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PREFACE

Evolutionary Computation has developed during the past years into a mature research field with a steadily improving theoretical basis and impressive results in challenging application domains. Computer algorithms gleaned from the model of organic evolution, such as genetic algorithms, evolutionary programming, evolution strategies, genetic programming, and classifier systems, have clearly demonstrated their capabilities in a wide range of tasks in optimization and adaptation as well as simulation of biological systems, in order to better understand the modeled real world.

Although these are the Proceedings of the Seventh International Conference on Genetic Algorithms, the title just reflects historical reasons rather than a scope limitation: The Seventh ICGA is a conference on evolutionary computation, just as the other main conference series in the field (namely, the Parallel Problem Solving from Nature Conferences, the Conferences on Evolutionary Programming, and the IEEE International Conferences on Evolutionary Computation). The ICGA is the conference with the longest history in the field, however, and that is one of the reasons why I feel particularly proud to be in the position of the program chair for ICGA-97.

For this year's conference, 208 papers were submitted by the international scientific community, and I would like to thank all researchers for their submissions to ICGA-97. The papers were generally on a level of quality that I found exceptionally high, such that it was easy for the program committee, the executive program committee, and the program chair to arrive at the final decisions for the 102 accepted papers contained in this volume (and not so easy to reject the rest). Fortunately, I had a good estimate of the expected number of submissions, and the 87 program committee members did a good job by providing timely, fair, and helpful reviews for the authors and the program chair. We all know that reviewing scientific work of other members of the community is a time-consuming, difficult honorary business that steals our free time and requires our most honest and fair attention—but if we all give our best to achieve this, it still turns out to be the most reasonable way of obtaining an adequate assessment of quality. For this conference, I would like to give my special thanks to the program committee for their help in reviewing the submissions.

I would also like to thank the members of the executive program committee for their help in making acceptance decisions in some more difficult cases. Jörg Ziegenhirt, one of my colleagues at the Informatik Centrum Dortmund, was an invaluable help in handling all the organizational details connected with the management of a submission and review procedure involving 208 papers, 87 reviewers, and 1248 review forms. He faced this challenge in the most reliable and precise form I could imagine.

The conference committee members handled all the other aspects of the conference organization, and I would like to emphasize that it was a real pleasure for me to work with them. Erik Goodman (Conference Chair), Bill Punch (Local Arrangements Chair), Gil Syswerda (Financial Chair), Dave Schaffer (Publications Chair), Ian Parmee (Publicity Chair), Marc Schoenauer (Tutorials Chair), and Dave Levine (Workshops Chair) have done excellent jobs in organizing the details of this conference.

For the publishing process at Morgan Kaufmann, Marilyn Uffner Alan was again responsible for the proceedings, and it was a pleasure to cooperate with her on this project.

Finally, the conference committee would like to thank the following institutions for their financial support:

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International Society for Genetic Algorithms
Genetic Algorithms Research and Applications Group (MSU GARAGe)

And now: Enjoy the proceedings! You will find plenty of material both for the interest of practicians and theoreticians, reflecting a strong progress of our knowledge about evolutionary computation.

Thomas Bäck, Program Chair

Option Pricing with Genetic Algorithms: The Case of European-Style Options

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Abstract

The cross-fertilization between artificial intelligence and computational finance has resulted in some of the most active research areas in *financial engineering*. One direction is the application of *machine learning techniques* to pricing financial products, which is certainly one of the most complex issues in finance. In the literature, when the interest rate, the mean rate of return and the volatility of the underlying asset follow general stochastic processes, the exact solution is usually not available. In this paper, we shall illustrate how *genetic algorithms* (**GAs**), as a numerical approach, can be potentially helpful in dealing with pricing. In particular, we test the performance of basic genetic algorithms in the determination of prices of *European call options*, whose exact solution is known from *Black-Scholes option pricing theory*. The solutions found by basic genetic algorithms are compared with the exact solution. The results show that **GAs** can be a powerful tool for option pricing.

1 Introduction

One of the most difficult issues in finance is the valuation of complex financial products, such as *financial derivatives*.[1] This valuation usually requires knowledge of the statistics of the *underlying security*, such as the *mean return* and *standard deviation of the return* (the *volatility*). Given that these parameters as well as the interest rate are constant, Black and Scholes (1973) established a formula (an exact solution) for determining the value of *European-style options*.[2] However, if these parameters are not constant and other styles of options, such as *American-style options*, are considered, then the exact solution may not be feasible. For the latter case, several numerical techniques based on massive computation have been developed.[3] Recenlty, techniques from *machine learning*, such as *artificial neural networks* (**ANNs**), have also been used to derive exact numerical solutions to option pricing. For example, Barucci, Cherubini and Landi (1995) and Chen and Lee (1997a) used **ANNs** as a *semi-nonparametric* technique to approximate *prices* of European-style options.[4]

To our best knowledge, this paper is one of the first few applications of the *evolutionary computation paradigm* to option pricing.[5] The particular style of evolutionary computation considered in this paper is *genetic algorithms*. As we learned from the recent discussion on the *no-free-lunch* (**NFL**) *theorem* (Wolpert and Macready, 1995), when comparing two machine learning tecniques, instead of asking "Which technique is better?", the right question should be "*When is each technique at its best?*". Motivated by the **NFL** theorem, this paper makes an *initial attempt* to test whether we can benefit more by using genetic algorithms instead of artificial neural networks to solve

[1] A financial derivative is a financial instrument that is based on another more elementary financial instrument. The value of this financial derivative obviously depends on the price of the instrument on which the derivative is based.

[2] The solution is obtained from the equation known as the *Black-Scholes partial differential equation*.

[3] For a review of these techniques, see Freeman and Di Giorgio (1996).

[4] For those who are not familiar with *option pricing*, the *price* referred to in this field is the price defined by a *no-arbitrage partial differential equation*. Whether this price reflects the true value of the asset or whether the assumptions used to derive this partial differential equation hold in the real world is a philosophical issue known as the *efficient market hypothesis* in finance. We shall not dwell on this topic here. For recent reflections on this hypothesis, the interested reader is referred to Chen and Yeh (1996), Chen and Tan (1996), and Dowe and Korb (1996).

[5] The only paper we know in this area is Trigueros (1997). The technique employed by Trigueros (1997) is genetic programming.

the option pricing issue. As there is no standard way to apply ANNs to option pricing, the way to apply genetic algorithms to option pricing is also varied.[6] Thus, by "initial attempt", we mean that this work is just a *straightforward application* of GAs to option pricing. At the end of this paper, some thoughts on enriching this initial attempt are provided. The rest of the papaer is organized as follows. Section 2 briefly reviews the Black-Scholes option pricing model. Section 3 shows the relevance of genetic algorithms to option pricing by means of the *function approximation theorem*. Section 4 discusses the parameters set in this study and describes the simulations based on GAs. The results are compared with the exact solution from the Black-Scholes partial differential equation. Limitations of this application of GAs and its possible extensions, along with concluding remarks, are given in Section 5.

2 Option Pricing Theory: A Brief Review

2.1 Terminology

The two most common types of option contracts are *puts* and *calls*. A *call* is an option to buy, and a *put* is an option to sell. Ownership of a *call option* (*put option*) gives the owner the right to buy (sell) a particular good at *a certain price*, with that right lasting until *a particular date*. The *certain price* is known as the *exercise price*, *strike price* or the *selling price*. The *particular date* is known as the *maturity* or the *expiration date*. The option has no validity after its expiration date. The options that can be exercised at any time *before* or *on* the expiration date are called *American options*. The options that can only be exercised *at their maturity* are called *European options*.

Consider a call option with a striking price of E on a stock price of S. If the stock price is above the striking price ($S > E$), the call option is said to be *in-the-money*, but if the stock price is below the striking price ($S < E$), the option is said to be *out-of-the-money*. If the striking price is closest to the current value of the underlying stock ($S \approx E$), the option is *at-the-money*. When options are first written, the striking price usually is set at-the-money. Prior to expiration, an in-the-money option will normally be worth more than $S - E$. This difference ($S - E$), known as the *intrinsic value* of the option, is sometimes called the *when-exercised value*. For a call, the intrinsic value is $MAX(S - E, 0)$. For a put, the intrinsic value is the $MAX(E - S, 0)$. The intrinsic value of the option does not measure its market value. Typically, an option sells for more than its intrinsic value.

[6] For variants of using ANNs to option pricing, see Hutchinson, Lo and Poggio (1994), Lajbcygier, Boek, Flitman and Palaniswami (1996) and Liu (1996).

2.2 The Black-Scholes Option Pricing Model

Black and Scholes (1973) were the first to provide a closed-form solution for the valution of *European options*. The Black-Scholes option pricing model is based on the principle known as the *no-arbitrage condition* in economics. Given a few asumptions[7], Black and Scholes recognized that it is possible to form a *risk-free hedge portfolio* consisting of a long position in the *stock* and a short position in the *European call* written on that stock. If the stock price changes over time, the risk-free hedge can be maintained by continuously readjusting the proportions of stock and calls. In the following, we shall briefly review the derivation of the famous *Black-Schole partial differential equation*.

Let Q_S denote the number of shares of a stock, S the price per share, and Q_C the quantity of calls and C the price per call, then V_H, the value of the hedge portfolio, is simply,

$$V_H = SQ_S + CQ_C. \tag{1}$$

The change in the value of the hedge portfolio is the total derivative of Equation (1)

$$dV_H = Q_S dS + Q_C dC. \tag{2}$$

We assume that the stock price follows a *geometric Brownian motion process*, i.e., its rate of return can be described as

$$\frac{dS}{S} = \mu dt + \sigma dz \tag{3}$$

where μ is the instantaneous expected rate of return (*drift*), σ the instantaneous standard deviation of the rate of return (*volatility*), dt denotes a small increment of time, and dz is a Wiener process. Since the option's price is a function of the stock's price, its movement over time must be related to the stock's movement over time. To make this relation explicit, we shall, sometimes, use the notation $C(S, \tau)$ to denote the price of the call, where τ is time to maturity. Employing *Ito's Lemma*, $C(S, \tau)$ can be expressed as the following stochastic differential equation:

$$dC = \frac{\partial C}{\partial S} dS + \frac{\partial C}{\partial t} dt + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 dt \tag{4}$$

Replacing dC in Equation (2) with the RHS of Equation (4), we can rewrite Equation (2) as follows.

[7] The derivation of the original version of Black-Scholes model rests on six well-known assumptions. However, subsequent modifications of the basic model have shown that it is quite robust with respect to relaxations of many of these assumptions.

$$dV_H = Q_s dS[\frac{\partial C}{\partial S}dS + \frac{\partial C}{\partial t}dt] + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 dt \quad (5)$$

One of the most important insights revealed by Black-Scholes option pricing model is that it can be used as a *hedging vehicle*, i.e., it is possible to continuously adjust the hedge portfolio, V_H, so that it becomes *risk free*. More precisely, the relation in Equation (6) should sustain in the riskless situation.

$$dV_H = Q_s dS + Q_c dC = 0 \quad (6)$$

Without loss of generality, we can normalize Equation (6) by setting $Q_S = 1$ and derive Equation (7) from (6).

$$Q_C = -Q_S\frac{dS}{dC} = -\frac{dS}{dC} \quad (7)$$

The risk-free hedge portfolio will earn the *risk-free rate* in equilibrium if *capital markets are efficient* and the equilibrium relationship is expressed as Equation (8).

$$\frac{dV_H}{V_H} = r_f dt \quad (8)$$

Substituting Equations (8) and (7) into Equation (5), we obtain

$$\begin{aligned} dV_H &= r_f V_H dt \quad (9) \\ &= dS - \frac{\partial S}{\partial C}[\frac{\partial C}{\partial S}dS + \frac{\partial C}{\partial t}dt + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 dt] \end{aligned}$$

Equation (9) can be rearranged as follows.

$$\frac{\partial C}{\partial t} = r_f V_H \frac{-\partial C}{\partial S} - \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 \quad (10)$$

Substituting equation (1) for V_H ,we have

$$\begin{aligned} \frac{\partial C}{\partial t} &= r_f(SQ_S + CQ_C)(-\frac{\partial C}{\partial S}) - \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 \\ &= r_f C - r_f S\frac{\partial C}{\partial S} - \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 \quad (11) \end{aligned}$$

Equation (11) is the famous *Black-Scholes partial differential equation*. This partial differential equation can be solved with the following two boundary conditions:

$$C(S,0) = MAX(S-E,0) \quad (12)$$

and

$$C(S=0,\tau) = 0 \quad (13)$$

Equation (12) simply says, at expiration ($t = T, \tau = 0$), a call option must have a value that is equal to zero or to the difference between the stock price and the exercise price whichever is greater; otherwise there will be *arbitrage opportunities* awaiting exploitation. Equation (13) says that the call option price is worthless when $S = 0$ even if there is a long time to expiry.[8] Black-Scholes(1973) transforms the equation into the heat exchange equation from physics to find the following solution:

$$C = SN(d_1) - Ee^{-r_f\tau}N(d_2) \quad (14)$$

where $d_1 = \frac{ln(S/E)+r_f\tau}{\sigma\sqrt{\tau}} + \frac{1}{2}\sigma\sqrt{\tau}$, $d_2 = d_1 - \sigma\sqrt{\tau}$, and $N(d)$ is the cumulative distribution function for the standardized normal distribution. Equation (14) says that the price of an option on a stock without cash dividends depends on only five directly observable variables:

- the stock's price (S)
- the exercise price (E)
- the time to maturity (τ)
- the risk-free rate of interest (r_f)
- the volatility of the stock (σ)

Furthermore, it can be shown that

$$\frac{\partial C}{\partial S} > 0, \frac{\partial C}{\partial E} < 0, \frac{\partial C}{\partial \tau} > 0, \frac{\partial C}{\partial r} > 0, \frac{\partial C}{\partial \sigma} > 0. \quad (15)$$

3 Use GAs to Solve OPM

Assuming that an asset price S follows a stochastic process with $v(S)S^2$ denoting the *diffusion term* and rS the *risk-adjusting drift*, the partial differential equation characterizing all the contingent claims defined on the asset price is

$$L(C(S,\tau)) = \frac{1}{2}v(S)S^2\frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} + \frac{\partial C}{\partial \tau} \quad (16)$$

with the boundary conditions

$$C(S,0) = MAX(S-E,0), \quad C(0,t) = 0. \quad (17)$$

For example, to satisfy the no-arbitrage condition, the price of a European call, is given by

$$L(C(S,\tau)) - rC(S,\tau) = 0, \quad (18)$$

where $\tau = T - t$ is the time to expiration of the call.

The call price C can be approximated by C_a,[9]

$$C_a(S,\tau) = C_0(S,\tau) + \sum_{i=1}^{N}\psi_i(\tau)\phi_i(S), \quad (19)$$

[8] These conditions were originally proven rigorously by Merton (1973).

[9] Here, the *weight residuals method* extensively used in the numerical partial differential equation is applied. For reference, see Barucci et al. (1995).

where

$$C_0(S,\tau) = S - Ee^{-r\tau}$$

Or, alternatively, C_a can be written as follows.

$$C_a(S,\tau) = S - Ee^{-r\tau} + \sum_{i=1}^{N} \psi_i(\tau)\phi_i(S) \quad (20)$$

ψ_i and ϕ_i, i=1,...,N, are known analytic functions and are called *trial functions*, $C_0(S,\tau)$ is a function chosen properly to satisfy the boundary and initial conditions.

The trial functions chosen to approximate $C(S,\tau)$ in this paper are the *linear functions* for $\psi_i(\tau)$ and *sigmoid functions* with slope $3.5i$ for functions $\phi_i(S)$.[10] More precisely,

$$\psi_i(\tau) = a_i\tau \quad (21)$$

and

$$\phi_i(S) = \frac{1}{1+e^{3.5iS}} \quad (22)$$

In addition to the boundary and initial conditions, it is desirable to have C_a which can also satisfy the signs of the five partial derivatives in Equation (15). Among them, the most important one is $\frac{\partial C}{\partial S} > 0$. $\frac{\partial C}{\partial S}$ is called the *Black-Scholes delta* or *hedge ratio*. It tells us how the call price will change in response to the change in the stock price. In the Black-Scholes model, the hedge ratio is $N(d1)$, which is between 0 and 1.

Given the choices of Equations (21) and (22), $\frac{\partial C}{\partial S}$ implies,

$$\frac{\partial C_a}{\partial S} = 1 + \sum_{i=1}^{N} \psi_i(\tau)\frac{-3.5iSe^{3.5iS}}{1+e^{3.5iS}} > 0 \quad (23)$$

Based on the no-arbitrage condition, i.e.,

$$L(C(S,\tau)) - rC(S,\tau) = 0, \quad (24)$$

we shall define the error of our approximation R in terms of the linear operator L,

$$R = L(C_a(S,\tau)) - rC_a(S,\tau) \quad (25)$$

By the chosen trial functions, R can be derived analytically as follows.

$$\begin{aligned} R = & \sum_{i=1}^{N} \frac{\partial\psi_i(\tau)}{\partial\tau}\phi_i(S) + \sum_{i=1}^{N} \psi_i(\tau)[(1/2)v(S)S^2 \\ & \sum_{i=1}^{N} \frac{\partial^2\phi(S)}{\partial S^2} + rS\sum_{i=1}^{N}\frac{\partial\phi_i(S)}{\partial S}] \\ & - r\sum_{i=1}^{N}\psi_i(\tau)\phi_i(S) + 2rEe^{-r\tau} \\ = & \sum_{i=1}^{N} a_i\frac{1}{1+e^{3.5iS}} + (1/2)v(S)S^2 \\ & [\sum_{i=1}^{N} a_i\tau\frac{(3.5i)^2e^{3.5iS}(1+e^{3.5iS})^2}{(1+e^{3.5iS})^4} + \\ & \sum_{i=1}^{N} a_i\tau\frac{-2(3.5i)^2(1+e^{3.5iS})(e^{3.5i})^2}{(1+e^{3.5iS})^4}] + \\ & rS\sum_{i=1}^{N} a_i\tau\frac{-3.5ie^{3.5iS}}{(1+e^{3.5iS})^2} - r\sum_{i=1}^{N} a_i\tau\frac{1}{1+e^{3.5iS}} \\ & + 2rEe^{-r\tau} \end{aligned} \quad (26)$$

In the next section, genetic algorithms are applied to the search for $\{a_i\}_{i=1}^{N}$.

4 Simulation Description and Results

Table 1: The Setting of Controlling Parameters

ITEM	SETTING
Number of chromosome	25000
Population size	50
Length of string	15
Selection mechanism	roulette-wheel selection
Crossover style	two-point crossover
Crossover rate	0.6
Mutation rate	0.001
Interval of parameters	[0,5]
Fitness function (1)	$\sum_{S=0.6}^{5} R_S^2$
Fitness function (2)	$\sum_{S=0.6}^{5}(C_{a,S} - C_{BS,S})^2$

The interval of parameters $\{a_i\}_{i=1}^{N}$ is set to satisfy the condition $\frac{\partial C_a}{\partial S} > 0$ given that the interval of stock price is set to be [0.6, 5] (See Table 2)

The software used in this paper is *GENESIS 5.0*, written by John Grefenstette (Grefenstette, 1990) to promote the study of genetic algorithms for function optimization. The package is a set of rountines written in C. It requires users to have some programming ability to link it with a rountine of the fitness function

[10] The choice of the trial functions and the signoid functions can have effect on the performance of genetic algorithms. In earlier studies (Chen and Lee, 1997b,c), we also choose the second-order polynominal functions for $\psi_i(\tau)$ and trigometric functions for $\phi_i(S)$, but the result seems to be inferior to the one given in this paper.

provided by the user. The chosen parameters to run *Genesis 5.0* are shown in Table 1.

Notice that there are two fitness functions considered in this study. The first one is based on the residuals defined by the Black-Scholes partial differential equation under different stock prices, i.e., the one defined in Equation (25). We shall denote these residuals by R_S where S are stock prices. The second one is simply based on the residuals defined by the difference between the approximating price C_a and the true price (the Black-Scholes price) C_{BS}. The second one is also frequently used in the application of **ANNs** to option pricing. The difference between these two measurements is that to have the former one, we must know the true model, e.g., the Black-Scholes model, while the latter does not require this knowledge. Therefore, by taking both fitness functions into account, we can evaluate the pricing performance of GAs not only for the case when the true model is known but also for the case when it is unknown. Given these two defined residuals, our chosen fitness functions are simply the sum of squared errors (**SSE**), namely, $\sum_S R_S^2$ and $\sum_S (C_{a,S} - C_{BS,S})^2$ (Table 1).

Table 2: The Setting of Controlling Parameters

ITEM	SETTING
Stock price (S)	[0.6, 5]
Exercise pirce (E)	1
Time to maturity (τ)	1
Risk-free rate of interest (r_f)	0.1
Volatility of stock (σ^2)	0.1

Table 3: Coefficient Estimates of the Trial Functions: Fitness Function 1

PARA.	a_1	a_2	a_3	a_4	a_5
GAs (N=1)	2.30				
GAs (N=3)	2.30	2.30	2.3026		
GAs (N=5)	2.30	2.30	2.30	2.30	2.3158

Table 4: Coefficient Estimates of the Trial Functions: Fitness Function 2

PARA.	a_1	a_2	a_3	a_4	a_5
GAs (N=1)	2.8727				
GAs (N=3)	2.6167	2.30	2.30		
GAs (N=5)	2.6114	2.30	2.30	2.30	2.3026

The test problem is European call options with the five parameters described in Table 2. In this study, **GAs** are applied to apporximating the continuous call price function $C_{BS}(S)$ given that the other four parameters are fixed. The domain of S is set to be [0.6,5]. Respresentative points $\{S_i\}_{i=1}^{l}$ are sampled from this domain in the following manner: $S_1 = 0.6, S_{i+1} - S_i = 0.1, S_{l=45} = 5, \forall i$. Given E, τ, r_f, σ, the *no-arbitrage* prices can be obtained directly from Equation (14) for each S_i ($i = 1, ..., 45$) and they are depcited as the solid line in Figures 1 and 2. The performance of genetic algorithms is tested with the number of trial functions increasing from 1 to 3 and then to 5. The C_as computed from the five trial functions with the fitness functions 1 and 2 are depicted as a dash line in Figures 1 and 2 respectively. The coefficients estimated from different numbers of trial functions with the fitness functions 1 and 2 are also exhibited separately in Tables 3 and 4.

From Tables 3 and 4, we can see that the coefficient estimates under different fitness functions are pretty close. When the number of trial functions increases from one to three, the **SSE** derived from both fitness functions drops quite significantly (See Table 5). However, further increase in N may not help very much. In terms of the *absolute error*, the **GA** can approximate the function $C_{BS}(S)$ pretty well under both fitness functions (Figures 1 and 2). In fact, the call prices estimated by the GA are difficult to distinguish visually from the true Black-Scholes values.

Table 5: Fitness of the GA Option Pricing

N	$\sum_S R_S^2$	$\sum_S (C_{a,S} - C_{BS,S})^2$
GAs (N=1)	8.8×10^{-3}	5.533×10^{-4}
GAs (N=3)	3.0×10^{-3}	4.805×10^{-4}
GAs (N=5)	2.9×10^{-3}	4.873×10^{-4}

Nevertheless, since the call price goes down with the stock price, the same error experienced under different stock prices cannot be treated equally. Therefore, in addition to the absolute error, we consider a relative measure, the *absolute percentage error* (**APE**), is also taken into account. The **APE** is defined to be $\frac{|C_{a,S} - C_{BS,S}|}{|C_{BS,S}|}$. The **APEs** under the fitness functions 1 and 2 are depicted in Figures 3 and 4 respectively. It is clear from these two figures that the **APE** distribution is *asymmetric*. When the option price is *in-the-money* ($S > E$), the **APE** is almost nil, and when the option price is *out-of-the-money* ($S < E$), the **APE** is high up to 70% to 80% (Figure 4). This asymmetric distribution may be due to the chosen fitness functions, which are *quadratic*; hence, errors are penalized asymmetrically. The asymmetric phenomenon can also be found in **ANNs** and **GP** applications to option pricing (Hutchinson, et al, 1994, Figure 5-c; Barucci, et al, 1995, Figure 1; Trigueros, 1997, Table 10.).

In addition to $C(S)$, it is also interesting to see how well **GAs** perform in terms of the *hedge ratio (delta)* and the *daily rate of erosion* of the call price (θ), i.e., $\frac{\partial C}{\partial S}$ and $-\frac{\partial C}{\partial \tau}$. By Equation (7), the hedge ratio is the *risk-free* portfolio of stocks and call options. Overestimating or underestimating this ratio will put investors in a risky position. In Figures 5 and 6, the correct hedge ratio (the Black-Scholes hedge ratio) is plotted against S as the solid line. Associated with the solid line is the **GA** hedge ratio derived from the case of using five trial functions (the dash line). From these figures, we can see that, while the hedge ratio is estimated rather accurately when the option is *in-the-money*, it is underestimated when the option is *at-the-money* ($\frac{S}{E} \approx 1$) and overestimated when the option is *deep out-of-the-money*.

Given Equations (20)-(22), the time derivative of the call price $-\frac{\partial C}{\partial \tau}$ can be derived as follows.

$$\frac{\partial C_a}{\partial \tau} = rEe^{-r\tau} + \sum_{i=1}^{N} \frac{\partial \psi_i(\tau)}{\partial \tau} \frac{1}{1+e^{3.5iS}} \tag{27}$$

By an appropriate choice of the coeficients in $\psi_i(\tau)$, $\frac{\partial C_a}{\partial \tau}$ is positive, which says that the longer the time to maturity, the higher the call price. Since the *daily rate of erosion* θ is defined as the negative of $\frac{\partial C_a}{\partial \tau}$, θ is negative, which means that a decrease in time to maturity decreases the call price. For example, if the call option has a θ of -0.05, then each day the call price should erode by 0.05. θ is a very usefull tool for measure the potential varitions of options prices and is generally employed by options traders.

In Figures 7 and 8, the Black-Scholes θ is depicted as the solide line, and the **GA** θ is in the dotted line. Agian, we see that, when the option is *in-the-money*, the **GA** θ fits the Black-Scholes θ pretty well. However, when option price is *out-of-the-money*, θ is generally underestimated and it could be further underestimated when the option is *deep out-of-the-money*.

5 Concluding Remarks

While genetic algorithms seem to be a promising tool for option pricing, there are several issues deserving serious work to bring this approach to maturity.

First of all, it is not clear how to add *boundary conditions* to our approximating function. In this paper, the boundary conditions is arbitrarily imposed and it can only satisfy the case when $S > E$. When $S < E$ at the expiration date, this imposed condition fails to hold and the call price can be negative, which is certainly not acceptable. There should be better ways to handle this issue. For example, the *penalty methods* extensively applied in *constrained optimization* or the *niched Pareto method* in multiobjective genetic algorithms may be a good substitutes. [11]

Secondly, *representation*, in particular, the representation of trial functions needs to be improved. The Black-Scholes model has an analytical solution, but there are lots of derivatives whose analytical function is unknown. Therefore, to facilitate option pricing, a more flexible approach to handling trial functions should be considered. In this study, **GAs** are used with a *fixed* set of sigmoid functions. As a generalization, one may consider using *evolutionary artificial neural networks* (**EANNs**) to tackle more difficult issues, such as the *American-style options*, *convertible bonds*, and fixed-income securities.

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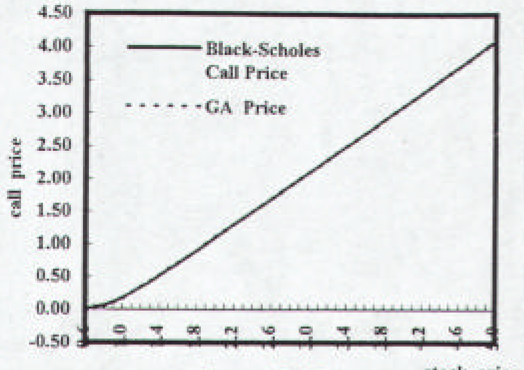

Figure 1:Call Price
[Fitness Function 1,N=5]

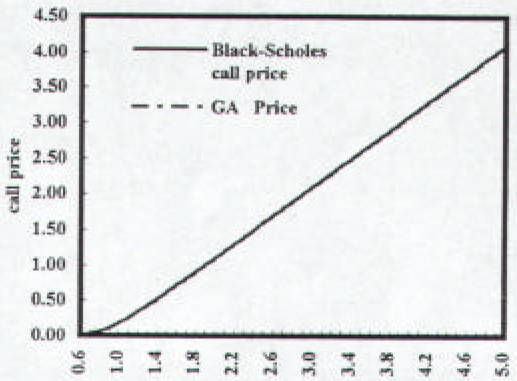

Figure 2:Call Price
[Fitness Function 2,N=5]

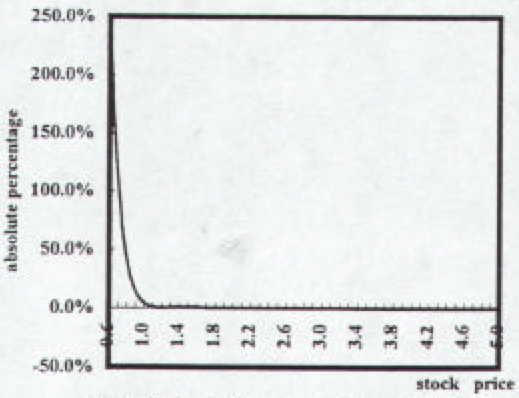

Figure 3:Absolute Percentage Error of Call Price
[Fitness Function 1,N=5]

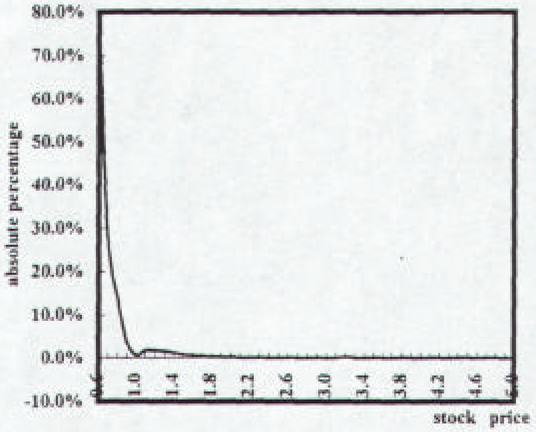

Figure 4:Absolute Percentage Error of Call Price
[Fitness Function 2,N=5]

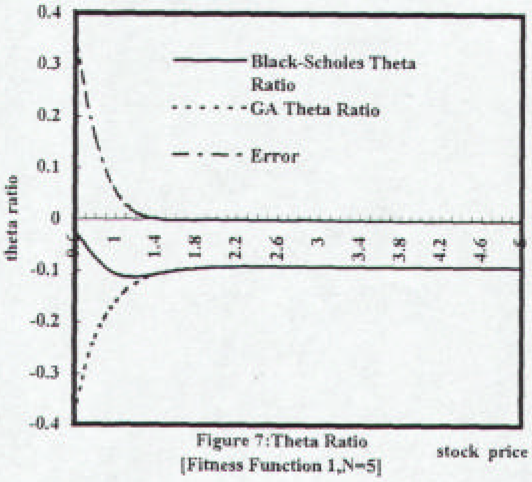

Figure 7:Theta Ratio
[Fitness Function 1,N=5]

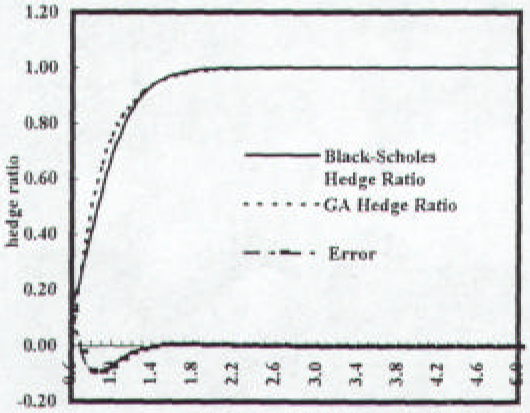

Figure 5:Hedge Ratio
[Fitness Function 1,N=5]

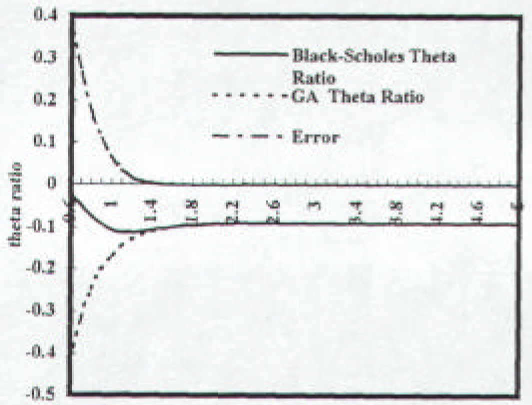

Figure 8:Theta Ratio
[Fitness Function 2,N=5]

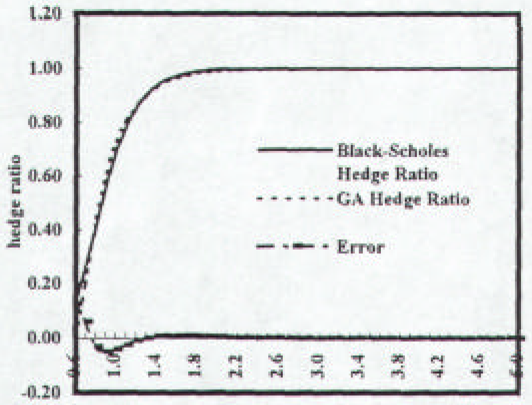

Figure 6:Hedge Ratio
[Fitness Function 2 ,N=5]

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