Inequality and Growth: What are the Tradeoffs?

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Background and Overview

- Growth and inequality relationship dates back to Kuznets (AER, 1955)

    --Inverted-U between inequality and development
    --Controversial and inconclusive
    --Kuznets originally associated it with "dual economy dynamics"
    --transition from agricultural to industrial economy

- Subsequent empirical research focused on the running regressions between income inequality and growth.

- Anand and Kanbur (JDE, 1993), Alesina and Rodrik (QJE, 1994), Persson and Tabellini (AER, 1994), Perotti (JEG, 1996), and others obtain **negative** relationship. The various explanations for this include:

    -- Political economy consequences of inequality
    -- Negative impact of inequality on education
    -- Capital market imperfections and credit constraints, etc.

- Other studies find **positive**, or more ambiguous, relationship; see e.g. Li and Zou (RDE, 1998), Forbes (AER, 2000), and Barro (JEG, 2000).

    Explanations include

    -- Relative savings propensities of rich vs. poor
    -- Investment indivisibilities
    -- Incentives.

- From theoretical perspective, empirical controversy unsurprising.
- Growth rate and income distribution are both endogenous outcomes.
- Income inequality-growth relationship will reflect the underlying set of forces to which both are simultaneously reacting.
- Need to examine within context of a consistently specified general equilibrium growth model.
- “Association” rather than “causality” is more appropriate characterization.

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- Key element is heterogeneity
- Many sources (tastes, endowments, technology, progressive taxes….)
- In most general context to solve for growth and inequality simultaneously is intractable (see Sorger, *ET*, 2000).
- Under certain assumptions, including homogeneity of utility function, then aggregation is possible (Gorman, *Econometrica*,1953)
- Assumption of homogeneity is routine throughout modern growth theory
- Leads to "representative agent theory of distribution" (Caselli - Ventura, 2000).
- Macroeconomic equilibrium has a simple recursive structure:
  --Aggregate equilibrium is independent of distribution,
  Distribution depends upon aggregate.

- My work in this area has followed this approach

- Source of heterogeneity typically initial endowments of capital (assets) and in some cases ability

- Extensive literature taking this approach (Piketty, Stiglitz, most prominent).

- Alternative approach: assume people start out identical and that heterogeneity is endogenized through idiosyncratic random shocks (Krusell and Smith, JPE 1998, and others).

- Fall into three categories, structurally

   1. Economies always on balanced growth path (One-sector endogenous growth model)

      --No dynamics either in aggregates or distributions.

   2. Models with transitional dynamics (e.g. neoclassical growth models; two-state variable endogenous growth model)

      --Transitional dynamics of aggregates and distribution, but no mobility (inequality may expand or contract but no changing in rank)

      --Introduces **intertemporal** as well as **intratemporal** tradeoffs.

   3. Two sources of heterogeneity in endowments (e.g. capital and skills)

      --Permits both wealth and income inequality dynamics and mobility. These need not move together.

Applications

- Effects of structural changes (e.g. productivity increases)
- Tax and expenditure policies (e.g. government investment vs. government consumption and financing)
- Extensions to international economy (e.g. distributional consequences of international transfers, tariffs)
- Path dependence of structural changes and their distributional consequences (particularly relevant for issues relating to foreign aid)
- Human capital vs. physical capital
- Heterogeneity vs. demographic distribution

Remainder of Talk

- Focus on two models dealing with interaction between public expenditure, growth, and inequality

    1. Based on “Representative agent theory”

    2. Idiosyncratic shocks model

Public Investment, Growth, Inequality

- Several emerging-market countries e.g. India, China, Brazil have undertaken extensive public investment, contributed to recent high growth rates.
- Several European countries have reduced public spending as austerity measure.
- Contemporaneously with these diverse policies, we have witnessed rising income inequality, both in emerging markets and in most OECD countries.
- Raises the important question: Relationship between public investment directed toward growth enhancement and its consequences for inequality.

- Interest in link between public investment, output/growth dates back to Arrow and Kurz (1970).
- Since Barro (1990), an extensive literature has evolved addressing issue in an endogenous growth framework Futagami et al. (1993), Glomm and Ravikumar (1994), Turnovsky (1997) and more recently Agénor (2011).
- Empirical literature focuses on estimating the productive elasticity of government expenditure in output.
- Consensus is that infrastructure contributes positively and significantly to output; Bom and Ligthart (2014) provide an exhaustive review of the literature and place the elasticity at between 0.10 and 0.20.

- By impacting relative factor returns, public investment is critical in the evolution of wealth and income distributions.
- Diverse nature infrastructure $\Rightarrow$ significant redistributive consequences.
- Empirical evidence on the relationship between infrastructure investment and inequality is less definitive and more anecdotal.
    1. Several studies find that public investment has both promoted growth and helped mitigate inequality.
    2. Other studies find that government spending on infrastructure has increased regional disparities.
- Diversity of empirical findings emphasizes need for a well-specified analytical framework, within which the interaction between infrastructure spending, economic growth, and inequality can be systematically addressed.
- Present two alternative models offering different perspectives and conclusions.

Model of Public Capital, Growth, Distribution

Firms

- Identical production condition.

$$Y_j = A\left[\alpha\left(X_P L_j\right)^{-\rho} + (1-\alpha)K_j^{-\rho}\right]^{-1/\rho} \tag{1a}$$

$$X_P = K^{\varepsilon} K_G^{1-\varepsilon}, \tag{1b}$$

Letting $z \equiv K_G/K$, average product of aggregate private capital $y \equiv Y/K$ is:

$$y \equiv y(z,l) = A\left[1-\alpha+\alpha\left\{(1-l)z^{1-\varepsilon}\right\}^{-\rho}\right]^{-1/\rho} \tag{2}$$

- the economy-wide returns to capital and labor are

$$r = r(z,l) \equiv (1-\alpha)A^{-\rho}y(z,l)^{1+\rho} \tag{3a}$$

$$w = \omega(z,l)K;\ \ \omega(z,l) \equiv \alpha A^{-\rho}y(z,l)^{1+\rho}z^{-\rho(1-\varepsilon)}(1-l)^{-(1+\rho)} \tag{3b}$$

Consumers

- Identical consumers except for initial endowments of capital, K_{i0}.

  --Agent i's share of total capital stock is $k_i \equiv K_i / K$.
  --Initial distribution of capital $G_0(k_i)$, mean $\sum_i k_i = 1$; coeff. of variation σ_k.

- Agents allocate unit of time between leisure, l_i and work, $1 - l_i \equiv L_i$.

- The agent's decision problem: choose C_i, l_i, $\dot{K}_i$ to

$$\max U_i = \int_0^\infty \frac{1}{\gamma}\left[C_i^{-\upsilon} + \theta\left(X_U l_i\right)^{-\upsilon}\right]^{-\gamma/\upsilon} e^{-\beta t} dt \tag{4a}$$

$$X_U = K^{\varphi} K_G^{1-\varphi}, \tag{4b}$$

subject to initial endowment of capital, $K_{i,0}$, and flow budget constraint

$$\dot{K}_i = \left(1 - \tau_k\right) r K_i + \left(1 - \tau_w\right) w (1 - l_i) - \left(1 + \tau_c\right) C_i - T \tag{5}$$

- Optimality conditions

$$\left[C_i^{-\upsilon}+\theta\left(X_U l_i\right)^{-\upsilon}\right]^{-\frac{\gamma}{\upsilon}-1} C_i^{-\upsilon-1}=\lambda_i\left(1+\tau_c\right) \tag{6a}$$

$$\theta X_U^{-\upsilon}\left[C_i^{-\upsilon}+\theta\left(X_U l_i\right)^{-\upsilon}\right]^{-\frac{\gamma}{\upsilon}-1} l_i^{-\upsilon-1}=\lambda_i\left(1-\tau_w\right)\omega(z,l) K \tag{6b}$$

$$\left(1-\tau_k\right) r(z,l)=\beta-\frac{\dot{\lambda}_i}{\lambda_i} \tag{6c}$$

together with the transversality condition

$$\lim_{t\to\infty} \lambda_i K_i e^{-\beta t}=0 \tag{6d}$$

Dividing (6b) by (6a), yields

$$\frac{C_i}{l_i}=\Omega(z,l) K \equiv\left[\left(1-\tau_w\right)\omega(z,l) z^{\upsilon(1-\varphi)} \big/ \theta\left(1+\tau_c\right)\right]^{1/(1+\upsilon)} K \tag{7}$$

Individual’s rate of capital accumulation, (5), is

$$\frac{\dot{K}_i}{K_i}=(1-\tau_k) r(z,l)+\left[(1-\tau_w)\omega(z,l)(1-l_i)-(1+\tau_c)\Omega(z,l) l_i-\frac{T}{K}\right]\frac{K}{K_i} \tag{8}$$

Government

- Stock of public capital, (non-rival and non-excludable), evolves as

$$\dot{K}_G = G = gY,\ 0 < g < 1 \tag{9}$$

- Government budget constraint:

$$G = \tau_k rK + \tau_w w(1-l) + \tau_c C + T \tag{10}$$

- Dividing (10) by *K*:

$$gy(z,l) = \tau_k r(z,l) + \tau_w \omega(z,l)(1-l) + \tau_c \Omega(z,l)l + \tau y(z,l) \tag{10'}$$

Macroeconomic equilibrium

- Economy-wide *average* of a variable, X_i: $\left(1/N\right)\sum_i^N X_i \equiv X$.
- Homogeneity of utility function and perfect factor markets $\Rightarrow$ all individuals choose the same growth rates for consumption and leisure

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C}; \frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l} \qquad \text{for each } i \tag{11}$$

- System can be aggregated perfectly over agents.
- Each individual will choose different *levels* of consumption and leisure:

$$l_i = \pi_i l \quad \text{and} \quad \frac{1}{N}\sum_i^N \pi_i = 1 \tag{12}$$

- Aggregate consumption-capital ratio

$$\frac{C}{K} = \Omega(z,l)l \qquad (7')$$

- Growth rate of aggregate private capital.

$$\frac{\dot{K}}{K} = (1-\tau_k)r(z,l) + [(1-\tau_w)\omega(z,l)(1-l) - (1+\tau_c)\Omega(z,l)l - \tau y(z,l) \qquad (13)$$

- Combining with (10') yields aggregate goods market clearing condition

$$\frac{\dot{K}}{K} = (1-g)y(z,l) - \Omega(z,l)l \qquad (13')$$

- Long-run equilibrium of this economy is a balanced growth path along which all aggregate variables grow at a common rate and average leisure is constant.

- The transitional dynamics of the aggregate economy are driven by z and l.

$$\frac{\dot{z}}{z} = g\frac{y(z,l)}{z} - \left[(1-g)y(z,l) - \Omega(z,l)l\right] \tag{14a}$$

$$\frac{\dot{l}}{l} = \frac{H(z,l)}{J(z,l)} \tag{14b}$$

$$H(z,l) \equiv (1-\tau_k)r(z,l) - \beta - (1-\gamma)\frac{\dot{K}}{K}$$

$$+\left\{\frac{\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}(\upsilon+\gamma)(1-\varphi)}{1+\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}} - \left[\frac{(1-\gamma)+(1+\upsilon)\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}}{1+\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}}\right]\frac{\Omega_z z}{\Omega}\right\}\frac{\dot{z}}{z},$$

$$J(z,l) \equiv 1-\gamma+\left[\frac{(1-\gamma)+(1+\upsilon)\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}}{1+\theta\left[z^{-(1-\varphi)}\Omega\right]^{v}}\right]\frac{\Omega_l l}{\Omega}$$

Steady State and Aggregate Dynamics

- Assuming stability, aggregate economy converges to a balanced growth path:

$$\frac{\left(1-\tau_k\right)r\left(\tilde{z},\tilde{l}\right)-\beta}{1-\gamma}=g\frac{y\left(\tilde{z},\tilde{l}\right)}{\tilde{z}}=(1-g)y\left(\tilde{z},\tilde{l}\right)-\Omega\left(\tilde{z},\tilde{l}\right)\tilde{l}\equiv\tilde{\psi} \qquad (15)$$

- Determine $\tilde{z}$ and $\tilde{l}$, such that consumption, public and private capital grow at $\tilde{\psi}$

- Given $\tilde{z}$ and $\tilde{l}$, (7') determines the steady-state $\tilde{c}\equiv\frac{\tilde{C}}{\tilde{K}}$ ratio

$$\tilde{c}=\Omega(\tilde{z},\tilde{l})\tilde{l} \qquad (13a')$$

- Transversality condition implies $\tilde{\psi}<\tilde{r}(1-\tau_k)$, and hence

$$\tilde{c}>\frac{(1-\tau_w)\omega(\tilde{z},\tilde{l})(1-\tilde{l})-\tau y(\tilde{z},\tilde{l})}{1+\tau_c} \qquad (16)$$

- For sustainable long-run growth some (net) capital income is allocated to consumption.

- Linearizing (14), yields a local approximation to aggregate dynamics:

$$\begin{bmatrix} \dot{z} \\ \dot{l} \end{bmatrix} = \begin{bmatrix} \overset{(-)}{a_{11}} & \overset{(+)}{a_{12}} \\ \overset{(+)}{a_{21}} & \overset{(+)}{a_{22}} \end{bmatrix} \begin{bmatrix} z - \tilde{z} \\ l - \tilde{l} \end{bmatrix} \tag{18}$$

- Stable transition path of the aggregate economy can be described by

$$z(t) = \tilde{z} + \left[z(0) - \tilde{z} \right] e^{\mu t} \tag{19a}$$

$$l(t) = \tilde{l} + \frac{a_{21}}{(\mu - a_{22})} \left[z(t) - \tilde{z} \right] \tag{19b}$$

- Simulations always yields negatively sloped saddle path.
- Increase in public to private capital raises productivity of private capital, raising wage rate and inducing agents to increase labor supply and to reduce leisure.
- Consumption-private capital ratio evolves according to

$$c(t) - \tilde{c} = \left[\Omega_z\left(\tilde{z}, \tilde{l}\right)\tilde{l} + \left\{ \Omega_l\left(\tilde{z}, \tilde{l}\right)\tilde{l} + \Omega\left(\tilde{z}, \tilde{l}\right) \right\} \left(\frac{a_{21}}{\mu - a_{22}} \right) \right] \left[z(t) - \tilde{z} \right] \tag{19c}$$

Distributional Dynamics

- Aggregate dynamics are independent of distributional aspects is a consequence of the homogeneity of utility function and the perfect aggregation that this permits.
- Next step: Characterize the behavior of a cross-section of agents, and to determine the evolution of that cross-section relative to that of the average.

Distribution of Private Capital (Wealth)

- To derive dynamics of relative capital stock of individual i, $k_i \equiv K_i/K$ (relative wealth) combine (8) and (13). Define:

$$\Delta(z,l) \equiv (1-\tau_w)\omega(z,l) - \tau y(\tilde{z},l); \;\; \Gamma(z,l) \equiv \left[(1+\tau_c)\Omega(z,l) + (1-\tau_w)\omega(z,l)\right] > 0$$

- Evolution of relative wealth (capital):

$$\dot{k}_i(t) = -\Gamma(z,l)(l_i - l) + \left[\Gamma(z,l)l - \Delta(z,l)\right](k_i(t) - 1) \tag{20}$$

- Viability condition (16) can be expressed as $\Gamma(\tilde{z},\tilde{l})\tilde{l} > \Delta(\tilde{z},\tilde{l})$, implying that the dynamic equation (20) is locally unstable near the steady state.

- A key element of a stable (bounded) solution includes steady-state to (20).

$$\tilde{l}_i - \tilde{l} = \left[\tilde{l} - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})} \right] (\tilde{k}_i - 1) \tag{21}$$

- Transversality condition implies that an individual who in the long run has above-average private capital, $\tilde{k}_i - 1$, enjoys above-average leisure, $\tilde{l}_i - \tilde{l} > 0$.

- This equation also yields agent *i's* (constant) allocation of leisure time:

$$\pi_i - 1 = \left(1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}} \right) (\tilde{k}_i - 1) \tag{22}$$

Again, using (12) and substituting (22) into (20), yields

$$\dot{k}_i = -\Gamma(z,l)l \left[1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}} \right] (\tilde{k}_i - 1) + \left[\Gamma(z,l)l - \Delta(z,l) \right] (k_i - 1) \tag{20'}$$

- Linearizing (20') around steady-state levels $\tilde{z}$, $\tilde{l}$, and $\tilde{k}_i$, yields the following equation of motion for relative wealth

$$\dot{k}_i = \delta_1(\tilde{z},\tilde{l})(\tilde{k}_i - 1)\left[z(t) - \tilde{z}\right] + \delta_2(\tilde{z},\tilde{l})\left[k_i(t) - \tilde{k}_i\right] \tag{23}$$

$$B_1(\tilde{z},\tilde{l}) \equiv \frac{1}{\Gamma(\tilde{z},\tilde{l})}\left(\Delta(\tilde{z},\tilde{l})\Gamma_z(\tilde{z},\tilde{l}) - \Delta_z(\tilde{z},\tilde{l})\Gamma(\tilde{z},\tilde{l})\right), \quad B_2(\tilde{z},\tilde{l}) \equiv \left(\frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\left(\Gamma(\tilde{z},\tilde{l}) + \Gamma_l(\tilde{z},\tilde{l})\tilde{l}\right) - \Delta_l(\tilde{z},\tilde{l})\right)$$

$$\delta_1(\tilde{z},\tilde{l}) \equiv B_1(\tilde{z},\tilde{l}) + B_2(\tilde{z},\tilde{l})\left(\frac{a_{21}}{\mu - a_{22}}\right), \quad \delta_2(\tilde{z},\tilde{l}) \equiv \Gamma(\tilde{z},\tilde{l})\tilde{l} - \Delta(\tilde{z},\tilde{l}) > 0$$

- Evolution of economy-wide ratio of public to private capital impacts the evolution of relative wealth, both directly, and indirectly through $l(t)$.

- $\delta_2 > 0 \Rightarrow$ the bounded solution to (23) is of the form

$$k_i(t) - 1 = (\tilde{k}_i - 1)\left[1 + \frac{\delta_1}{\mu - \delta_2}\left[z(t) - \tilde{z}\right]\right] = (\tilde{k}_i - 1)\left[1 + \frac{\delta_1}{\mu - \delta_2}\left(z_0 - \tilde{z}\right)e^{\mu t}\right] \tag{24}$$

Setting $t = 0$ in (24) gives

$$k_i(0) - 1 \equiv k_{i,0} - 1 = (\tilde{k}_i - 1)\left[1 + \frac{\delta_1}{\mu - \delta_2}\left(z_0 - \tilde{z}\right)\right] \tag{24'}$$

- Evolution of agent i's relative capital stock is determined as follows.
- Given (i) steady-state of the aggregate economy, and (ii) agent's initial endowment, $k_{i,0}$, (24') determines steady-state relative stock of capital, $(\tilde{k}_i - 1)$, which together with (24) then yields entire time path, $k_i(t)$.
- Since all distributional variables are expressed relative to the mean, we can measure their dispersion in canonical form, by using the coefficient of variation.
- Given linearity of (24), (24') in terms involving k_i, we can transform these equations into relationships for coefficient of variation for distribution of capital.

$$\sigma_k(t) = \left[1 + \frac{\delta_1}{\mu - \delta_2}\left(z(t) - \tilde{z}\right)\right]\tilde{\sigma}_k, \tag{25a}$$

$$\tilde{\sigma}_k = \left[1 + \frac{\delta_1}{\mu - \delta_2}\left(z_0 - \tilde{z}\right)\right]^{-1}\sigma_{k,0} \tag{25b}$$

- (25a) and (25b) completely characterize the evolution of wealth inequality, given initial distribution of $\sigma_{k,0}$ and aggregate ratio $\tilde{z}_0$.

Labor Income Inequality

- No heterogeneity in labor skills $\Rightarrow$ labor income inequality is driven by the distribution of hours worked by households.

- Wage income for i-th household relative to the mean wage income is

$$w_i(t) - 1 = \frac{l(t) - l_i(t)}{1 - l(t)} = \frac{l(t)}{1 - l(t)}(1 - \pi_i) \tag{26}$$

- Evolution of inequality in labor income is

$$\sigma_w(t) = \frac{l(t)}{1 - l(t)}\left(1 - \frac{\Delta(\tilde{z}, \tilde{l})}{\Gamma(\tilde{z}, \tilde{l})\tilde{l}}\right)\tilde{\sigma}_k \tag{26'}$$

- Labor income inequality is determined by (i) the evolution of the average labor-leisure choice and (ii) the steady-state distribution of wealth.

Distribution of Pre-tax Income

- Gross income of individual i: $Y_i = r(z,l)K_i + \omega(z,l)(1-l_i)K$
  Average income: $Y = \left[r(z,l) + \omega(z,l)(1-l)\right]K$.

- Relative pre-tax income of agent i:

$$y_i(t) - 1 = s_k(t)(k_i(t)-1) - \left[1 - s_k(t)\right]\frac{l(t)}{1-l(t)}\left[1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right]\left(\tilde{k}_i - 1\right) \qquad (27)$$

$$s_k(t) \equiv r(z,l)/y(z,l) = (1-\alpha)[Ay(z,l)]^{-\rho}.$$

- Distribution of pre-tax income:

$$\sigma_y(t) = \zeta(t)\sigma_k(t) \qquad (28a)$$

where $$\zeta(t) \equiv s_k(t) - \left[1 - s_k(t)\right]\frac{l(t)}{1-l(t)}\left[1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right]\left[1 + \frac{\delta_1}{\mu - \delta_2}\left(z(t) - \tilde{z}\right)\right]^{-1} \qquad (28b)$$

  - Wealth inequality, $\sigma_k(t)$, evolves gradually, the initial jump in $l(0)$, which impacts on $\sigma_y(0)$ through its effect on labor income inequality, means that any shock, causes an initial jump in $\sigma_y(t)$, after which it too evolves continuously.

Distribution of Post-tax Income

- After-tax relative income

$$y_i^N(t) = \frac{(1-\tau_k)rK_i + (1-\tau_w)wK(1 - l_i(t))}{(1-\tau_k)rK + (1-\tau_w)wK(1 - l)} \qquad (29)$$

- After-tax income inequality can be written as

$$\sigma_y^N(t) = \left[\zeta(t) + \frac{(\tau_w - \tau_k)s_k(t)}{(1-\tau_k)r + (1-\tau_w)w(1 - l)}(1-\zeta(t))\right]\sigma_k(t) \qquad (29a)$$

- After-tax income distribution will be more (less) equal than the before-tax income distribution, according to whether $\tau_k > (<)\tau_w$.
- Income tax rates, τ_k and τ_w, exert two effects on the after-tax income inequality.
    1. By influencing $\zeta(t)$, they influence *gross* factor returns, and therefore the before-tax distribution of income.
    2. They have direct redistributive effects that are captured by $(\tau_w - \tau_k)$.

Distribution of Welfare

- Individual i and average economy-wide level of instantaneous welfare at time t:

$$W_i = \frac{1}{\gamma}\left[\Omega(z,l)^{-\upsilon} + \theta z^{\upsilon(\varphi-1)}\right]^{-\frac{\gamma}{\upsilon}}(l_i K)^{\gamma}; W = \frac{1}{\gamma}\left[\Omega(z,l)^{-\upsilon} + \theta z^{\upsilon(\varphi-1)}\right]^{-\frac{\gamma}{\upsilon}}(lK)^{\gamma} \quad (30)$$

- Hence along the equilibrium growth path

$$\frac{U_i}{U} = \frac{W_i}{W} = \left(\frac{l_i}{l}\right)^{\gamma} = \pi_i^{\gamma} \quad (31)$$

- Using (22) we can express relative welfare in the form

$$w_i \equiv \frac{U_i}{U} = \frac{W_i}{W} = \left(\frac{l_i}{l}\right)^{\gamma} = \left[1 + \left(1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right)(\tilde{k}_i - 1)\right]^{\gamma} \quad (32)$$

- Applying the monotonic transformation $(w_i)^{1/\gamma} \equiv u(v_i)$, we obtain an expression for the relative welfare of individual i expressed in terms of equivalent units of wealth. The dispersion of welfare across agents is then given by:

$$\sigma_u = \left(1 - \frac{\Delta(\tilde{z},\tilde{l})}{\Gamma(\tilde{z},\tilde{l})\tilde{l}}\right)\tilde{\sigma}_k \quad (33)$$

Path Dependence and Inequality

- Effect of policy change on dynamics of wealth and income inequality, depends critically upon initial response of leisure (labor supply) to shock.
- This initial response depends upon time path that change is expected to follow.
- Impacts of a change of a **given** magnitude on **long-run distributions** of both wealth and income depend upon the time path that change is assumed to follow.
- Sharp contrast to the aggregate economy. In that case, the path followed by the change affects only the transitional path of the aggregate economy.

- Important aspect

    --Model can admit a diverse set of distributional equilibria for countries having similar levels of aggregate development (per capita income), in accordance with empirical evidence.

    --Important in foreign aid which typically is granted over time

Modification to Model

- Macrodynamic equilibrium is described by:

$$\frac{\dot{z}}{z} = g(t)\frac{y(z,l)}{z} - \left[(1-g(t))y(z,l) - \Omega(z,l)l\right] \tag{34a}$$

$$\frac{\dot{l}}{l} = \frac{H(z,l)}{J(z,l)} \tag{34b}$$

$$\dot{g}(t) = \kappa\left(\tilde{g} - g(t)\right) \tag{34c}$$

- Contrast between how κ affects **aggregate** economy and its consequences for **distribution**.

- While it affects the transitional path of the aggregate economy, it has no effect on the aggregate steady state.

- But the choice of κ has profound impacts on **both** the time paths and the steady-state equilibria of wealth and income distributions.

Distributional dynamics

- Dynamic equation for the relative capital stock is now:

$$\dot{k}_i(t) = \left[\tilde{k}_i - 1\right]\left[B_1(\tilde{z},\tilde{l})(z(t)-\tilde{z}) + B_2(\tilde{z},\tilde{l})\left(\frac{a_{21}}{\mu - a_{22}}\right)(l(t)-\tilde{l})\right] + \delta_2(\tilde{z},\tilde{l})(k_i(t)-\tilde{k}_i)$$

- The bounded solution is

$$k_i(t) - 1 = \theta(t)(\tilde{k}_i - 1)$$

$$\theta(t) \equiv 1 - B_1(\tilde{z},\tilde{l})\int_t^{\infty}\left(z(\tau)-\tilde{z}\right)e^{-\delta_2(\tau - t)}\,d\tau$$

$$-B_2(\tilde{z},\tilde{l})\left(\frac{a_{21}}{\mu - a_{22}}\right)\int_t^{\infty}\left(l(\tau)-\tilde{l}\right)e^{-\delta_2(\tau - t)}\,d\tau$$

$$k_{i,0} - 1 = \theta(0)(\tilde{k}_i - 1)$$

- Long run depends upon time path of *l* and *z*.

Fiscal Policy, Growth, Inequality: Numerical Simulations

- Compare dynamic adjustment under four financing schemes: (i) lump-sum tax; (ii) capital income tax, (iii) labor income tax, (iv) consumption tax.
- Benchmark Specification of Structural Parameters

Preferences	γ = -1.5, β = 0.04, θ = 1.75, υ = 0
Production	A = 0.6, α = 0.6, ρ = 0
Externalities	$\varphi = \varepsilon = 0.6$
Fiscal	g = 0.05 $\tau = 0.05$

Increase in Government Spending on Infrastructure

- Unanticipated and permanent increase in g from 5% to 8% of GDP.
- Economy starts from initial benchmark equilibrium in which government expenditure is fully financed by lump-sum taxes.
- For the distortionary taxes, the corresponding tax rate is set such that it fully finances the long-run *change* in government expenditure.
- During transition fiscal imbalance financed by lump-sum taxes.

TABLE 1

A. Benchmark Steady-State Equilibrium: $g = 0.05$

$\varepsilon = \varphi = 0.6$ (composite externality), $\rho = \upsilon = 0$

Financing Policy	$\tilde{z}$	$\tilde{l}$	$\tilde{y}$	$\tilde{\psi}$ (%)
Lump-sum tax-financing $\tau = 0.05$	0.531	0.714	0.243	2.29

B. Increase in Government Spending:Benchmark Specification

$g = 0.05$ to 0.08

i. Steady-State Aggregate Effects

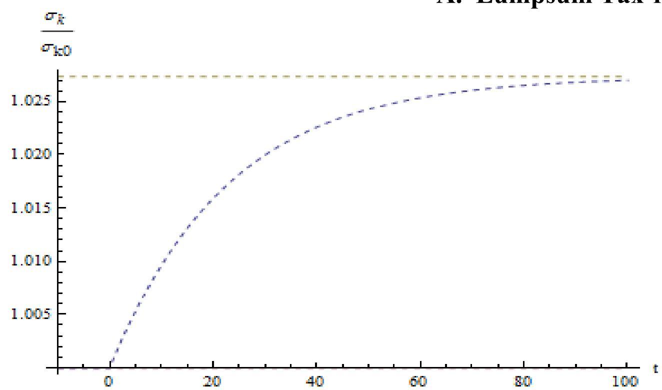
Policy Change	$d\tilde{z}$	$d\tilde{l}$	$d\tilde{\psi}$
Lump-sum tax-financed increase in g $d\tau = 0.030$	0.259	-0.01	0.206
Capital income tax-financed increase in g $d\tau_k = 0.075$	0.353	-0.006	0.101
Labor income tax-financed increase in g $d\tau_w = 0.05$	0.268	0.002	0.168
Consumption tax-financed increase in g $d\tau_c = 0.096$	0.265	-0.001	0.179

ii. Distributional Effects

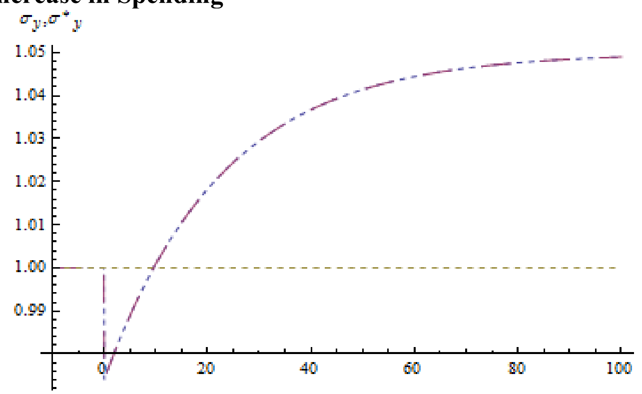
Policy Change	Wealth Inequality		Pre-tax Income Inequality		Post-tax Income Inequality	
	Short-run	**Long-run**	**Short-run**	**Long-run**	**Short-run**	**Long-run**
Lump-sum tax-financed increase in g	0	2.736	- 2.602	4.996	-2.602	4.996
Capital income tax-financed increase in g	0	3.527	2.707	11.976	-9.174	-0.149
Labor income tax-financed increase in g	0	2.805	-8.256	-0.331	-0.110	7.933
Consumption tax-financed increase in g	0	2.952	-3.117	4.955	-3.117	4.955

Figure 1. Increase in Government Spending, g = 0.05 to 0.08
Benchmark Case: $s = 1$, $q = 1$, φ = ε= 0.6

A. Lumpsum Tax-financed Increase in Spending

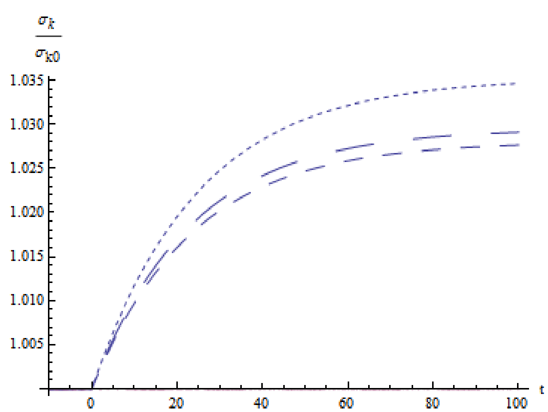


i. Wealth Inequality

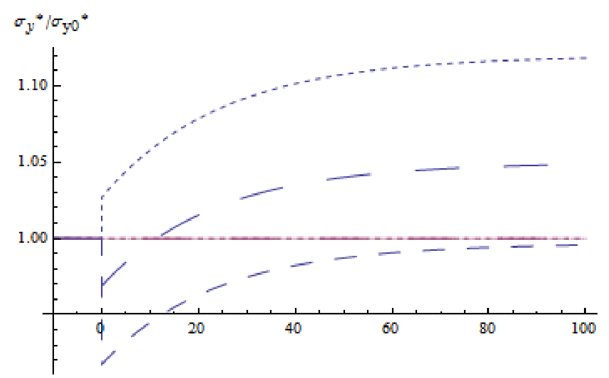


ii. Pre- and Post-tax Income Inequality

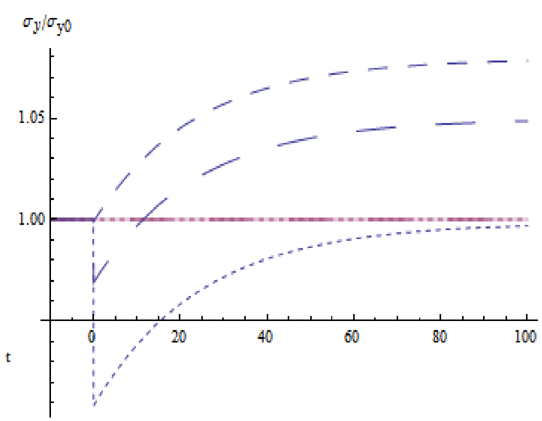
B. Distortionary Tax-financed Increase in Spending



i. Wealth Inequality



ii. Pre-tax Income Inequality



iii. Post-tax Income Inequality

………………. Capital income tax-financed _ _ _ _ _ _ _ **Labor income tax-financed** __ __ __ __ __ **Consumption tax-financed**

Aggregate Effects

- Direct stimulus to public investment increases equilibrium ratio $\tilde{z}$.
- Except when g is financed by tax on labor income, leisure falls in long run; higher spending raises marginal product of labor via production externality.
- When g is financed by a tax on labor income, time allocated to leisure increases, as higher tax rate reduces the after-tax return on labor.
- In all cases the effects are small.
- For all forms of financing, productive benefits of public capital spending and the consequent private capital accumulation ensure that equilibrium growth rate increases.
- Overall, differential impacts on growth, leisure, and ratio of public to private capital reflect the varying degrees of distortions associated with the different tax rates.

Distributional Effects

- **Lump-sum tax financing**: isolates pure effect of a government spending increase on the distributional measures.
- Wealth inequality changes only gradually, increasing by 2.7% in long run.
- Pre-tax and post-tax distributions of income are identical.
- Sharp intertemporal trade-off for income distribution.

  --In short run income inequality declines by 2.6%;

  --Over time decline reversed; long run income inequality increases by 5%,
- Increase in long-run income inequality exceeds increase in wealth inequality $\Rightarrow$ labor income inequality also increases in the long-run.

- **Dynamics:** During transition, increasing public capital raises marginal product of private capital, $\Rightarrow$ private capital accumulation.
- Private capital is unequally distributed $\Rightarrow$ capital-rich agents derive larger increase in income from capital investment than do capital-poor.
- Wealth inequality therefore increases during transition.
- By raising the expected long-run return to capital and labor, the higher government spending has a productivity impact on labor supply, causing the real wage to rise and labor supply to increase.
- In short run, since capital-poor agents supply more labor relative to capital-rich, their higher wage income compresses dispersion of labor supply, leading to an instantaneous decline in income inequality on impact.
- During transition, this trend is reversed due to gradual accumulation of wealth.

- **Distortionary tax financing:** Impacts depend on interaction between two counter-acting effects along the transitional path.

  1. Public spending tends to increase productivity of both capital and labor, affecting labor-leisure choice, and raising average factor incomes.

  2. Each distortionary tax permanently reduces after-tax return on the variable it impinges on, $\Rightarrow$ a dampening effect on productivity and the labor-leisure allocation decision.

- Response of pre-tax and post-tax income inequality now distinct, [except for the consumption tax].

- **Long-run wealth inequality:** increases in all three cases.
- Largest increase (3.5%) occurs when financed by **taxing capital income**,

  --One effect reduce after-tax return on capital and average capital stock.

  --Combined with higher spending on public good, leads to a large increase in the ratio of public to private capital, which more than offsets decline in after-tax return on capital.
- Capital-rich agents experience higher long-run returns on capital than the capital poor, and wealth inequality increases.
- For **labor tax**, same effect now operates through after-tax return on labor.
- **Consumption tax** qualitatively similar to that of lump-sum tax-financing.

- Capital tax-financing raises **pre-tax income inequality** both in short run and the long run; exactly opposite effect on **post-tax income inequality**.
- Long-run decline in post-tax income inequality under capital tax-financing reflects redistributive effects of the financing policy, since wealth is the primary source of inequality in this economy.
- Labor income tax financing reduce pre-tax income inequality both in short run and long run; after a small initial decline, post-tax inequality increases.
- Labor tax-financing policy increases long-run post-tax income inequality by reducing after-tax labor income.

The Growth-Income Inequality Relationship

- Simulations confirm ambiguity in the direction of the growth-inequality relationship that is characteristic of recent empirical studies.
- Transition paths of GDP growth and post-tax income inequality are positively correlated.
- But short-run and steady-state relationships depend critically on

  (i) Composition of the different externalities in terms of private capital-public capital mix,

  (ii) Tax policy used to finance government investment,

  (iii) Time horizon, namely short run or long run.

TABLE 2
Increase in Government Spending: The Growth-Inequality Relationship
g = 0.05 to 0.08
$\rho = \upsilon = 0$

A. Composite Externality in Utility and Production, $\varepsilon = \varphi = 0.6$ (Benchmark Case)

Policy Change	**Short Run Change**		**Long Run Change**	
	Growth	**Post-tax Income Ineq.**	**Growth**	**Post-tax Income Ineq.**
Lump-sum tax-financed increase in g	0.129	-2.602	0.206	4.996
Capital income tax-financed increase in g	0.044	-9.174	0.101	-0.149
Labor income tax-financed increase in g	0.096	-0.110	0.168	7.933
Consumption tax-financed increase in g	0.106	-3.117	0.179	4.955

B. Public Good Externality in Utility Function: $\varphi = 0$, $\varepsilon = 1$

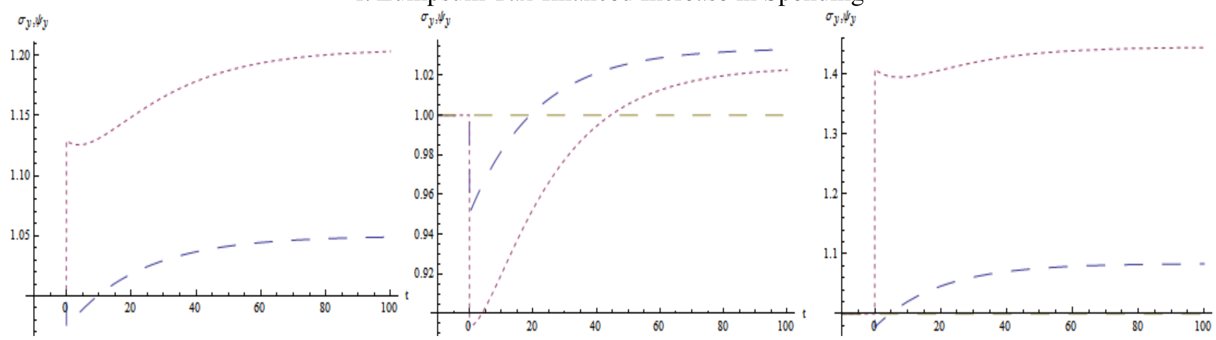
Policy Change	**Short Run Change**		**Long Run Change**	
	Growth	**Post-tax Income Ineq.**	**Growth**	**Post-tax Income Ineq.**
Lump-sum tax-financed increase in g	-0.107	-4.964	0.025	3.373
Capital income tax-financed increase in g	-0.215	-11.631	-0.102	-2.199
Labor income tax-financed increase in g	-0.136	-2.511	-0.010	6.210
Consumption tax-financed increase in g	-0.128	-5.468	-0.0002	3.315

C. Public Good Externality in Production Function: $\varphi = 1$, $\varepsilon = 0$

Policy Change	**Short Run Change**		**Long Run Change**	
	Growth	**Post-tax Income Ineq.**	**Growth**	**Post-tax Income Ineq.**
Lump-sum tax-financed increase in g	0.409	-2.287	0.446	8.392
Capital income tax-financed increase in g	0.377	-9.087	0.386	4.060
Labor income tax-financed increase in g	0.375	0.113	0.408	11.531
Consumption tax-financed increase in g	0.385	-2.938	0.419	8.479

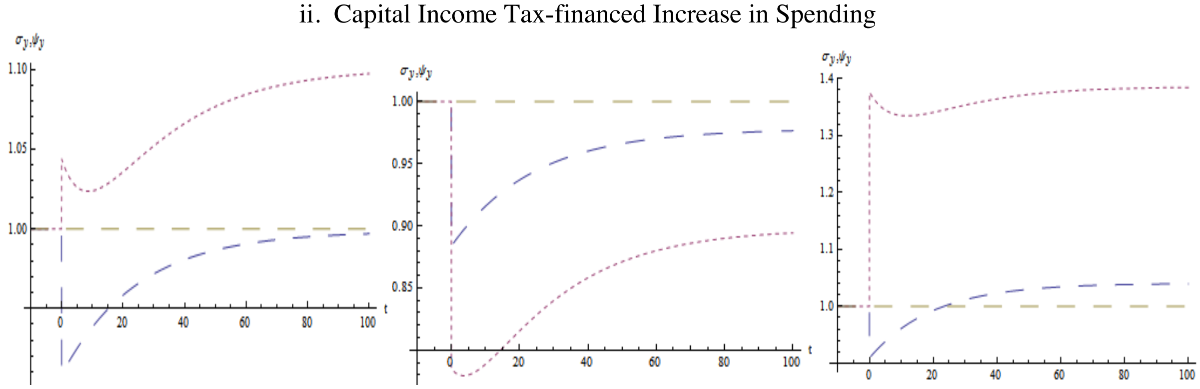
Figure 2. The Growth-Income Inequality (post-tax) Relationship
Sensitivity to Externality Parameters (ϕ, ε)

i. Lumpsum Tax-financed Increase in Spending



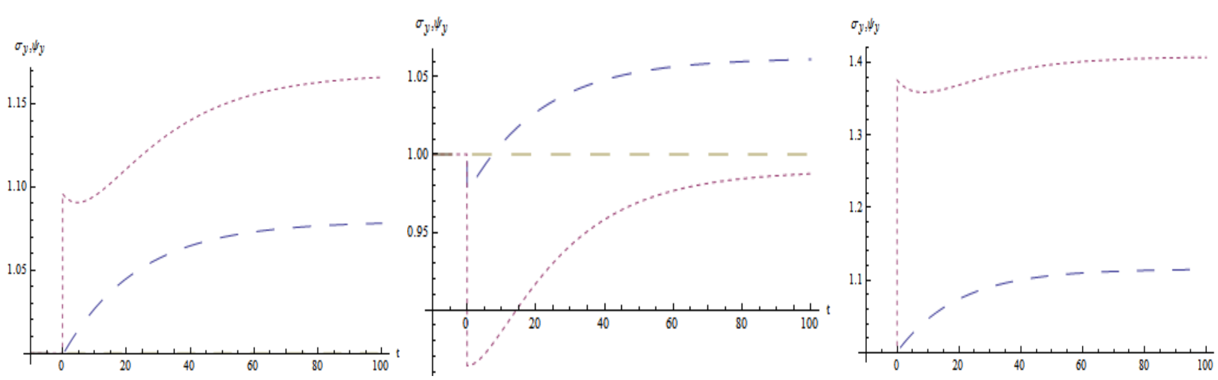
$\phi = 0.6, \varepsilon = 0.6$ **$\phi = 0, \varepsilon = 1$** **$\phi = 1, \varepsilon = 0$**

ii. Capital Income Tax-financed Increase in Spending



$\phi = 0.6, \varepsilon = 0.6$ **$\phi = 0, \varepsilon = 1$** **$\phi = 1, \varepsilon = 0$**

iii. Labor Income Tax-financed Increase in Spending



$\phi = 0.6, \varepsilon = 0.6$ **$\phi = 0, \varepsilon = 1$** **$\phi = 1, \varepsilon = 0$**

.... GDP Growth _ _ _ _ _ _ _ Post-tax Income Inequality

Average Welfare and its Dispersion

- With heterogeneous agents, we consider two elements:
  - (i) average welfare (welfare of the mean or representative agent)
  - (ii) its dispersion (welfare inequality).

- .In all cases increasing government investment raises average welfare, but also increases welfare inequality.

- May evaluate the financing options in terms of increased inequality per unit of average welfare gain.

- On this basis ranking is:
  - (i) consumption tax-financing
  - (ii) labor income tax-financing,
  - (iii) capital income tax-financing.

- Increase in welfare inequality per unit of welfare gain is highly sensitive to the structure of the externality.

TABLE 3
Trade-off between Aggregate Welfare and its Dispersion

A. Composite Externality in Utility and Production, $\varepsilon = \varphi = 0.6$ (Benchmark Case)

Policy Change	$d\tilde{W}$ (%)	$d\tilde{\sigma}_u$ (%)
Lump-sum tax-financed increase in g	4.012	5.415
Capital income tax-financed increase in g	1.790	3.620
Labor income tax-financed increase in g	3.139	2.996
Consumption tax-financed increase in g	3.398	2.946

B. Public Good Externality in Utility Function: $\varphi = 0$, $\varepsilon = 1$

Policy Change	$d\tilde{W}$ (%)	$d\tilde{\sigma}_u$ (%)
Lump-sum tax-financed increase in g	6.830	5.773
Capital income tax-financed increase in g	5.041	3.872
Labor income tax-financed increase in g	5.930	3.312
Consumption tax-financed increase in g	6.198	3.299

C. Public Good Externality in Production Function: $\varphi = 1$, $\varepsilon = 0$

Policy Change	$d\tilde{W}$ (%)	$d\tilde{\sigma}_u$ (%)
Lump-sum tax-financed increase in g	3.384	6.300
Capital income tax-financed increase in g	1.227	4.929
Labor income tax-financed increase in g	2.554	3.926
Consumption tax-financed increase in g	2.801	3.902

OLG Model with Idiosyncratic Productivity Shocks

- Two sources of heterogeneity:

  (i) initial endowments of private capital,
  (ii) idiosyncratic productivity shocks.

- The two sources of heterogeneity are lognormal distributions.

- CES production function CRS in public and private capital.

- Facilitates aggregation, enabling us to derive the joint distributional and aggregate dynamics of the CES economy in a very tractable closed form.

- There are no credit or insurance markets; absence is a key element generating inequality, [Loury (1981), and Bénabou (2000, 2002)].

- The equilibrium dynamics has a simple recursive structure.

  --The dynamics of inequality drives the growth of aggregate variables – private capital, public capital, output – but not vice versa.

- Increase in public investment will decrease or increase inequality, both in the short run and over time, according to whether elasticity of substitution between public and private capital, ε, is greater than, or less than, unity.

- Large ε favors poor leading to more equitable distribution of wealth.

- Small ε favors rich and will exacerbate inequality.

- Investment in infrastructure impact the growth rate through two channels.

    (i) Direct productivity effect, emphasized by Barro (1990).
    (ii) Indirect impact through its effect on inequality.

- The presence of the second channel influences the choice of optimal tax, which now also depends on the degree of wealth heterogeneity as well as ε.

- This model is a sharp contrast to previous one,

Analytical framework

- Individuals live for two periods; households consist of a young and old.
- Each parent of initial generation (t=0) is endowed with private capital, k_0^i.
- Access to public infrastructure, g_0, equally available to all.
- Initial distribution of wealth given; evolves endogenously over time.
- In the first period, when agents are young, they acquire capital from their parents, while their consumption is included in that of their parents.
- All decisions are made during the second period, when children are adults.
- Parents earn income by supplying capital and labor to privately-held firms, which produce output, by combining these factors with public capital.
- Government taxes income at a flat rate to finance public investment good
- Agents allocate after-tax income between consumption and saving; accumulated capital at the end of the second period endowed to offspring.
- Logarithmic preferences

$$W_t^i \equiv \ln c_t^i + \eta \ln\left(1 - l_t^i\right) + \beta \ln k_{t+1}^i \tag{36}$$

- Households choose c_t^i, s_t^i, and l_t^i to maximize (36) subject to

$$c_t^i + s_t^i = (1-\tau) y_t^i \tag{37a}$$

$$y_t^i = a\xi_t^i \left((1-\alpha)\left(k_t^i\right)^{\rho} + \alpha\left(g_t\right)^{\rho} \right)^{\frac{1-\theta}{\rho}} \left(l_t^i k_t\right)^{\theta} \tag{37b}$$

$$k_{t+1}^i = s_t^i \tag{37c}$$

- Two-level production function
- Two capital goods are cooperative in production implies

$$\rho' \equiv \frac{\rho}{1-\theta} < 1 \Rightarrow \varepsilon < \frac{1}{\theta} \tag{37d}$$

- Production subject to an idiosyncratic productivity shock, ξ_t^i:
  --i.i.d. and lognormal with mean one: $\ln \xi_t^i \sim N(-\upsilon^2/2, \upsilon^2)$.
- Initial distribution of capital is lognormal: $\ln k_0^i \sim N(\mu_0, \sigma_0^2)$.
- Private capital fully depreciates within period,
- Government budget is always balanced. Public capital also completely depreciates within the period,

$$g_{t+1} = \tau \int_i y_t^i di \equiv \tau y_t \tag{38}$$

- Aggregate consumption and private capital stock are respectively

$$c_t = (1-\tau) y_t - s_t \tag{39}$$

$$k_{t+1} = s_t \tag{40}$$

where $c_t = \int_i c_t^i di,\ s_t = \int_i s_t^i di,\ k_t = \int_i k_t^i di$.

Individual optimal capital accumulation

- Household i's first order conditions

$$s_t^i = \chi(1-\tau)y_t^i \tag{41a}$$

$$l_t^i = \frac{\theta}{\eta(1-\chi)+\theta} \equiv l \qquad \chi \equiv \beta/(1+\beta) \tag{42b}$$

- Dynamics of capital accumulation for the ith individual

$$k_{t+1}^i = (1-\tau)a'\chi\xi_t^i\left((1-\alpha)\left(k_t^i\right)^\rho + \alpha\left(g_t\right)^\rho\right)^{\frac{1-\theta}{\rho}}\left(k_t\right)^\theta \tag{43}$$

where $$a' \equiv a\left(\frac{\theta}{\eta\left(1-\chi\right)+\theta}\right)^\theta$$

Infrastructure, inequality and aggregate capital dynamics

(i) Inequality Dynamics:

$$\sigma_{t+1}^2 = \upsilon^2 + \frac{1}{\rho'^2}\ln\left[1+\left(\frac{z_t}{1+z_t}\right)^2\left(e^{\rho^2\sigma_t^2}-1\right)\right] \tag{44a}$$

where $$z_t \equiv \frac{1-\alpha}{\alpha}\phi^{\rho}e^{\rho(\rho-1)\sigma_t^2/2} \tag{44b}$$

$$\phi \equiv \frac{k_t}{g_t} = \chi\left(\frac{1-\tau}{\tau}\right) \tag{44c}$$

(ii) Growth dynamics

$$\begin{aligned}\gamma_{t+1}^k \equiv \ln g_{t+1} - \ln g_t &= \ln(a'\chi^{\theta}\alpha^{1/\rho'}) + \theta\ln(1-\tau) + (1-\theta)\ln\tau + \frac{1}{\rho'}\ln(1+z_t) \\ &+ \frac{1-\rho'}{2\rho'^2}\ln\left[1+\left(\frac{z_t}{1+z_t}\right)^2\left(e^{\rho^2\sigma_t^2}-1\right)\right]\end{aligned} \tag{45a}$$

- Causality contrasts sharply with that obtained in previous model.

Steady state

$$\left(\frac{\tilde{z}}{1+\tilde{z}}\right)^2\left(e^{\rho^2\tilde{\sigma}^2}-1\right)=e^{\rho'^2(\tilde{\sigma}^2-\upsilon^2)}-1 \tag{46a}$$

$$\tilde{\gamma}=\ln(a'\chi^{\theta}\alpha^{1/\rho'})+\theta\ln(1-\tau)+(1-\theta)\ln\tau+\frac{1}{\rho'}\ln(1+\tilde{z})+\frac{1-\rho'}{2}\left(\tilde{\sigma}^2-\upsilon^2\right) \tag{46b}$$

$$\tilde{z}\equiv\frac{1-\alpha}{\alpha}\phi^{\rho}e^{\rho(\rho-1)\tilde{\sigma}^2/2} \tag{46c}$$

- Relate long-run distribution of wealth with that using other approaches.
- Li and Sarte (2004) [agents have heterogeneous endowments of capital].

    1. Progressive taxes plus common rate of time discount $\Rightarrow$ long-run degenerate wealth distribution [cf (13a) in absence of supply shocks].
    2. Long-run wealth distribution will not degenerate if agents have different rates of time preference. [That too turns out to be case here].

- Krusell and Smith (1998) and others, show if agents are initially identical but are subject to idiosyncratic shocks, then this will lead to long-run non-degenerate wealth distribution, [as (13a) implies].
- In our first model only source of heterogeneity is initial endowments of capital $\Rightarrow$ non-degenerate long-run distributions of wealth and income that are directly tied to the initial distribution.

  [Hysteresis arises because of assumption of complete financial markets that these models assume, making distributional dynamics path-dependent].

Proposition 1: (i) An increase in the variance of the idiosyncratic productivity shocks will increase inequality and will lead to a lower growth rate, both in the short run and in steady state.

(ii) An increase in the inequality in initial endowments will increase inequality and reduce the growth rate temporarily, declining over time, and vanishing in the long run.

Local stability

- System is locally stable with (overly) fast convergence
- Local approximation to dynamics

$$\sigma_{t+1}^2 = \upsilon^2 + \left(\frac{(1-\theta)z_t}{1+z_t} \right)^2 \sigma_t^2 \tag{47}$$

Effects of public investment on growth and inequality

Steady-state effects

$$\frac{\partial\tilde{\sigma}^2}{\partial\tau} = -\frac{(1-\theta)^2}{\rho}\frac{\left(e^{\rho^2\tilde{\sigma}^2}-1\right)}{(1-D)\left[1+\left(\tilde{z}^2/(1+\tilde{z})^2\right)\left(e^{\rho^2\tilde{\sigma}^2}-1\right)\right]}\frac{2(\tilde{z})^2}{(1+\tilde{z})^3}\frac{1}{\tau(1-\tau)} \tag{48a}$$

$$\frac{\partial\tilde{\gamma}}{\partial\tau} = \frac{1}{\tau(1-\tau)}\left[\frac{(1-\theta)}{(1+\tilde{z})}-\tau\right]+\frac{1}{2}\left[\left(1-\frac{\rho}{1-\theta}\right)-\frac{(1-\theta)(1-\rho)\tilde{z}}{1+\tilde{z}}\right]\frac{\partial\tilde{\sigma}^2}{\partial\tau} \tag{48b}$$

Proposition 2: (i) An increase in the rate of public investment will increase (decrease) both short-run and long-run inequality according to whether the elasticity of substitution is less (greater) than one.

(ii) To the extent that more public investment increases inequality this will tend to increase the growth rate both in the short run, and in the long run, although in all cases the net overall effect will also depend critically upon whether current rate of expenditure $\tau \gtrless (1-\theta)(1+\tilde{z})^{-1}$.

Growth versus welfare maximization

- Setting $\partial\tilde{\gamma}/\partial\tau = 0$, growth-maximizing rate of public investment, τ^*, and corresponding ratio of private to public capital, z^*, are related by

$$\frac{1}{\tau^*(1-\tau^*)}\left[\frac{(1-\theta)}{(1+z^*)}-\tau^*\right]+\frac{1}{2}\left[\left(1-\frac{\rho}{1-\theta}\right)-\frac{(1-\theta)(1-\rho)z^*}{1+z^*}\right]\frac{\partial\tilde{\sigma}^2}{\partial\tau}=0 \tag{49}$$

Proposition 3: To extent that government investment in infrastructure increases (decreases) inequality, it will set the long-run growth-maximizing rate of public investment at rate $\tau^* > (1-\theta)(1+z^*)^{-1}$, the optimality condition characterizing a riskless economy.

- Choice of τ^* depends upon presence and degree of inequality in economy.

- In comparing (49) to riskless economy, we need to take account of the fact that the equilibrium ratio of private to public capital will also be affected.

- Optimal expenditure in the analogous riskless economy is characterized by $\bar{\tau} = (1-\theta)(1+\bar{z})^{-1}$ in which case (49) implies

$$\frac{\tau^*}{\bar{\tau}} > \frac{1+\bar{z}}{1+z^*}$$

- Fact that government investment increases inequality and growth rate causes them to reduce investment. This increases z, enabling them to reduce tax rate.

- Growth-maximizing tax rate is relatively lower in an economy with no idiosyncratic risk, and therefore no inequality, if and only if $\varepsilon < 1$

- Key issue in assessing consequences of structural changes and policy responses is impact on social welfare.

- Defining such a measure as the discounted sum of the expected utility of all future generations, $W = E\sum_{t=0}^{\infty} W_t (1+R)^{-t}$, in steady state the implied social welfare function is

$$W = \frac{(1+R)(1+\beta)}{R}\left(C + \ln\left(1-\tau\right) + \frac{\tilde{\gamma}}{R} - \frac{1}{2}\tilde{\sigma}^2 \right) \tag{50}$$

where C is constant.

- Steady-state social welfare involves tradeoff between the equilibrium growth rate, the degree of income inequality, and the loss in average income due to the tax rate used to finance government investment.

- Welfare-maximizing rate of government investment satisfies

$$\frac{\partial \tilde{\gamma}}{\partial \tau} = R\left[\frac{1}{1-\tau} + \frac{1}{2}\frac{\partial \tilde{\sigma}^2}{\partial \tau} \right] \tag{51}$$

Conclusions

- Relationship between growth and inequality/distribution is complicated!
- Multidimensional
- Hopefully has presented how one might study the issue in a tractable way that provides some insights