $Z_t = [Y_t, Y_{t-1}, Y_{t-2}, Y_{t-3}]'$. Although being parsimonious, our VAR specification captures about 94% of output gap volatility and the Portmanteau test reports no autocorrelation in the residuals up to the 20th lag ($p$-value $Q(20) = 0.796$).

**NLS estimation**

As standard in empirical works on the NKPC, we fix the discount factor $\delta = 0.99$, and we select a number of $K = 4$ lags for measuring past performance.[13] That is, if fundamentalists (naive) have a more accurate inflation forecast over the past year, more firms will follow the fundamentalist (naive) expectation formation rule. Model (2.11) is then estimated using non-linear least squares (NLS). Table 1 presents the results and diagnostic checks of the residuals are reported in Appendix B.

All coefficients have the correct sign and are significant at least at the 5% level. The positive sign and the significance of the intensity of choice parameter $\beta$ implies that agents switch towards the better performing forecasting rule, based on its past

[13]From a behavioral point of view it seems a sensible choice to pick $K = 4$ for quarterly data, meaning that the fitness measure takes an average of the forecast errors over the past year. Experimentation with different values of $K$ shows that our results are robust to the choice of the number of lags in the performance measure.

Table 1: NLS estimates of model (2.11)

| Parameter | $\beta$ | $\gamma$ |
|---|---|---|
| Estimate | $4.685^{***}$ | $0.004^{**}$ |
| Std. error | 1.341 | 0.001 |
| $R^2$ from Inflation Equation | 0.774 | |

*Notes*: Standard errors are computed using White's heteroskedasticity-consistent covariance matrix estimator (HCCME). *, **, *** denote significance at the 10%, 5%, and 1% level.

performance.[14] The positive sign and the significance of the parameter $\gamma$ is a rather interesting result. It has been quite difficult to obtain parameter estimates with the correct sign and of a plausible magnitude when the output gap is used as driving variable for the inflation process. Fuhrer and Moore (1995) and Galí and Gertler (1999), for example, find a negative and insignificant estimate of $\gamma$ when real marginal costs are approximated by detrended output. The results in Table 1 show that taking into account non-rational heterogeneous expectations helps to establish a plausible link between output and inflation dynamics via the NKPC. Interestingly, Adam and Padula (2011) reach the same conclusion by estimating a NKPC using data from the Survey of Professional Forecasters as proxy for expected inflation.

The time series of inflation predicted by (2.11) for the estimated values of $\beta$ and $\gamma$ is plotted in Fig. 1, as dashed line, versus the actual series (solid line).[15] Overall the predicted inflation path tracks the behavior of actual inflation quite well (the $R^2$ from inflation equation (2.11) is about 0.78, see Table 1).

Our results are, in some respects, similar to findings obtained in previous empirical works. In particular, Galí and Gertler (1999) and Sbordone (2005) find that models derived from the assumption of heterogeneous price setting behavior are capable of fitting the level of inflation quite well. However, Rudd and Whelan (2005a) and Rudd and Whelan (2006) show that this good fit reflects the substantial role that these models still allow for lagged inflation, and that forward-looking

[14]The order of magnitude of $\beta$ is more difficult to interpret as it is conditional on the functional form of the performance measure $U$.

[15]The series are in deviation from the mean.

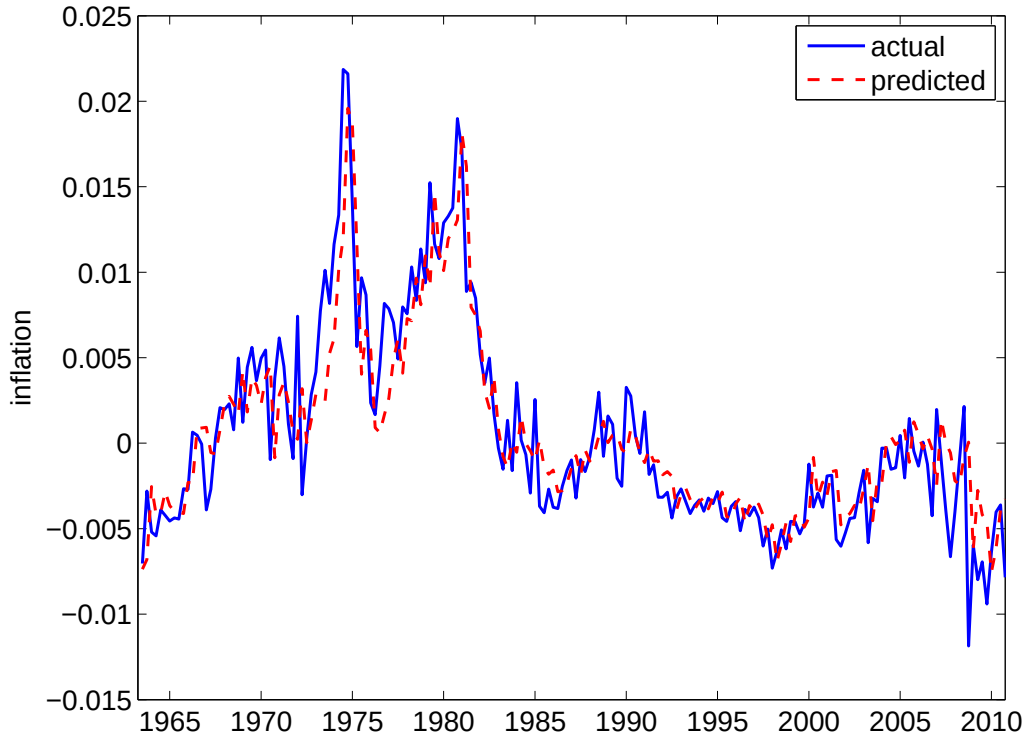


Figure 1: Actual vs. predicted inflation

components play no discernable empirical role in determining inflation.

Our NKPC specification allows for time-varying weights assigned to fundamentalists and naive price setters. Having estimated model (2.11), we are now ready to assess the relative importance over time of forward-looking versus backward-looking components in inflation dynamics. Table 2 displays descriptive statistics of the weight of the forward-looking component $n_f$.

Table 2: Descriptive statistics of weight $n_f$

| | |
|---|---|
| Mean | 0.334 |
| Median | 0.265 |
| Maximum | 0.945 |
| Minimum | 0.021 |
| Std. Dev. | 0.272 |
| Skewness | 0.489 |
| Kurtosis | 1.832 |
| Auto-corr. Q(-1) | 0.899 |

On average, the majority of agents use the simple backward-looking rule (with average fraction $1 - 0.33 = 0.67$). However, the spread between the minimum and the maximum indicates that the market can be dominated by either forward-looking or backward-looking agents. Moreover the autocorrelation of the series $n_f$, about 0.9, indicates that agents do not change their strategy quickly, suggesting a relatively high degree of inertia in the updating process.

Fig. 2 shows the time series of the fraction of fundamentalists, i.e., the forward-looking component in our NKPC specification, the time series of the distance of actual inflation from the fundamental forecast, and a scatter plot of the fraction of fundamentalists against the relative forecast error of the naive rule.

It is clear that the fraction of fundamentalists varies considerably over time with periods in which it is close to 0.5 and other phases in which it is close to either one of the extremes 0 or 1. For example, immediately after the oil crisis of 1973, the proportion of fundamentalists drops almost to 0. Soon after the difference between inflation and fundamental value reaches its peak in 1974:Q2/1974:Q3, the estimated weight of the forward-looking component shoots back up to about 0.65.

During the second oil crisis inflation was far above the fundamental, causing more and more agents to adopt a simple backward-looking rule to forecast inflation. Our findings suggest that, in reaction to large shocks pushing inflation away from the fundamental, a large share of agents adopt random walk beliefs causing therefore high inflation persistence. This result is in line with the analysis of Branch and Evans (2013) showing that innovations to inflation can lead agents adaptively learning in the economy to temporarily believe that inflation follows a random walk. Fundamentalists dominated the economy in the late-80s and early-90s, while in the late stage of the Great Moderation (from 1993 until 2004), inflation stayed continuously well below the fundamental, causing the weight of random walk believers to increase. A somewhat similar conclusion is reached by Stock and Watson (2007) who show that, for the Great Moderation, using an activity-based Phillips curve does not improve forecasts obtained using simple univariate models. From 2005 until the early stages of the recent global financial crisis, the proportion of fundamentalists stayed, on average, around 0.5, reaching peaks of about 0.75. In the aftermath of the crisis we observe that $n_f$ declines with the fundamentalist rule losing its forecast accuracy when actual inflation falls below its fundamental value.

The bottom panel of Fig. 2 presents a scatter plot of the relative forecast error of the random walk rule, $(FE^n - FE^f)/(FE^n + FE^f)$, versus the fraction of fundamentalist agents, $n_f$. Due to the positive estimated value of $\beta$ this line slopes upwards, such that a more accurate fundamentalist forecast results in a higher weight $n_f$. The S-shape is induced by the logit function in Eq. (2.4).

The analysis conducted in this section shows that the evolutionary switching model fits the data quite nicely. The positive sign and the significance of the intensity of choice parameter, $\beta$, imply that the endogenous mechanism that relates predictors choice to past performance is supported by the data. We also find that the ability of the discounted sum of expected future output gap values to predict the empirical inflation process varies considerably over time. In fact the spread between the minimum and the maximum value of $n_f$, i.e., the fraction of fundamentalists,

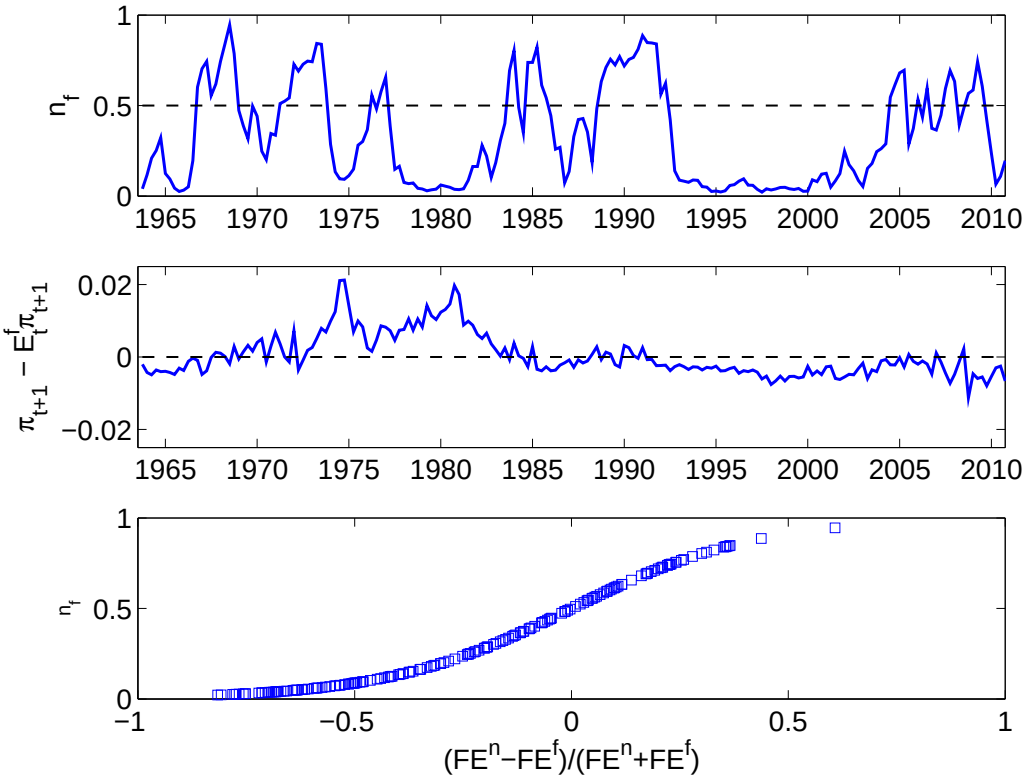


Figure 2: **Top panel:** Time series of the fraction of fundamentalists $n_{f,t}$. **Middle panel:** Distance between actual inflation and fundamental forecast. **Bottom panel:** Scatter plot of the weight $n_{f,t}$ versus the relative forecast error of the naive rule.

shows that the economy can be dominated by either forward-looking or backward-looking behavior. Moreover, even though the market is, on average, dominated by agents using a simple heuristic to predict inflation, fundamentalists, or forward-looking components, still have a significant impact on inflation dynamics.

## 3.3 Specification tests and model selection

In order to assess the validity of our baseline model, which will be denoted by $H_1$ for the purposes of this section, we test it against four alternative specifications: a model with heterogeneous agents and exogenous estimated fixed weights ($n_{f,t} \equiv \widehat{n}_f$), which is similar to the model estimated by Rudd and Whelan (2006) and Sbordone (2005), and it is denoted by $H_2$; a static model with heterogeneous agents in which we let $\beta = 0$ ($n_{f,t} \equiv 0.5$), which is similar to the model of Fuhrer and Moore (1995), denoted by $H_3$; a model with homogeneous fundamentalists agents ($n_{f,t} \equiv 1$), which resembles the the RE closed form solution of the standard NKPC without backward looking component, denoted by $H_4$;[16] a model with homogeneous naive agents ($n_{f,t} \equiv 0$), which recalls the old backward-looking Phillips curve and it is denoted by $H_5$. Given that, with the exception of model $H_3$ which obtains by setting $\beta = 0$, the competing models are nonnested, we will use nonnested hypothesis testing procedures. In particular, we construct the $P$ test for the adequacy of our nonlinear specification with endogenous switching in explaining inflation dynamics (null hypothesis) against the alternative specifications mentioned above. Nonnested hypotheses tests are appropriate when rival hypotheses are advanced for the explanation of the same economic phenomenon. We will follow the proce-

[16] In fact, in the presence of homogeneous firms we have that $\xi_t = 0$ and, substituting the fundamental forecast in Eq. (2.3), we get $\pi_t = \delta\gamma \sum_{k=1}^{\infty} \delta^{k-1} E_t^f mc_{t+k} + \gamma mc_t = \gamma \sum_{k=0}^{\infty} \delta^k E_t^f mc_{t+k}$ which corresponds to the inflation path implied by the RE closed form solution in Eq. (2.5), when the discounted sums of current and future expected marginal costs are estimated in the same way. The difference between the model with only fundamentalists and the standard model under RE is the fact that typically in the latter the matrix $A$ in the VAR model has constant coefficients and is estimated using the full sample, while consistently with our behavioural model, fundamentalists only use available information in each period implying coefficients of matrix $A$ estimated period by period. Qualitative results of the tests reported below do not change if we substitute model $H_4$ with the actual RE closed form solution.

dure described in Davidson and MacKinnon (1981) and Davidson and MacKinnon (2009) and compute a heteroskedasticity-robust $P$ test of $H_1$ against the alternatives $H_2$, $H_3$, $H_4$, and $H_5$. We report the results of the test in Tables 3 and 4, and refer the reader to Davidson and MacKinnon (2009), p. 284 and p. 669, for details on the construction of the heteroskedasticity-robust test. Table 3 reports

Table 3: Paired nonnested hypotheses tests

| $H_r$ vs. $H_c$ | $H_1$ | $H_2$ | $H_3$ | $H_4$ | $H_5$ |
|---|---|---|---|---|---|
| $H_1$ | - | 0.061 | 0.961 | 0.265 | 0.065 |
| $H_2$ | 0.008 | - | 0.651 | 0.651 | 0.651 |
| $H_3$ | 0.000 | 0.000 | - | 0.000 | 0.000 |
| $H_4$ | 0.000 | 0.000 | 0.000 | - | 0.000 |
| $H_5$ | 0.000 | 0.016 | 0.016 | 0.015 | - |

*Notes:* p-values from $P$ tests of each model $H_r$ (rows) against each model $H_c$ (columns).

the results of paired nonnested tests in which each model of $H_1$, $H_2$, $H_3$, $H_4$, and $H_5$ are tested against each other.[17] The first row of Table 3 shows that we reject with a 95% confidence level all alternative models ($H_2$, $H_3$, $H_4$, and $H_5$) when tested against our baseline switching model ($H_1$). In other words, there is no statistically significant evidence of departure from the null hypothesis, i.e., adequacy of the nonlinear switching model, in the direction of these alternative explanations of inflation dynamics. On the contrary, the first column of Table 3 shows that we never reject the switching model when tested against each of the alternative models.

For the sake of completeness, we also test the joint significance of the alternative models against our benchmark nonlinear switching model.

Table 4: Joint nonnested hypotheses test

| | | | |
|---|---|---|---|
| test-statistic | 6.484 | Prob. $\chi^2(4)$ | 0.166 |

*Notes:* The null hypothesis is the joint insignificance of alternative models $H_2$, $H_3$, $H_4$, and $H_5$ against $H_1$.

[17] For completeness, we also compared the switching model to the nested static model without switching ($\beta = 0$) using a likelihood ratio test. We rejected the null of a restricted static model at the 1% level.

The results reported in Table 4 reject the significance of the competing models against the baseline model.

The results of the nonnested hypotheses tests show that the data support our model of inflation dynamics and provide evidence in favor of the switching model when tested against alternative models of the inflation process. However, because nonnested hypotheses tests are designed as specification tests, we complete the analysis by reporting the Bayesian Information Criterion (BIC) for model selection in Table 5.

Table 5: Bayesian Information Criteria

| | $H_1$ | $H_2$ | $H_3$ | $H_4$ | $H_5$ |
|---|---|---|---|---|---|
| BIC | **-1685** | -1674 | -1589 | -1407 | -1671 |

*Notes:* Best model shown in bold.

The BIC chooses the baseline switching model as the best model among all competing models, confirming the results of the nonnested hypotheses tests.

# 4 Robustness analysis

The empirical analysis that we presented is conditional upon the assumption that the output gap is well forecasted by our baseline VAR specification, and that the output gap itself is a good approximation to real marginal costs. In this section we address the issue of how sensitive our results are to alternative specifications of the VAR forecasting model, and to different measures of real marginal costs.

## 4.1 Robustness to the specification of the VAR model

In order to choose the baseline forecasting system, we started from a broad model that recalls the baseline specifications of previous empirical works (see, e.g., Woodford (2001) and Rudd and Whelan (2005a)) and then restricted the number of variables to include in our VAR as documented in Section 3.2. However, one does

not necessarily have to exclude from the information set other variables that may help forecasting the output gap beyond the contribution of the rate of change of the labor share of income. Therefore, to investigate how sensitive our results are to the specification of the fundamentalists' forecasting system, we augmented our baseline VAR model by including lagged inflation, hours of work and consumption-output ratio.[18] These variables have been used in the VAR specifications considered by Rudd and Whelan (2005a) and Sbordone (2002). Table 6 reports estimation results from alternative VAR forecasting models for the output gap.

Table 6: Estimation results using alternative VAR for output gap

| | VAR specification | | | | |
|---|---|---|---|---|---|
| | $\begin{bmatrix} y_t \\ \Delta lsi_t \end{bmatrix}$ | $\begin{bmatrix} y_t \\ \Delta lsi_t \\ \pi_{t-1} \end{bmatrix}$ | $\begin{bmatrix} y_t \\ \Delta lsi_t \\ \pi_{t-1} \\ h_t \end{bmatrix}$ | $\begin{bmatrix} y_t \\ \Delta lsi_t \\ \pi_{t-1} \\ c_t/y_t \end{bmatrix}$ | $\begin{bmatrix} y_t \\ \Delta lsi_t \\ \pi_{t-1} \\ h_t \\ c_t/y_t \end{bmatrix}$ |
| $\beta$ | 4.685*** | 4.747*** | 4.746*** | 4.468*** | 4.848*** |
| | (1.341) | (1.234) | (1.357) | (1.337) | (1.374) |
| $\gamma$ | 0.004** | 0.003 | 0.001 | 0.0005** | 0.001 |
| | (0.001) | (0.002) | (0.001) | (0.0002) | (0.001) |
| $R^2$ from Inflation Equation | | | | | |
| | 0.774 | 0.770 | 0.769 | 0.767 | 0.770 |

*Notes*: $y_t \equiv$ output gap, $\Delta lsi_t \equiv$ labor share growth, $\pi_{t-1} \equiv$ (past) inflation, $h_t \equiv$ detrended hours of work, $c_t/y_t \equiv$ detrended consumption-output ratio. Optimal lag length ($l_i$) in VAR specifications ($i = 1...5$): $l_1 = l_2 = l_3 = l_4 = l_5 = 4$. Standard errors are computed using White's heteroskedasticity-consistent covariance matrix estimator (HCCME). *, **, *** denote significance at the 10%, 5%, and 1% level.

As shown in Table 6, point estimates of the coefficients $\beta$ and $\gamma$ do not change substantially, though the significance levels of $\gamma$ in some of the alternative specifications are slightly above 10%.

## 4.2 Robustness to alternative measures of marginal costs

Our benchmark model considers a traditional output gap measure, defined as the deviation of log real GDP from a quadratic trend, as the driving variable in the

[18] Hours are quadratically detrended total hours of work in the non-farm business sector, while consumption-output ratio is linearly detrended.

inflation process. Previous tests of sticky-price models under RE have reported that the NKPC provides a poor description of the actual inflation process when output gap is used as a proxy for real marginal costs (see, e.g., Fuhrer and Moore (1995) and Rudd and Whelan (2005a, 2006) among others). As an alternative to the standard approach, a number of researchers have suggested using the labor's share of income as driving variable in the NKPC. The motivation for this measure stems from the fact that the micro-foundations underpinning the NKPC imply that the correct driving variable for inflation is actually real marginal cost. Some theoretical restrictions are then required in order for real marginal costs to move with the output gap. Using average unit labor costs (nominal compensation divided by real output) as a proxy for nominal marginal cost results in the labor share of income (nominal compensation divided by nominal output) as a proxy for real marginal cost. Even though empirical implementations of this variant of the NKPC generated mixed evidence,[19] we estimate, as a second robustness exercise, the evolutionary switching model using the (log of) labor's share of income as driving variable. As noted in Section 3, the labor share process presents a unit root when considered over the full sample 1960:Q1-2010:Q4. In order to avoid spurious correlations and facilitate comparison with earlier works, we restrict the estimation sample to 1960:Q1-2001:Q4.[20] The estimation results reported in Table 7 show that the estimated coefficients have the correct sign and are mostly significant. Moreover, the point estimates are of the same order of magnitude as in the output gap VAR specification.

Overall, the results presented in this section suggest that our analysis is robust to different VAR forecasting models for the driving variable in the inflation process. Moreover, using the labor share of income as an alternative measure of real marginal costs, does not significantly alter the main results.

[19] Galí and Gertler (1999), Woodford (2001), Sbordone (2002) and others report that predicted inflation series based on labor share fit actual inflation well, while Rudd and Whelan (2005a,b, 2006) and others show that even the labor share version of the model provides a poor description of the inflation process.

[20] Standard unit root tests motivate this choice.

Table 7: Estimation results using alternative VAR for labor share of income

| | VAR specification | | | | |
|---|---|---|---|---|---|
| | $\begin{bmatrix} lsi_t \\ y_t \end{bmatrix}$ | $\begin{bmatrix} lsi_t \\ y_t \\ \pi_{t-1} \end{bmatrix}$ | $\begin{bmatrix} lsi_t \\ y_t \\ \pi_{t-1} \\ h_t \end{bmatrix}$ | $\begin{bmatrix} lsi_t \\ y_t \\ \pi_{t-1} \\ c_t/y_t \end{bmatrix}$ | $\begin{bmatrix} lsi_t \\ y_t \\ \pi_{t-1} \\ h_t \\ c_t/y_t \end{bmatrix}$ |
| $\beta$ | $4.830^{***}$ | $4.673^{***}$ | $4.729^{***}$ | $4.950^{***}$ | $4.775^{***}$ |
| | (1.635) | (1.542) | (1.507) | (1.695) | (1.570) |
| $\gamma$ | $0.001^{**}$ | 0.0002 | $0.001^{***}$ | $0.0002^{*}$ | 0.001 |
| | (0.0003) | (0.0003) | (0.003) | (0.0001) | (0.001) |
| $R^2$ from Inflation Equation | | | | | |
| | 0.803 | 0.800 | 0.809 | 0.799 | 0.803 |

*Notes*: $lsi_t \equiv$ labor share of income, $y_t \equiv$ output gap, $\pi_{t-1} \equiv$ (past) inflation, $h_t \equiv$ detrended hours of work, $c_t/y_t \equiv$ detrended consumption-output ratio. Optimal lag length ($l_i$) in VAR specifications ($i = 1...5$): $l_1 = l_3 = l_5 = 2$ and $l_2 = l_4 = 4$. Standard errors are computed using White's heteroskedasticity-consistent covariance matrix estimator (HCCME). *, **, *** denote significance at the 10%, 5%, and 1% level.

# 5 Conclusions

Over the past decade it has become relatively well accepted that the purely forward-looking NKPC cannot account for the degree of inflation inertia observed in the data. In response, the profession has increasingly adopted hybrid models in which lagged inflation is allowed to have an explicit role in pricing behavior. This reformulation of the basic sticky-price model has recently provoked a heated debate as to the extent of forward- versus backward-looking behavior, with little consensus after years of investigation. Most of the empirical studies on the topic take the distribution of heterogeneous pricing behavior as fixed and exogenously given. Recent works on structural stability in short-run inflation dynamics in the U.S. have provided statistical evidence of multiple structural breaks in the relative weights of forward- and backward-looking firms. Moreover, empirical studies based on survey data as well as experimental data, provided evidence that the proportions of heterogeneous forecasters evolve over time as a reaction to past forecast errors. In the light of this empirical evidence, we have proposed a model of monopolistic price setting with nominal rigidities and endogenous evolutionary switching be-

tween different forecasting strategies according to their relative past performances. Importantly, in light of the recent criticisms to model-consistent RE in the NKPC on both theoretical and empirical grounds, heterogeneous firms in our model hold optimizing behavior given their *subjective* expectations of future inflation. In our stylized framework, fundamentalist firms believe in a present-value relationship between inflation and real marginal costs, as predicted by standard RE models, while naive firms use a simple rule of thumb to forecast future inflation. Although with a different behavioral interpretation, our measure of fundamental expectation is similar to the measure of forward-looking expectations in commonly estimated RE models, while the expectations of random walk believers account for the lagged value of inflation in the hybrid specification of the NKPC. The difference with traditional tests of sticky-price models arises from the introduction of time-varying weights and endogenous switching dynamics.

We estimated our behavioral model of inflation dynamics on quarterly U.S. data from 1960:Q1 to 2010:Q4. Our estimation results show statistically significant behavioral heterogeneity and substantial time variation in the weights of forward- and backward-looking price setters. The data gave considerable support for the parameter restrictions implied by our theory. In particular, the intensity of choice was found to be positive, indicating that agents switch towards the better performing rule according to its past performance, and inflation was positively affected by real marginal costs. These results were found to be independent from whether detrended output or the labor share of income were used as a measure of real marginal costs.

Our results suggest that large shocks, such as the oil crises, pushed inflation away from the fundamental, triggering therefore a large share of agents to adopt random walk beliefs. Moreover, during the Great Moderation, forecasts obtained in accordance to a Phillips curve relationship between inflation and real activity, did not perform better than simple univariate models.

Our findings have important monetary policy implications. Recent papers have

shown that multiple equilibria and complex dynamics can arise in New Keynesian models under dynamic predictor selection, even if the model under RE has a unique stationary solution. Given the statistical evidence found in our empirical results for heterogeneous expectations and evolutionary switching, determinacy under RE may not be a robust recommendation and monetary policy should be designed to account for potentially destabilizing heterogeneous expectations.

# Appendix

## A Data sources

Below we describe the data sources and the data definitions used in the paper.

*Inflation* is constructed using the quarterly Price Indexes for GDP from the March 2011 release of the NIPA Table 1.1.4, 1960:Q1 - 2010:Q4, which can be downloaded at http://www.bea.gov/national/nipaweb/SelectTable.asp.

*Output gap* is constructed using the quarterly real GDP from the March 2011 release of the NIPA Table 1.1.3, 1960:Q1 - 2010:Q4, which can be downloaded at http://www.bea.gov/national/nipaweb/SelectTable.asp. To construct our measure of the output gap we take logs and quadratically detrend.

*Unit labor costs* are constructed using the Bureau of Labor Statistics quarterly Unit Labor Costs series PRS85006113, 1960:Q1 - 2010:Q4, for the nonfarm business sector. The series can be downloaded at http://data.bls.gov/, under the heading Major Sector Productivity and Costs Index.

*Labor share of income* is constructed using the Bureau of Labor Statistics quarterly Labor Share Income series PRS85006173, 1960:Q1 - 2010:Q4, for the nonfarm business sector. The series can be downloaded at http://data.bls.gov/, under the heading Major Sector Productivity and Costs Index.

*Hours of work* are constructed using the Bureau of Labor Statistics quarterly Hours series PRS85006033, 1960:Q1 - 2010:Q4, for the nonfarm business sector. The series can be downloaded at http://data.bls.gov/, under the heading Major Sector Productivity and Costs Index. To construct our measure of the hours of work we take logs and quadratically detrend.

*Consumption-output ratio* is constructed using the quarterly real GDP from the March 2011 release of the NIPA Table 1.1.3, 1960:Q1 - 2010:Q4, which can be downloaded at http://www.bea.gov/national/nipaweb/SelectTable.asp. To construct our measure of the consumption-output ratio we take logs and linearly detrend.

# B Diagnostic checks

Here we report diagnostic checks on the residuals of the NLS estimation of model (2.11). Fig. 3 reports the time series of the residuals.

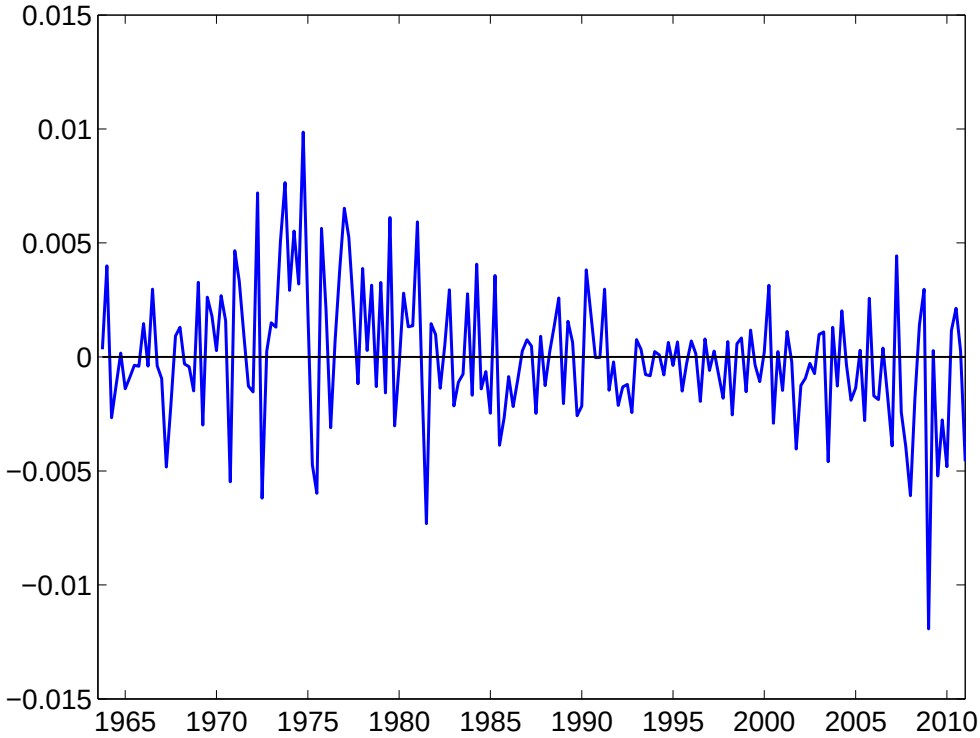


Figure 3: Time series of residuals $u_t$

The results of the Breusch-Pagan-Godfrey test, reported in Table 8, reveal the presence of heteroskedasticity. Standard errors are thus computed using White's heteroskedasticity-consistent covariance matrix estimator (HCCME).

Table 8: Heteroskedasticity test

| $H_0$: homoskedasticity | | | |
|---|---|---|---|
| $F$-statistic | 2.664 | Prob. | 0.003 |
| Obs*$R^2$ | 29.06 | Prob. | 0.004 |

Given the presence of heteroskedasticity, we perform the heteroskedasticity-robust $F$ test for serial autocorrelation proposed by Davidson and MacKinnon (2009), p.284. The

results reported in Table 9 show the absence of serial correlation in the residuals up to lag $p = 20$.

Table 9: Serial correlation test

| $H_0$: no serial correlation | | | |
|---|---|---|---|
| test-statistic ($p = 20$) | 19.82 | Prob. | 0.469 |

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