Testing Rational Expectations Hypothesis with Agent-Based Model of Stock Markets*

Shu-Heng Chen, Chia-Hsuan Yeh and Chung-Chih Liao
AI-ECON Research Group
Department of Economics
National Chengchi University
Taipei, Taiwan, 11623

Abstract ***Using agent-based models of stock markets, this paper examines the rational expectations hypothesis from a bottom-up perspective. We apply standard linear and nonlinear econometric tests to artificial time series generated from two artificial stock markets composed of bounded-rational traders. The two artificial stock markets differs in their architectures: one has a business school, and one does not. While the linear test shows that the market with the business school fails to reject the rational expectations hypothesis quite often, the nonlinear one does not. Therefore, strictly speaking, these two agent-based markets of bounded-rational traders do not collectively behave as what rational expectations hypothesis predicts, and hence do not lend support to rational expectations hypothesis.***

Keywords: Genetic Programming, Agent-Based Stock Markets, Rational Expectations Hypothesis, Bounded Rationality, Business School, LISP Trees

1 Motivation and Introduction

Rational expectations hypothesis (**REH**) plays an extremely important role in the development of modern macroeconomics. [4] provided an annotated bibliography of over 470 significant books and articles on this research area. Nonetheless, most of these studies were conducted based on the assumption of the *representative agents* rather than based on a collection of *interacting heterogeneous agents*. Under the assumption of the representative agent, macroeconomic time series, such as consumption, saving, and income, are treated as the behavior of *representative agent*, and the rational expectations hypothesis is then formulated and tested by examining the statistical behavior of this representative agent.

However, the main problem of this research strategy is that, since the representative agent does not really exist, rejecting or accepting rational expectations hypothesis does not imply anything about the rational or irrational behavior of the individuals. In this paper, we took a different approach. We first constructed the market from an bottom-up approach so that we know very well how each traders was born and how their mental power shall develop. By designs, we can make traders behave in bounded-rational style in the sense that they are prudent and adaptive. But, then we asked: *will a market composed of these bounded-rational traders collectively behave as what rational expectations hypothesis predict.* In other words, we test the REH by using the observations from the agent-based stock markets. The purpose of doing this is the following. When REH at the top (as an aggregate phenomenon) is rejected, it does not mean traders at the bottom (as an individuals) behave not rationally. In fact, we know that they all ra-

*Research support from NSC grants No.88-2415-H-004-008 and No.87-I-2911-15 is gratefully acknowledged.

tional in a sense. Therefore, the REH, to some extent, is irrelevant.

The rest of the paper is organized as follows. Section 2 describes the agent-based stock markets considered in this paper, and how the artificial data was generated. Section 3 provides some basic statistics about traders' behavior in these artificial stock markets. Section 4 then gives the econometric tests of the rational expectations hypothesis based on the artificial data generated as described. Section 5 makes concluding remarks.

2 Experimental Designs and Data Description

The data used in this paper were generated from two agent-based stock markets, coded as Market C and Market D in this paper. The analytical model on which these two agent-based stock markets were built can be found on the website:
http://econo.nccu.edu.tw/staff/csh/
course/grad-mac/lec12/lec12.htm
The artificial time series data of Market D was from CASE 2 in [1], who also gave a description of Market D. As opposed to Market D, the distinguishing feature of Market C is the addition of the *business school* to the artificial stock market. The motivation to do this is given in [2] and is briefly summarized in [3] in this volume.

2.1 Business School and Single-Population GP

The business school in our model functions as usual business schools in the real world. It mainly consists of faculty, and their different kinds of models (schools of thoughts). Let F be the number of faculty members (forecasting models). These models are propagated via a competition process driven by the faculty through publications. In this *academic* world, a scholar can ill afford to keep something serious to herself if she wants to be well-acknowledged. If we consider business school a collection of forecasting models, then we may well use single-population GP to model its adaptation.

At the evaluation date, say t, each forecasting model (faculty member) will be reviewed by a *visitor*. The visitor is another model which is generated randomly from the collection of the existing models in the business school at $t-1$, denoted by $GP_{i,t-1}$, by one of the following three genetic operators, reproduction, crossover and mutation, each with probability p_r, p_c, and p_m (Table 1). In the case of reproduction or mutation, we first randomly select two GP trees, say, $gp_{j,t-1}$ and $gp_{k,t-1}$. The MAPE of these two trees over the last m_2 days' forecasts are calculated. A *tournament selection* is then applied to these two trees. The one with lower MAPE, say $gp_{j,t-1}$, is selected. We then apply Schwefel's *1+1 strategy* over the host $gp_{i,t-1}$ and the visitor $gp_{j,t-1}$ (in the case of reproduction) or $gp'_{j,t-1}$ (in the case of mutation) based on the criterion MAPE, and $gp_{i,t}$ is the outcome of this *1+1* competition.

In the case of crossover, we first randomly select two pairs of trees, say $(gp_{j_1,t-1}, gp_{j_2,t-1})$ and $(gp_{k_1,t-1}, gp_{k_2,t-1})$. The tournament selection is applied separately to each pair, and the winners are chosen to be parents. The children, say (gp_1, gp_2), are born. One of them is randomly selected to compete with $gp_{i,t-1}$, and the winner is $gp_{i,t}$.

2.2 Traders and Business School

Given the adaptive process of the business school, the adaptive process of traders can be described as a sequence of two decisions. First, should she go back to the business school to *take classes*? Second, should she *follow* the lessons learned at school? In the real world, the first decision somehow can be more *psychological* and has something to do with *peer pressure*. One way to model the influence of peer pressure is to suppose that each trader will examine how well she has performed over the last n_2 trading days, when compared with other traders. Suppose that traders are ranked by *the net change of wealth* over the last n_2 trading days. Let $W_{i,t}^{n_2}$ be this net change of

Table 1: Parameters of the Stock Market: Market C

The Stock Market	
Shares of the stock (H)	100
(M_1) per capital	100
Interest rate (r)	0.1
Stochastic Process (D_t)	$Normal(10, 2)$
Price adjustment function	$tanh$
Price adjustment (β_1)	10^{-5}
Price adjustment (β_2)	0.2×10^{-5}
Business School	
Number of faculty members (F)	500
Number of trees created by the full method	50
Number of trees created by the grow method	50
Function set	$\{+, -, Sin, Cos, Exp, Rlog, Abs, Sqrt\}$
Terminal set	$\{P_{t-i}\}_{i=1}^{10}$ $\{P_{t-i}+D_{t-i}\}_{i=1}^{10}$
Selection scheme	Tournament selection
Tournament size	2
Probability of creating a tree by reproduction	0.10
Probability of creating a tree by crossover	0.70
Probability of creating a tree by mutation	0.20
Probability of mutation	0.0033
Probability of leaf selection under crossover	0.5
Mutation scheme	Tree Mutation
Replacement scheme	(1+1) Strategy
Maximum depth of tree	17
Number of generations	20,000
Maximum number in the domain of Exp	1700
Criterion of fitness (Faculty members)	MAPE
Evaluation cycle (m_1)	20
Sample Size (MAPE) (m_2)	10
Traders	
Number of Traders (N)	500
Degree of RRA (λ)	0.5
Criterion of fitness (Traders)	Increments in wealth (Income)
Sample size of $\sigma^2_{t\|n_1}$ (n_1)	10
Evaluation cycle(n_2)	1
Sample size (n_3)	10
Search intensity (I^*)	5
(θ_1, θ_2, θ_3)	(0.5, 10^{-5}, 0.0133)

wealth of trader i at time period t, i.e.,

$$\Delta W_{i,t}^{n_2} \equiv W_{i,t} - W_{i,t-n_2}, \qquad (1)$$

and, let $R_{i,t}$ be her rank. Then, the probability that trader i will go to business school at the end of period t is assumed to be determined by

$$p_{i,t} = \frac{R_{i,t}}{N}. \qquad (2)$$

In addition to peer pressure, a trader may also decide to go back to school out of a sense of *self-realization.* Let the growth rate of wealth over the last n_2 days be

$$\delta_{i,t}^{n_2} = \frac{W_{i,t} - W_{i,t-n_2}}{\mid W_{i,t-n_2} \mid}, \qquad (3)$$

and let $q_{i,t}$ be the probability that trader i will go back to business school at the end of the tth trading day, then it is assumed that

$$q_{i,t} = \frac{1}{1 + exp^{\delta_{i,t}^{n_2}}}. \qquad (4)$$

The choice of this density function is also straightforward. The traders who have made great progress will naturally be more confident and hence have little need for schooling, whereas those who suffer devastating regression will have a strong desire for schooling.

Once a trader decides to go to school, she has to make a decision on what kinds of classes to take. Since we assume that business school, at period t, consists of 500 faculty members (forecasting models), let us denote them by $gp_{j,t}$ ($j = 1, 2, ..., F$.) The class-taking behavior of traders is assumed to follow the following sequential search process. The trader will randomly select one *forecasting model* $gp_{j,t}$ ($j = 1, ..., F$) with a uniform distribution. She will then *validate* this model by using it to fit the stock price and dividends over the last n_3 trading days, and compare the result (MAPE) with her original model. If it outperforms the old model, she will discard the old model, and put the new one into practice. Otherwise, she will start another random selection, and do it again and again until either she has a successful search or she continuously fail I^* times.

Based on the experiment design given in Table 1, a single run with 20,000 generations was

Table 2: Number of Martingale Believers and Traders with Successful Search in Markets C and D

	N_1		N_3	
T	Mkt C	Mkt D	Mkt C	Mkt D
1	0.65	20.02	191.98	352.74
2	0.86	19.67	187.43	352.43
3	3.34	21.44	179.39	352.20
4	3.82	21.16	165.16	351.97
5	3.68	21.10	190.20	351.98
6	2.08	21.00	175.65	351.92
7	0.98	20.31	188.92	351.86
8	4.15	22.74	184.07	351.98
9	3.67	21.54	189.53	352.14
10	5.97	21.46	178.55	352.25

"T" refers to the Tth 2000-observation. For example, "1" refers to the period "1-2000", and "5" refers to the period "8000-10000".

conducted for Market C. This time series data and that of Market C was then used to test the rational expectations hypothesis. Nevertheless, before we proceed future, let us take a closer look at traders' behavior from an evolutionary perspective.

3 Experimental Results: Traders

There are three questions which we would like to ask. First, what do our traders believe? Do they believe in the *efficient market hypothesis* (**EMH**)? Second, do our traders actively search for new ideas? If so, do they benefit from such an adventure? Third, will traders evolve to be more and more sophisticated as time goes on?

First, are they believers of the EMH?. That is, do they believe that

$$E_t(P_{t+1} + D_{t+1}) = P_t + D_t? \qquad (5)$$

Or, technically speaking, are they *martingale believers*? To see this, the time series of the number of martingale believers ($N_{1,t}$) of Markets C and D are drawn in Figures 1 and 2. In addition, we also averaged the number of martingale believers at every 2000 periods, and the result are shown in Table 2. While there were

Table 3: Complexity of Trees (Depth & Node) in Markets C and D

	k		κ	
T	Mkt C	Mkt D	Mkt C	Mkt D
1	8.62	2.96	26.31	3.91
2	7.80	2.89	16.50	3.74
3	8.03	3.01	15.95	3.93
4	7.97	2.89	16.58	3.76
5	9.77	2.88	29.28	3.74
6	8.34	2.92	21.20	3.80
7	8.91	2.83	22.69	3.67
8	8.21	2.95	18.21	3.85
9	8.59	2.99	19.21	3.88
10	8.76	2.90	20.17	3.79

not many martingale believers in both markets, the difference between them is still quite significant. This number is almost nil in Market C, while it is quite steady around 20 in Market D. This result is somewhat interesting because that our PSC and BDS tests for the stock returns of Market C showed that Market C is very efficient. It is so efficient that returns are both linearly and nonlinearly uncorrelated.[1] On the contrary, from [1], we know that Market D, as opposed to Market C, is not that efficient, while there were many more martingale believers in this market.

This naturally brings up the second question: *if traders do not believe in the martingale hypothesis, what do they actually do?* Figures 3 and 4 is the time series plot of the number of traders with successful search, $N_{3,t}$. Due to the density of the plot and the wide range of fluctuation, this figure is somewhat complicated and difficult to read. We, therefore, report the average of $N_{3,t}$ over different periods of trading days in the fourth and the fifth columns of Table 7. From Table 7, it can be seen that the number of traders with successful search, on the average, fluctuates between 160 to 190 in Market C and stay even higher up to 350 in Market D. At a rough estimate, 30% to 70% of the traders benefit from search per trading

[1] By the PSC criterion, the ARMA(p,q) process extracted from the return series was ARMA(0,0). The BDS test was also failed to reject the null hypothesis that return series are *iid*.

day. Clearly, search is not futile.

Third, it is interesting to know what kind of useful lessons traders learn from their search. On way to see what traders may learn is to examine the forecasting models they employ. However, this is a very large database, and is difficult to deal with directly. But, since all forecasting models are in the format of LISP trees, we can at least ask *how complex these forecasting models are.* To do so, we give two definitions of the *complexity* of a GP-tree. The first definition is based on the *number of nodes* appearing in the tree, while the second is based on the *depth* of the tree. On each trading day, we have a profile of the evolved GP-trees for 500 traders, $\{f_{i,t}\}$. The complexity of each tree is computed. Let $k_{i,t}$ be *the number of nodes* of the model $f_{i,t}$ and $\kappa_{i,t}$ be the *depth* of $f_{i,t}$. We then average as follows.

$$k_t = \frac{\sum_i^{500} k_{i,t}}{500}, \quad and \quad \kappa_t = \frac{\sum_i^{500} \kappa_{i,t}}{500}. \qquad (6)$$

Figures 5-8 are the time series plots of k_t and κ_t of Markets C and D. One interesting hypothesis one may make is that *the degree of traders' sophistication is an increasing function of time (monotone hypothesis).* In other words, traders will evolve to be more and more sophisticated as time goes on. However, this is not the case here. Both figures evidence that, while traders can evolve toward a higher degree of sophistication, at some point in time, they can be simple as well (Also see Table 3). Despite the rejection of the monotone hypothesis, we see no evidence that traders' behavior will converge to the simple martingale model.

4 Experimental Results: Rational Expectations Hypothesis

To test the rational expectations hypothesis, we first constructed a *representative agent* by using the *market expectations.* The market expectations is defined as the *average* of all traders' expectations, i.e.,

$$E_t = \frac{\sum_{i=1}^{N} E_{i,t}}{N} \qquad (7)$$

Given E_t, the prediction error of the representative agent at time t is

$$e_t = E_t - (P_t + D_t) \qquad (8)$$

The time series plot of $\{e_t\}$ is given in Figures 9 and 10.

Then, in spirit of the conventional rational expectations hypothesis test, we assumed that rational expectations (market expectations) could not make *systematic errors.* By *systematic errors*, we mean that the time series $\{e_t\}$ is *patternless.* In other words, the time series $\{e_t\}$ is totally unpredictable, or $\{e_t\}$ is an independent series with mean 0. Therefore, to test whether rational expectations hypothesis holds in our artificial stock markets, it is required to test whether $\{e_t\}$ is independent.

To do so, we followed the procedure of [1]. This procedure is composed of two steps, namely, *PSC filtering* and *BDS testing.* We first applied Rissanen's predictive stochastic complexity (**PSC**) to filter out the linear process. Table 4 gives us the $ARMA(p,q)$ process extracted from the error series $\{e_t\}$. As we can see from Table 4, most subseries are not linearly uncorrelated. The linear processes selected by the PSC criterion are all AR(1) in Market C except Series 9. In Market D, the linear patterns found are even more structured. A careful examination of the *coefficient of determination* (R^2), however, reveals that for many series in Market C, R^2 is very low (lower than 0.1), which means linear patterns, even if they exist, are very weak. As opposed to *Market C*, systemic errors in Market D are more severe. Except for the first series, the R^2 of Market D is always higher than 0.1. Therefore, at this stage, the REH is already rejected in Market D.

Once the linear signals are filtered out, any signals left in the residual series must be *nonlinear.* Therefore, one of the most frequently used statistic, the BDS test, is applied to the residuals from the PSC filter. There are two parameters required to conduct the BDS test. One is the distance parameter (ϵ standard deviations), and the other is the *embedding dimension* (DIM). The result of $\epsilon = 1$ and $DIM = 4, 5$ are given in Table 5.

Table 4: Rational Expectations Hypothesis: PSC Testing

	Market C		Market D	
T	(p,q)	R^2	(p,q)	R^2
1	(1,0)	0.04	(1,0)	0.06
2	(1,0)	0.05	(1,2)	0.22
3	(1,0)	0.00	(1,3)	0.20
4	(1,0)	0.23	(1,2)	0.15
5	(1,0)	0.07	(1,2)	0.14
6	(1,0)	0.07	(1,2)	0.11
7	(1,0)	0.12	(1,2)	0.13
8	(1,0)	0.23	(2,1)	0.10
9	(0,0)	0.00	(2,2)	0.18
10	(1,0)	0.02	(2,1)	0.11

The orders p and q are selected based on the Rissanen's PSC criterion. R^2 is the coefficient of determination derived by running the PSC-selected $ARMA(p,q)$ regression.

In Market C, seven out of the ten periods *reject* the null hypothesis that *the filtered error series is nonlinear independent*, while, in Market D, only the first series rejects it. Therefore, while the representative agent of Market A did not make linear systematic errors, she did make nonlinear ones. Hence, strictly speaking, the representative agent of both markets are not satisfied with the rational expectations hypothesis. Alternatively speaking, rational expectations hypothesis is rejected in both agent-based artificial stock markets.

5 Conclusions

The essence of this line of research is the following. We consider the top-down approach (the representative agent), as the mainstream in conventional macroeconomics, can be quite misleading for the understanding of the so-called *emergent*, or *bottom up*, *properties.* In one example, we see that, while the data can support the EMH, it was generated from a group of traders who did not believe the EMH. In another example, the data can reject the rational expectations hypothesis, while each traders were perfectly rational in the sense that they were prudent and adaptive.

Table 5: Rational Expectations Hypothesis: BDS Testing

	Embedding Dim			
	Market C		Market D	
T	4	5	4	5
1	2.84	2.41	3.33	3.71
2	3.01	2.71	0.88	0.84
3	2.49	1.99	1.89	1.94
4	4.68	4.35	1.14	1.07
5	1.83	1.58	1.08	1.12
6	1.84	1.73	1.24	1.31
7	5.79	5.26	1.22	1.32
8	3.06	2.65	0.98	1.16
9	2.34	2.01	0.89	0.96
10	1.80	1.49	1.64	1.76

Since the BDS test is asymptotically normal, the critical value for rejection of the null hypothesis that *the series in question is iid* at 0.05 significance level is 1.96.

References

[1] Chen, S.-H. and C.-H. Yeh. On the Consequence of "Following the Herd": Evidence from the Artificial Stock Market. *Proceedings of the International Conference on Artificial Intelligence*, Computer Science Research, Education, and Application Press. 1999.

[2] Chen, S.-H. and C.-H. Yeh. Genetic Programming in Agent-Based Artificial Stock Markets: Simulations and Analysis. In *Proceeding of 1999 Congress on Evolutionary Computation*, IEEE Press. 1999.

[3] Chen, S.-H, C,-H. Yeh and C.-C. Liao. Testing for Granger Causality in the Stock Price-Volume Relation: A Perspective from the Agent-Based Model of Stock Markets. *Proceedings of the International Conference on Artificial Intelligence*, Computer Science Research, Education, and Application Press. 1999.

[4] Redman, D. A. *A Reader's Guide to Rational Expectations: A Survey and Comprehensive Annotated Bibliography.* Edward Elgar, 1992.

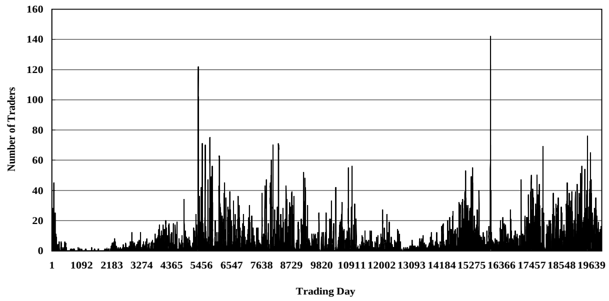

Figure 1 : The Number of Martingale Believers on Each Trading Day (Market C)

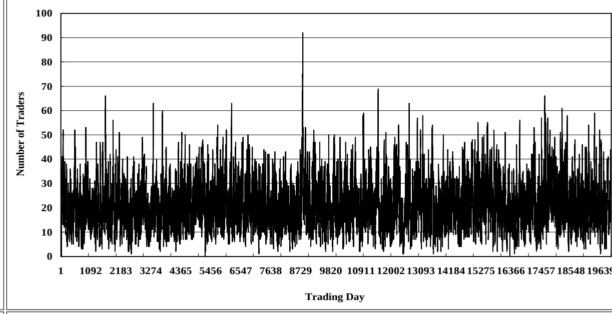

Figure 2 : The Number of Martingale Believers on Each Trading Day (Market D)

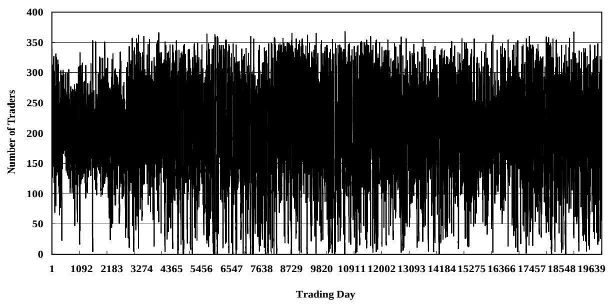

Figure 3 : The Number of Traders with Successful Search on Each Trading Day (Market C)

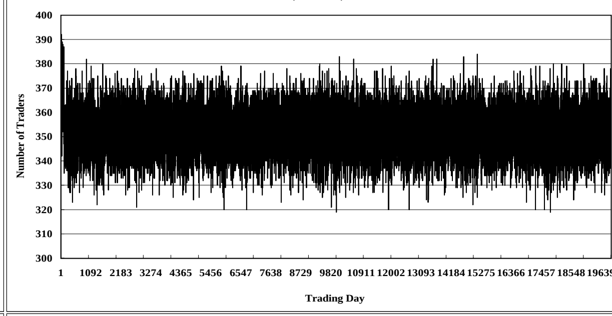

Figure 4 : The Number of Traders with Successful Search on Each Trading Day (Market D)

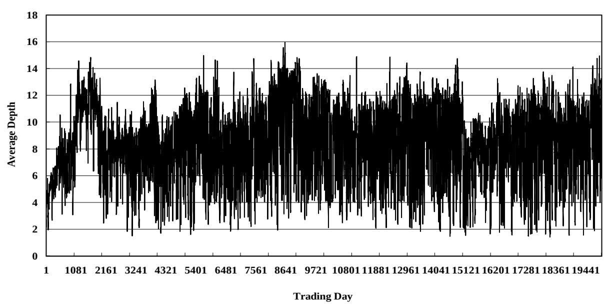

Figure 5 : Traders' Complexity : The Average of Depth of GP-Trees (Market C)

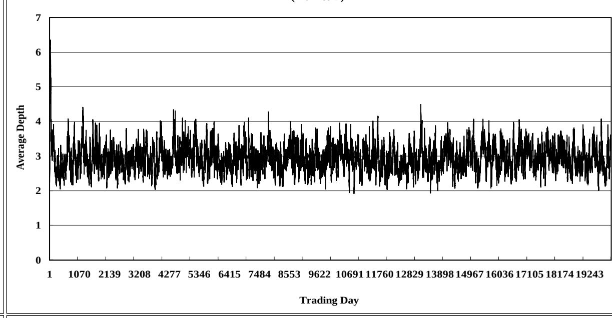

Figure 6 : Traders' Complexity : The Average of Depth of GP-Trees (Market D)

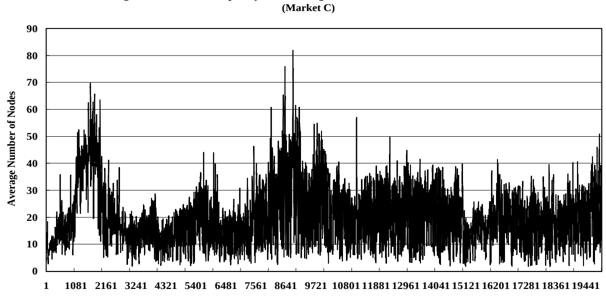

Figure 7 : Traders' Complexity : The Average of the Number of Nodes of GP-Trees (Market C)

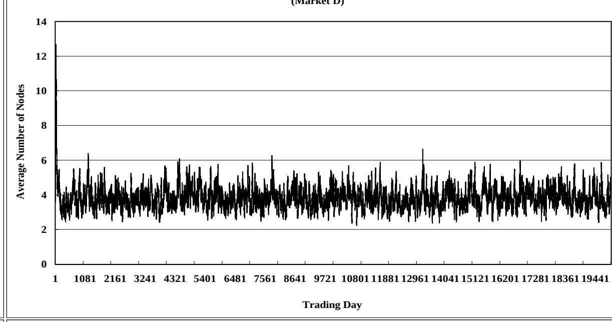

Figure 8 : Traders' Complexity : The Average of the Number of Nodes of GP-Trees (Market D)

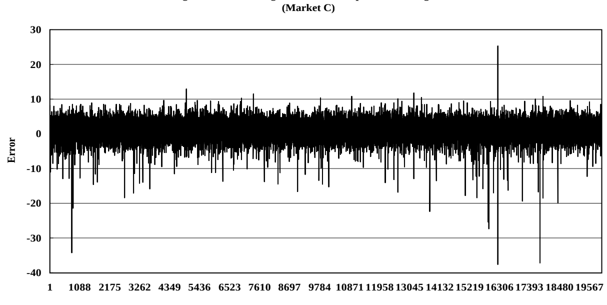

Figure 9 : Forecasting Errors of the Representative Agent (Market C)

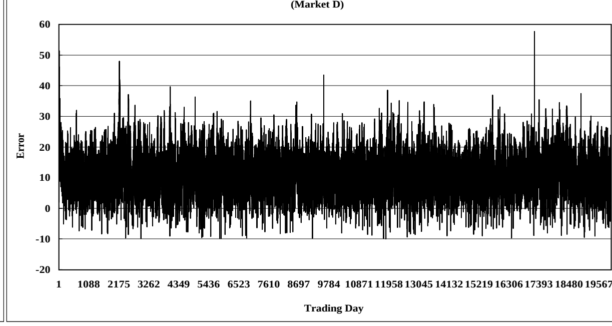

Figure 10 : Forecasting Errors of the Representative Agent (Market D)