AGENT-BASED MODELING OF LOTTERY MARKETS

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ABSTRACT

The lottery market modeled in this study cannot be explained by the conventional rational expectation approach. Clarifying lottery market behaviors is a daunting task. We apply an agent-based computational modeling technique in which each agent is modeled as autonomous with his or her own perceptions and actions. The objective is to use three empirical observations in lottery markets — the halo effect or lottomania, conscious selection of betting numbers, and aversion to regrets — to examine the effects of the lottery takeout rate on its revenue. Initial results show the Laffer curve, which indicates the existence of an optimal lottery takeout rate or range. This finding provides some insights to the empirical averaged rate for the 25 lottery markets examined.

Keywords: Lottery markets, agent-based computational modeling, Laffer curve, fuzzy system, genetic algorithms

1 INTRODUCTION

Economists find lottery market behavior to be an interesting subject. Many studies have used demographic and socioeconomic data to estimate lottery sales or demand. The standard econometric approach, however, primarily treats the demand decision as an individual rational choice problem. Within this framework, the number of tickets purchased by an individual is determined only by his or her personal profile; this choice has nothing to do with how other people would act. To model this aspect, an agent-based computational modeling approach is used to capture some aspects that cannot be described by using an analytical model.

Modern agent engineering techniques offer more advantages for capturing the idea of autonomous agents. Over the past years, these insights have extended to the economics analysis arena in some areas, such as the artificial financial market. As an extension of our earlier studies with an artificial stock market (Chen and Yeh, 2001; 2002), this paper addresses an agent-based model of lottery markets.

We survey the takeout rate of some lottery markets, which shows a wide distribution. This rate ranges from a low of 40% in Taiwan to a high of 68.4% in Brazil. Although these data are helpful in reaching a design, we observe that the takeout rate is only one dimension of the complex lottery design. Scoggins (1995), Hartley and Lanot (2000), and Paton, et al. (2002) have discussed this issue.

Empirical observations of psychological studies of the lottery market have motivated us to use an agent-based modeling approach. Actually, we find that gamblers are not so concerned

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with the probabilistic calculation of the odds of winning, because they often rely on heuristic strategies for handling situations. Even though the generation of the winning number is totally a random mechanism, gamblers still tend to pick nonrandom numbers; this process is called *conscious selection*. Griffiths and Wood (2002) reviewed various heuristics and biases involved in the psychology of the lottery market, such as hindsight bias, representation bias, and availability bias. It is not easy to capture such heuristics and biases by using standard rational models.

This paper is organized as follows. Section 2 briefly describes an agent-based model of the lottery market. Section 3 shows how the genetic algorithm is used. Section 4 outlines the experimental designs and Section 5 gives the results of the simulation. Finally, Section 6 provides concluding remarks.

2 AN AGENT-BASED MODEL OF THE LOTTERY MARKET

2.1 The Lottery Market and Its Design

In general, agent-based models consist of two essential parts — the *environment* and the *agent*. In this study, the environment comprises the rules or the design of a lottery game and the states of the market. Walker and Young (2001) conducted a well-known study of the design of the lottery game. Typically, the game is expressed by two parameters, x/X. In this game, gamblers pick x numbers from a total of X numbers without replacement; different prizes then are set for the various numbers that are matched on the drawing day. In a simple description of this process, let y denote the number matched. Clearly, $y = 0,1, \ldots, x$. Let S_y be the *prize pool* reserved for the winners who matched y numbers. The special term for the largest prize pool is called *Jackpot*, S_x.

A common feature of lotteries is that, if a given draw does not generate winners, the jackpot prize pool from that draw is added to the pool for the next draw; this is referred to as a *rollover*. Rollovers usually make the next draw, called the *rollover draw,* much more attractive. The prize pool is defined by the *lottery takeout rate*, τ, which is the proportion of sales that is not returned as prizes. Thus, the overall prize pool is $(1-\tau)S$, where S is sales revenue and $1-\tau$ is the *payout rate*. Therefore, a lottery game can be represented by the following x + 4-tuple vector: $L = (x, X, \tau, s_0,..., s_x)$, which is shown in the control panel of our agent-based lottery software (Figure 1).

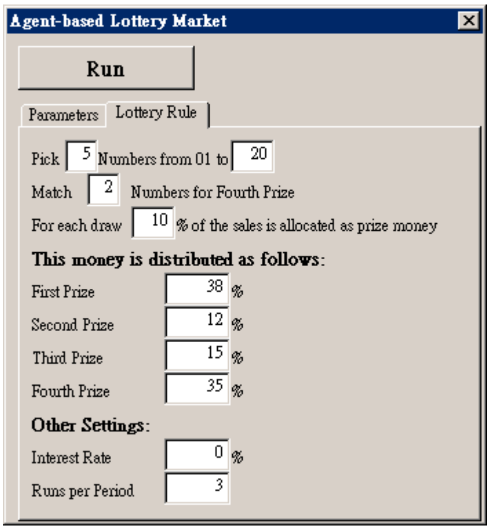

FIGURE 1 Control panel of parameter settings and lottery rules

One of the objectives for using agent-based simulation of the lottery market is to examine the effects of changes in the design L on lottery sales, and more important, on charity fund revenue. The literature shows two approaches for analyzing agents' participation in the lottery markets. In the first approach, the empirical data are used to model the principal features of the observed aggregate behavior (Farrell and Walker, 1999; Farrell, et al., 1999). In the second approach, a rational model of representative agents is used to aggregate these representative agents (Hartley and Lanot, 2000). The agent-based model is closer to the latter but does not use the attributes of rationality and homogeneity.

2.2 Agent Engineering

Since we do not know why people gamble, we do not think that a unique answer can be found to explain this issue. Therefore, many possibilities can be examined by using agent engineering. The basic principle is to ground agent engineering with theoretical and empirical observations. In this way, we minimize the degree of arbitrariness. In our agent-based model, we capture the following stylized facts of the lottery market: lottomania and the halo effect, conscious selection, and aversion to regret.

2.2.1 Lottomania and the Halo Effect

First we observe that lottery sales seem to be positively related to the size of the rollover or jackpot prize. By examining lottery market data, we find that this phenomenon is statistically significant. This phenomenon, called *halo effect* (Creigh-Tyte and Farrell, 1998; Walker and Yang, 2001), can create a bout of “lottomania,” which is propagated by the media. Therefore, we initially build the agents from a participation function, which is a measure of the participation level compared with the size of jackpot. In the standard rational analysis, the change between these two variables is in the expected value, or more generally, the expected utility, of the lottery ticket (Hartley and Lanot, 2000). However, we take a heuristic approach and assume that gamblers base their decisions on some heuristics rather than on the possibly demanding work on the computation of expectations.

The heuristic approach allows approximation of the relation by a few simple if-then rules. We represent the function of participation level by a set of fuzzy if-then rules, which are manipulated by the standard mathematical operations of the fuzzy sets as prescribed by fuzzy set theory.

2.2.2 Conscious Selection

The second important observation related to lottery markets is that gamblers are generally ignorant as to how probability operates. The phenomenon known as conscious selection refers to nonrandom selections of the combinations of numbers. Even more interesting is that there is a market for “experts,” who advise gamblers regarding which numbers to choose. To take conscious selection into account, let a vector be an X-dimensional vector, whose entities take either 0 or 1.

2.2.3 Aversion to Regret

The last feature of our model of agents is the utility function. For simplicity, most advanced-computing-environment models assume an exogenously given utility function that is homogeneous among agents. We have slightly departed from this tradition primarily because of the observation of aversion to regret. In the lottery market, regret simply refers to the utility that the decision not to gamble is based on whether there are winners. If nobody wins, gamblers do not feel regret; however, if somebody wins, they might feel regret (i.e., the prize could have been theirs if they had played the lottery).

In spirit, this consideration is in line with the regret theory proposed by Bell (1982) and Loomes and Sugden (1982). The regret theory offers explanations for numerous evident violations of the expected utility theory axioms. In regret theory, agents, after making decisions under uncertainty, may feel regret if their decisions prove to be wrong even if they seemed to be correct given the information available ex ante. This very intuitive assumption implies that an agent's utility function, among other things, should depend on the realization of alternatives not chosen and, in this sense, irrelevant.

3 GENETIC ALGORITHMS

3.1 Representation

Genetic algorithms (GAs) are motivated from the spirit of natural and are coded with the chromosomes, which is the unit of GAs. In our model, the chromosome is coded as the bit string, which is the vector ($\boldsymbol{a}_{i,t}$, $\boldsymbol{b}_{i,t}$, $\theta_{i,t}$). It fully characterizes an individual at time t. Since each component of the vector is associated with a different function, however, the coding and decoding schemes would be different. Figure 2 illustrates a fuzzy inference system with the corresponding binary string of $\boldsymbol{a}$, decoded as $\boldsymbol{a}$ = (0.2, 0.6, 0.8, 1.0) of real numbers. The input J

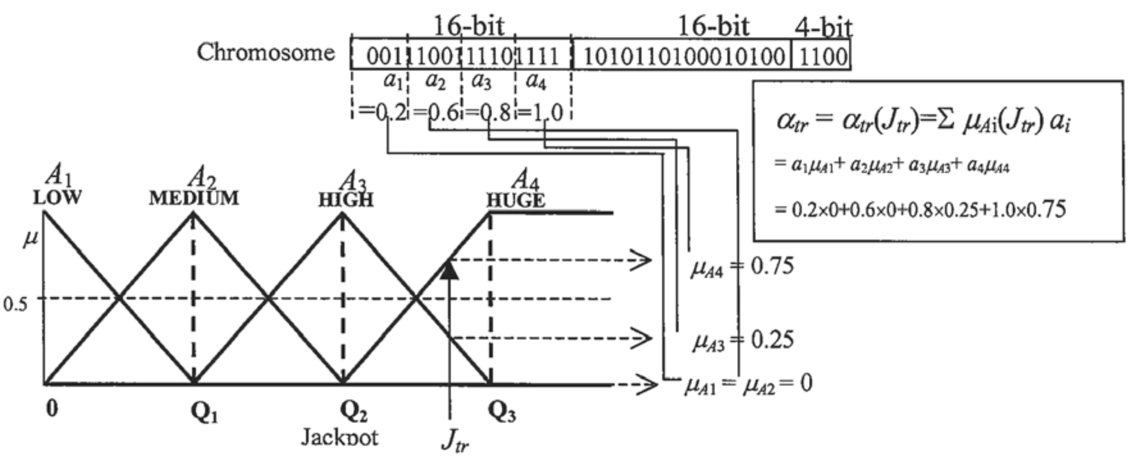

FIGURE 2 Betting heuristics based on the Sugeno fuzzy inference system

is perceived by the agent, and the membership degree of each fuzzy set is calculated as follows: $[\mu_{A1}(J), ..., \mu_{A4}(J)] = [0, 0, 0.75, 0.25]$. Therefore, the agent invests

$$\alpha = \sum_{i=1}^{4} \mu_{Ai}(J)a_i = 0.95$$

of his or her income to purchase lottery tickets.

It is straightforward to code ***b***, which is the number-picking vector. As mentioned earlier, ***b*** is simply an X-bit string. An example of the case $X = 20$ is shown in Figure 3.

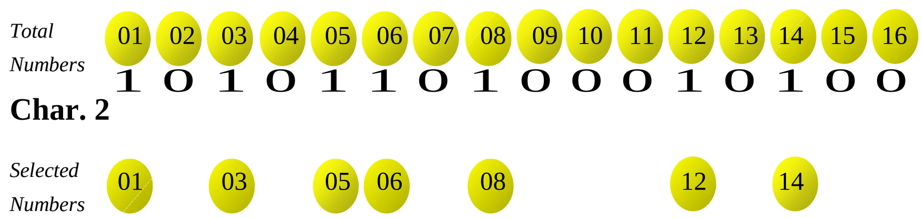

FIGURE 3 Example of numbers chosen by agents

Finally, the regret parameter θ, which also lies between 0 and 1, can be encoded in a similar fashion as binary coding by a l_θ-string bits. Therefore, the full characterization is encoded by a string with a total of $4 \times l_a + l_b + l_\theta$ bits.

3.2 Evolutionary Cycle

Genetic algorithms have two major selection schemes: *roulette-wheel selection* and *tournament selection*. Although these two selection schemes have been well studied in the GA literature, the scheme more suitable for agent-based economic modeling remains an open issue. The reason is because some of the advantages or disadvantages that are known to GA theorists may not be of relevance for social science-oriented studies. Chen (1997) argued that, for social scientists, the network behind the social dynamics is the primary criterion of the selection scheme. Generally, the roulette-wheel selection scheme implicitly assumes the existence of a well-connected global network, whereas the tournament selection scheme requires only the function of local networks. Lacking further evidence on which network assumption is appropriate, it would be beneficial to try both selection schemes to test for robustness. To narrow our focus here, we apply only tournament selection. We plan to include the other selection scheme at a later stage. The following describes the pseudo program of the evolutionary cycle:

```
begin
        Gen := 1;
        Pop := Population-Size;
        initialize(POP(Gen, Pop));
        evaluate(POP(Gen, Pop));
        while not terminate do
        begin
                for i := 1 to Pop step 2
                        Parent1 := Tournament-Select-1st(POP(Gen, Pop));
                        Parent2 := Tournament-Select-2nd(POP(Gen, Pop));
                        OffspringPOP(Gen, i) := Crossover-Mutation-1st(Parent1, Parent2);
                        OffspringPOP(Gen, i+1) := Crossover-Mutation-2nd(Parent1, Parent2);
          next i
                evaluate(OffspringPOP(Gen, Pop));
                POP(Gen+1) := OffspringPOP(Gen, Pop);
                Gen := Gen+1;
        end
end
```

4 EXPERIMENTAL DESIGNS

This paper studies the possible relation between the lottery takeout rate and the lottery sales by hypothesizing the existence of a Laffer curve and hence an optimal interior *T*. To do so, different values of *T* ranging from 10% to 90% are attempted. The remaining market parameters are treated as constants throughout the entire simulation. Figure 4 shows the parameter settings of the agent-based model of the lottery market.

The second set of parameters concerns the control parameters of the genetic algorithm. The parameter *T* (i.e., the tournament size) is unusually large ($T = 200$), which allows for greater interaction among gamblers; this approximates the intensive attention drawn to lottery results reported by mass media. In the future, we plan to apply this agent-based lottery market to some sensitivity issues that pertain to the choice of various selection schemes, market sizes, crossover styles, etc., including their economic significance and the effect on the simulation results.

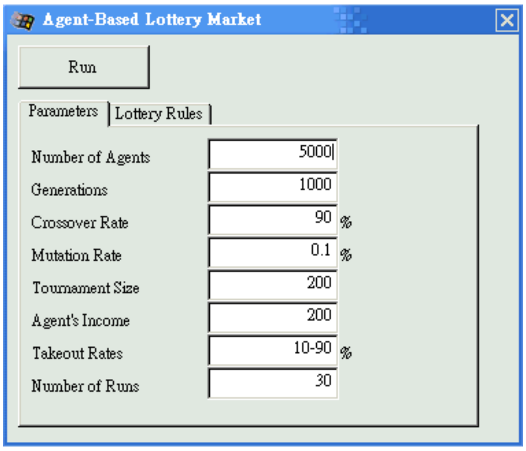

FIGURE 4 Parameter settings of agent-based lottery market

5 RESULTS

5.1 Takeout Rate and Tax Revenue

Figure 5 shows that initially, the normalized lottery revenue (effective takeout rate) increases with the lottery takeout rate τ and finally decreases with it. The highest revenue appears at τ = 40% with an effective takeout rate of 1.1%. However, the revenue curve is not unimodular; in addition to τ = 40%, it also peaks at τ = 60%. Hence, it is not a typical Laffer curve as one might suppose. The revenue does not monotonically decrease after τ = 40, and the jump at τ = 60% is not surprising. Certainly, this finding does not mean that the complex system used can have only one unique solution: τ = 0.40. Is it possible that different settings of the parameter values can lead to different results? Or are we by luck, for example, simulating a system with a set of parameters that has an optimal solution consistent with the empirical observation? This is indeed the robustness issue that must be addressed in agent-based computational modeling.

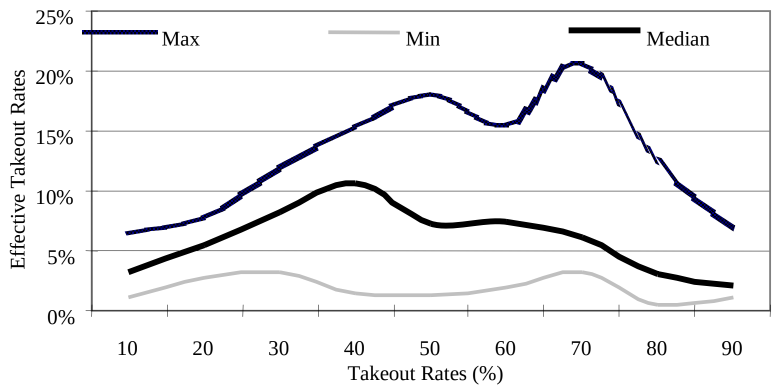

FIGURE 5 Effective tax rates statistics for 30 runs for each tax rate

5.2 Rollovers and Sales

Generally, large-sized rollovers tend to enhance the attractiveness of the lottery game. Statistics show that the mean sales that are conditional on the rollover draw are normally *higher* than those of the regular draw. For example, on the basis of the time series data for the U.K. lottery from November 19, 1994, to March 5, 2003, which comprises a total of 751 draws, the average sales are £56.0 million over the rollover draws, whereas they are £41.4 million over the regular draws. However, from a total of 112 rollover draws of the U.K. lottery, sales actually fell 25 times. On the basis of these statistics, it is interesting to see whether the patterns will be similar for our artificial lottery markets. Therefore, we use the same statistics for the simulated data.

The disappearance of the halo effect and the appearance of the anti-halo effect are certainly astonishing, especially because our agent engineering is based on the consideration of the halo effect. However, a comparison of the real data with the artificial data provides us the opportunity to reflect on something that we may take for granted. In particular, what is the essence of the phenomenon of the halo effect? Why did the agent-based system built on GA fail to deliver this feature? Is there a reasonable explanation for this?

5.3 Conscious Selection

In the real market, many “experts” who advise people on selecting numbers have analyzed the patterns of lottery numbers. In our simulations, the numbers favored by each agent are observable. The profile provides us with the opportunity to examine the behavior of conscious selection. In particular, it enables us to address the question as to whether the agents essentially believe that winning numbers are randomly selected.

5.4 Aversion to Regret

We examine the values of θ, which intensifies agents’ suffering when they do not bet in the last period, and take an average from this sample. We call the average $\bar{\theta}$. We see that a culture in which people are sensitive to what others have is nursed in this lottery in this environment. The statistic nearly reaches its maximum and is independent of the takeout rate.

6 CONCLUSIONS

We introduce an agent-based model of the lottery market. This market is composed of many highly interacting agents whose decisions are inevitably interdependent. A model must allow for imitation, fashion, and contagion. In general, an agent’s preference for the lottery should be adaptive and evolving rather than fixed. Agents should be modeled as a adaptive agents who, based on their past experiences, are continuously updating their anticipation of the value of lottery tickets and revising their decisions accordingly. By using GAs, we capture the decisions of lottery demand made by adaptive agents in a highly interactive environment and simulate the time series of the aggregate sales of a lottery tickets.

In this paper, the agents are primarily designed on the basis of two empirical phenomena known as the halo effect and the conscious selection of numbers. We also consider the agent’s utility function. The empirical observations of the aversion to regret motivated us to find an interdependent utility function for agents. These aspects, which included unsophisticated heuristic behavior, conscious number picking, and preference, are evolving over time via the canonical genetic algorithms.

This model is a starting point for conducting some initial evaluations of the impact of the lottery takeout rate on the lottery revenue. Two observations are made in this paper. First, the Laffer curve suggests an optimal lottery takeout rate τ^*. Second, the τ^* can be sensitive to how agents are modeled. Simulations show that when the regret effect is moved from agents’ preferences, the τ^* can go up. If so, the appearance of the interdependent utility function has an implication on the design of the lottery game. Empirical data from Taiwan, U.K., and South Africa national lotteries will be used to examine the performance of our agent-based model.

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